Passive and semi-active truck cabin suspension systems for driver comfort improvement

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Master's thesis

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Preface

The work presented in this thesis is a result of my graduation project, performed at the Dynamics & Control group of the faculty Mechanical Engineering of the Eindhoven University of Technology.

I would like to thank prof. dr. Henk Nijmeijer for his supervision and criticism during our monthly meetings. Also, I would like to thank dr. ir. Igo Besselink for his guidance and useful insights gained during and outside of these meetings. Especially, I would like to thank dr. ir. Willem-Jan Evers for his continuing coaching during the project and the inspiring discussions we had. Furthermore, I would like to thank dr. ir. Frans Veldpaus and ir. Arjan Teerhuis for participating in my graduation committee.

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While making long days on the road, truck drivers spend much time in their truck cabins. Vibrations induced by the road and cargo are transmitted to the cabin and consequently to the driver. These vibrations are can be uncomfortable and even unhealthy. To make the truck drivers ride as comfortable as possible truck cabins are equipped with cabin suspension systems. The main purpose of these systems is to minimize the vibrations transmitted to the driver.

Nowadays, most trucks are equipped with passive suspension systems. However, semi-active suspension system are gaining popularity, as can be seen in the passenger car industry. These semi-active suspension systems can alter their characteristics, using only a small amount of external energy. When these characteristics are controlled in a smart way the driver comfort can potentially be improved.

In this thesis research is done on the potential of different passive and semi-active cabin suspension systems to improve driver comfort in commercial vehicles. Therefore an extensive literature study is first performed on the topic of passive and semi-active automotive suspension systems. Since the amount of publications on cabin suspension systems is very limited, suspension concepts applied in primary vehicle suspensions are also regarded. From this literature study, several passive and semi-active suspension concepts are selected.

The performances in terms of driver comfort of the selected suspension concepts, applied to a truck cabin suspension, are compared. Therefore a 4 degree-of-freedom quarter-vehicle model is used to describe the dynamics in vertical direction of a truck with axle suspension, cabin suspension and engine suspension. To make a fair comparison, the various suspension concepts applied to the cabin suspension of this quarter-vehicle model are all numerically optimized to the same criterion. In this criterion the driver comfort is optimized, by minimizing the so-called ride index. Meanwhile, the suspension travel is kept within the absolute limits of the available working-space.

The suspension concepts selected from the literatures show a limited potential improvement of driver comfort. Therefore, in this thesis a new control strategy is designed for a semi-active cabin suspension system using a variable rate damper. This controller is based on the four degree-of-freedom quarter-vehicle suspension and uses a combination of Linear Quadratic (LQ) optimal control and Linear Parameter Varying (LPV) theory, in combination with a Kalman filter to estimate the required vehicle states. The semi-active suspension system using this controller is compared to the passive and semi-active suspension concepts selected from the literature. It is shown that the increase in comfort that can be achieved by the LQ/LPV controlled system is 12.4%, which is more than twice the 5% improvement achieved with the best systems selected from the literature. Finally it is shown that a road adaptive control strategy would improve all the investigated semi-active cab suspension systems. A road adaptive LQ/LPV controlled system even leads to a driver comfort improvement of 45%.
Samenvatting

Tijdens de lange dagen op de weg die vrachtwagenchauffeurs maken, brengen ze een groot deel van de tijd door in de cabine van hun trucks. Trillingen die veroorzaakt worden door het wegdek en de lading worden door geleid naar cabine en daardoor ook naar de chauffeur. Deze trilling kunnen oncomfortabel en zelfs ongezond zijn. Om de trip van een vrachtwagenchauffeur zo comfortabel mogelijk te maken zijn vrachtwagen cabines uitgerust met cabine suspensie systemen. Het belangrijkste doel van deze cabine-vering is het minimaliseren van de trilling die naar de chauffeur worden door geleid.

Tegenwoordig zijn de meeste trucks uitgerust met passieve cabine suspensie systemen. Semi-actieve suspensie systemen worden echter steeds populairder, zoals men kan zien in de passagiersvoertuigen industrie. Deze semi-actieve suspensie systemen kunnen met de toevoeging van slechts een kleine hoeveelheid energie hun karakteristiek veranderen. Als deze karakteristieken op een slimme manier geregeld worden kan het comfort van de chauffeur in potentie worden vergroot.

In dit proefschrift is onderzoek gedaan naar het potentieel van verschillende passieve en semi-actieve cabine suspensie systemen om het comfort van vrachtwagenchauffeurs te verbeteren. Hiervoor is eerst een uitgebreide literatuurstudie gedaan op het onderwerp van passieve en semi-actieve suspensie systemen voor motorvoertuigen. De hoeveelheid beschikbare literatuur over cabine-vering is uiterst beperkt, daarom zijn suspensie concepten toegepast op de as-vering van voertuigen ook beschouwd. Uit deze literatuurstudie zijn verschillende passieve en semi-actieve suspensie concepten geselecteerd voor verder onderzoek.

Van de geselecteerde suspensie concepten, toegepast op de cabine-vering van een vrachtwagen, zijn de prestaties wat betreft het chauffeurscomfort vergeleken. Hiervoor is een kwart-voertuig model met 4 graden van vrijheid geïntroduceerd. Dit model beschrijft de dynamica van een truck met as-vering, cabine-vering en motor-vering in de verticale richting. Om een eerlijke vergelijking te kunnen maken zijn de verschillende suspensie concepten, toegepast op dit kwart-voertuig model, numeriek geoptimaliseerd naar hetzelfde criterium. In dit criterium wordt het chauffeurscomfort geëxpimaliseerd door de zogenaamde ride index te minimaliseren. Ondertussen wordt de veerweg binnen de grenzen van de beperkte beschikbare ruimte voor het veersysteem gehouden.

De uit de literatuur geselecteerde suspensie concepten hebben een beperkt potentieel om het chauffeurscomfort te verbeteren. Daarom is er in dit proefschrift een nieuw regel-algoritme ontworpen voor een semi-actief suspensie systeem dat gebruikt maakt van een demper met een variabele karakteristiek. Deze regelaar is gebaseerd op het kwart-voertuig model met 4 vrijheidsgraden en gebruikt een combinatie van lineair kwadratisch (LQ) optimaal regelen en LPV (Linear Parameter Varying) theorie, in combinatie met een Kalman filter om de benodigde systeemtoestanden van de vrachtwagen te schatten. Het semi-actieve suspensie systeem dat gebruik maakt van deze regelaar is vergeleken met de passieve en semi-actieve systemen die uit de literatuur geselecteerd zijn. Er is aangetoond dat met het LQ/LPV geregeld suspensie systeem een verbetering in comfort gehaald kan worden van 12.4%. Dit is meer dan het dubbele van de 5% verbetering die gehaald kan worden met het beste uit de literatuur geselecteerde systeem. Tenslotte is er aangetoond dat een adaptieve regelstrategie die zich aan past aan verschillende wegedektypen alle onderzochte semi-actieve cabine veersystemen zou verbeteren. Een systeem met een adaptieve LQ/LPV regeling kan zelfs tot een een verbetering van 45% leiden.
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Chapter 1

Introduction

1.1 Background

Driver comfort, or ride quality, is an important aspect in truck design. Since drivers spend much time on the road, in their truck cabins, this environment should be as comfortable as possible. Driver comfort is, among things like noise, available space, etcetera, related to the level of whole-body vibration the driver is exposed to. Research in the past pointed out that truck drivers are common victims of lower back problems \[22\]. The origin of these health problems lies (at least partly) in the cabin vibrations resulting from the unevenness of the road profile.

One of the main reasons that vehicles are equipped with suspension systems is to minimize the vibrations transmitted to the driver, induced by the road and cargo. In a passenger car the suspension system consists of the front and rear axle suspension. Besides increasing driver comfort, these suspension systems should also ensure good performance in terms of vehicle handling and stability. The latter objectives are in conflict with the minimization of transmitted vibrations. Good road holding requires a hard suspension setting, while the suspension should be soft for optimal driver comfort.

In contrast to passenger cars, the suspension system in modern day trucks consists of a primary and a secondary suspension. The primary suspension is located between the axles and the chassis. The secondary suspension system includes the suspension of the cabin and possibly the driver seat suspension. The secondary suspension system can provide the driver with a comfortable ride, without the requirement of a soft axle suspension and consequent problems with vehicle handling, stability and static deflection \[17\].

To improve driver comfort, truck cabin suspensions have evolved over the past decades. Starting with the use of fixed cab mounts, the compliance in the cab-mount system has been increased to reduce the transmission from chassis vibrations to the cabin. At first this has been realized by the use of rubber bushings, after that steel springs were used and later came the introduction of air springs \[31\]. The air springs allow larger suspension travel at the cab mounting points, along with lower isolation frequencies. Furthermore the air spring suspension can adjust for changes in the cabin load, keeping the available suspension working space equal when the mass of the cabin changes.

A possible next step in the evolution of the cabin suspension is the use of (semi-)active suspension systems, instead of passive suspensions. Where passive suspension elements like springs and dampers can only store or dissipate energy, active suspensions can also add energy to the system. The active suspension systems incorporate actuators to apply forces to the suspended mass, computed by a system controller which is designed to produce desired ride characteristics \[3\]. However, these active suspension systems typically have a high energy consumption. When regarding energy consumption and performance potential, the group of semi-active suspension systems is positioned in between the passive and active suspensions. These semi-active systems use suspension components of which the characteristics can be changed using a relatively small amount of external energy \[45\], \[47\]. A semi-active suspension can therefore, like a passive suspension, only store and dissipate energy, but at a variable rate. In practice this means that, for example, a spring or a damper can change its charac-
characteristics almost instantaneously from a soft to a hard setting using only a small amount of external energy.

Fully active suspension systems have a large potential to increase the driver comfort [45]. However, a drawback of active suspension systems is the high energy consumption. In the context of the suspension system of a truck this means a higher fuel consumption. Therefore the semi-active suspension system may be a good compromise. Truck manufacturers are also interested in the potential of semi-active suspension systems, judging by the fact that e.g. DAF [54] and Scania [41] have built prototype trucks using these systems, albeit in the axle suspension and not the cabin suspension.

1.2 Problem statement and objectives

In this research, the potential of a semi-active cabin suspension system to improve driver comfort is investigated. To this end the following question is posed;

_How much can be gained in terms of driver comfort by using a semi-active instead of a passive suspension for truck cabins and how should it be controlled?_

To answer this question a semi-active cab suspension system and various control strategies for this system are compared to several passive suspension concepts. In this research the focus is on a semi-active cab suspension system using a variable rate damper. In order to make a fair comparison, these suspension concepts should all be applied in the same vehicle model, representing a truck with cabin suspension. Furthermore the various suspension concepts should be optimized to the same performance criteria and constraints.

An important part of a semi-active cab suspension system is the controller that actuates the semi-active suspension components. This controller prescribes the desired change of the characteristics of the semi-active suspension components, as a result of environmental disturbances on the vehicle. Examples of these disturbances are the vibrations caused by the unevenness of the road profile, manoeuvres like steering, braking and accelerating, load changes, aerodynamic forces, etc.

To get a good impression of the full potential of the semi-active suspension, it is essential that the system is controlled in such a way that the performance of the semi-active suspension system is optimal with respect to the chosen performance criteria. These criteria can regard driver comfort, energy consumption, suspension working space, etc. The design of the semi-active controller is therefore an important part of this research.

Regarding the problem stated above, the following research objectives are defined:

1. Get an overview of the research and development on cabin suspension systems, both passive and (semi-)active;
2. Select a representative criterion to serve as an objective measure for the performance of a cabin suspension;
3. Design a controller for the semi-active suspension system for optimal performance with respect to the selected criterion;
4. Compare the performance of several passive and semi-active suspension systems, and evaluate the performance potential of these systems.

1.3 Approach and main results

To get an overview of the research and development on cabin suspension done so far, a literature study is performed. This study concentrates on a number of different subjects. Modeling and optimization of the vehicle and the suspension components is reviewed as well as control strategies for the semi-active suspension system. However, publications of research on advanced secondary suspension systems are limited, and mainly discuss fully active systems [58], [64]. The amount of literature
available on the subject of primary (semi-)active suspension systems in truck and, especially, passenger cars is more extensive. These sources may also be valuable when designing a semi-active truck cabin suspension and are therefore also regarded. With the knowledge gathered from the literature study, choices are made regarding the performance criterion, the semi-active controller design and the passive suspension concepts. These passive suspension systems are used to position the performance of the semi-active system in the right perspective.

The selected passive suspension concepts all include a linear spring placed in parallel with various other suspension components. In this thesis a linear damper, a stroke dependent damper, a Frequency Selective Damping (FSD) damper and a suspension configuration including an inerter are analyzed. These concepts are compared to a cab suspension including a semi-active damper, using a quarter-vehicle model. This vehicle model describes the vehicle dynamics in the one dimensional vertical (or heave) direction.

Most of the regarded passive suspension concepts, as well as the semi-active suspension, are non-linear systems. Therefore the performance criterion is defined in the time-domain. The so-called ride index, a standardized ISO measure for driver comfort, is calculated for the quarter-truck model driving over a road surface for a certain amount of time. The time-domain approach also allows for a constraint on the maximum deflection of the cab suspension system. This is a constraint with more practical value than the minimization of the root mean square (RMS) value of the suspension deflection, which is often used in literature.

For the semi-active suspension system several control strategies are evaluated and compared using the selected optimization criterion. This has led to the proposal of the so-called LQ/LPV control algorithm. This algorithm is based on the $H_{\infty}$/LPV-algorithm presented in [72], but is using LQ-optimal control theory rather than optimal $H_{\infty}$ control. Furthermore the potential of road adaptive control is investigated.

1.4 Outline

This thesis is organized as follows. In Chapters 2 and 3 the results of the literature study are presented. Chapter 2 discusses the modeling and optimization of vehicles and passive and semi-active suspension components, while Chapter 3 discusses control strategies for semi-active suspension systems.

The selected passive suspension concepts are evaluated and compared in Chapter 4. The quarter-vehicle model, the modeling of the road surface and the selected optimization criterion are discussed first. Next the results of the optimization of the cab suspensions using the linear damper, the stroke dependent damper, the FSD damper and the inerter are regarded.

Chapter 5 starts with the introduction of the model used to represent the semi-active damper. The optimization of a semi-active suspension with this semi-active damper is discussed next for three different control strategies. Two of these strategies are selected from the literature, namely the so-called 2-state skyhook strategy and the Acceleration Driven Damping (ADD) strategy. The LQ/LPV strategy and a state-estimator for this strategy are presented in the final part of this chapter. Furthermore the possibility of adaptive control strategies is discussed. In Chapter 6 the conclusions and recommendations are given.
Chapter 2

Modeling and optimization of vehicle and suspension components, a literature review

2.1 Introduction

To be able to evaluate any passive or (semi-)active suspension system, models which accurately describe the dynamic behavior of the vehicle and suspension are often desirable. For a fair comparison of various suspension concepts, they should be applied to the same vehicle model. Furthermore it is important that the variables of the suspension component models are chosen such that the suspension systems perform optimal with respect to the same objectives and constraints.

When investigating a truck cabin suspension system to improve driver comfort, bounded by working-space constraints, the most important function of the vehicle model is to describe the cabin motion caused by to road irregularities and other disturbances. The most important directions of cabin motion in this case are heave, pitch and roll motion. Heave is the translational motion in vertical direction, pitch is the rotational motion around the lateral axis and roll is the rotational motion around the longitudinal axis. The complexity of the model depends on the number of directions of interest and the desired accuracy.

Besides the vehicle model, it is important to have representative suspension component models at disposal. A passive suspension system consists of passive elements like (linear or nonlinear) springs and dampers, bumpstops and possibly inerters, [82]. In [84], [87] and [93] nonlinear truck suspension components are investigated and optimized in the framework of the so-called CASCOV project. In the CASCOV project only axle suspensions are regarded. It may be worthwhile to evaluate the performance of some of these suspension concepts in a cabin suspension system.

Semi-active suspension components can be regarded as essentially passive, meaning they can only store or dissipate energy from the system. However, a relatively small amount of external energy can be used to change its characteristics [45], [47]. By doing so, the performance of the suspension system can be improved. These semi-active suspension components are typically variable rate springs and dampers.

Examples of variable rate stiffness components include pneumatic springs and other devices as can be found in [47] and references therein. It is stated that the concept of variable rate stiffness is a promising complement to semi-active suspension systems. Examples of variable rate damping components are also given in [47], among which are electrorheological (ER) dampers, magnetorheological (MR) dampers, variable orifice dampers and controllable friction braces.

After integrating the suspension component models in the vehicle model, the variables of the suspension system have to be optimized to the same set of objectives and constraints, in order to make a fair comparison. In the past the numerical optimization of a passive cabin suspension has
been performed by Besselink [11], as part of the numerical optimization of the total suspension system of a truck/semi-trailer, including axle and engine suspensions. In [11] all the springs and dampers in the suspension system are assumed to be linear. Numerous other publications on the subject of truck or car axle suspension optimization have appeared, e.g. [19], [6], [30], [37], [49] and [86]. These references use different criteria for the optimization of a suspension system. It is important that the criterion selected for the optimization of the cab suspension system can be applied to a non-linear passive or semi-active system. After choosing a suitable criterion, an optimization method has to be selected which is able to solve the optimization problem with the chosen criterion.

An overview is missing of literature addressing the modeling of truck cabin suspension systems and representative suspension elements for such a system. The same holds for the subject of optimization of truck cabin suspension systems. It must be mentioned that publications of research on cabin suspension system are limited. However, modeling and optimization techniques used in research on primary vehicle suspensions can also be valuable when regarding secondary suspension systems. Furthermore, an extensive amount of literature is available on suspensions concepts designed to improve driver comfort when applied to primary vehicle suspensions. It may be worthwhile to investigate the performance of some of these concepts, when applied to a cab suspension.

In this chapter the results of a literature study on vehicle models, suspension component models and suitable optimization methods are presented. Several truck and car models, with different levels of complexity, are discussed. The modeling of both passive and semi-active elements is addressed. The so-called FSD damper and the inerter discussed in this chapter are two passive suspension elements which have proven to be able to increase driver comfort, when used in a primary vehicle suspension. Regarding the semi-active elements, in this chapter the focus is on the components which have already proven to be feasible in the area of automotive suspensions. The modeling of MR and ER dampers and variable orifice dampers will be discussed. The Delphi MagneRide™ suspension system [21] uses MR dampers, while variable orifice dampers for automotive applications are produced, for example, by ZF Sachs [70]. The application of variable rate stiffness elements, mentioned before, is not further investigated. These elements often have a low bandwidth, are complex, expensive and unreliable. Furthermore it is difficult to accomplish stepless control of the stiffness independent from the damping of the suspension system [96]. Finally, several optimization criteria and optimization methods suitable for vehicle suspension systems are discussed.

This chapter is organized as follows. In Section 2.2 an overview of different relevant vehicle models that can be found in literature is given. Section 2.3 discusses the modeling of the so-called FSD damper and the inerter. In Section 2.4 different models for semi-active damper components are discussed and several optimization criteria and optimization methods found in literature are discussed in Section 2.5.

## 2.2 Vehicle models

In this section an overview is given of different vehicle models found in literature. Here a distinction is made between models with different levels of complexity. One dimensional (1D) models, which are frequently used to describe the vertical dynamics, are often referred to as quarter-car models and are discussed in Section 2.2.1. In Section 2.2.2 2D half-car models are discussed. These models can describe the dynamics of the vehicle in either heave and pitch direction or heave and roll direction. When the dynamics of the vehicle in heave, pitch and roll direction are of interest a full-car model is required. Section 2.2.3 discusses these full-vehicle models.

### 2.2.1 Quarter-car models

The quarter-car vehicle model only allows for 1D vertical or heave motion. The simplest form of the quarter-car model is the 1 degree-of-freedom (DOF) model. This model consists only of the vehicle or sprung mass, supported by a suspension system placed between the vehicle and ground as in Figure 2.1a. The equation of motion of this system is

\[
m_s \ddot{z}_s + d_s (\dot{z}_s - \dot{z}_r) + k_s (z_s - z_r) = F_c,
\]

(2.1)
where $m_s$ is the sprung mass, $k_s$ and $d_s$ are the suspension (constant) stiffness and damping coefficient respectively, $z_s$ and $z_r$ are the vertical displacement of the road and the sprung mass respectively and $F_c$ is the control force delivered by a (semi-)active actuator. It is common practice to assume that the vehicle is moving forward with a constant velocity $V$. In this case the vertical displacement of the road, which acts as a disturbance on the system, is proportional to $V$ and the spatial slope of the road unevenness [45]. A change in $V$ yields a change in the intensity of the disturbance.

However, the 1 DOF quarter-car model does not describe the so-called wheel-hop mode, which is the resonant mode of the wheel/tire-assembly. When a so-called unsprung mass element (representing a wheel and tire of the vehicle) is added to the 1 DOF model, as is depicted in Figure 2.1b, the result is a 2 DOF quarter car model. The majority of the research done on the optimal control of (semi-)active suspension systems makes use of the 2 DOF quarter-car model. Using this model the driver comfort, primary suspension working space and dynamic wheel load (relevant for vehicle handling) can be described. The equations of motion are

$$\begin{cases}
m_s \ddot{z}_s + d_s (\dot{z}_s - \dot{z}_u) + k_s(z_s - z_u) = F_c \\
m_u \ddot{z}_u - d_s (\dot{z}_u - \dot{z}_u) - k_s(z_s - z_u) + k_t(z_u - z_r) = -F_c,
\end{cases}$$

where, in addition to the symbols of (2.1), $m_u$ is the unsprung mass, $z_u$ is the vertical displacement of the unsprung mass and $k_t$ is the vertical stiffness of the tire. The damping of the rolling tire in vertical direction is often considered to be negligible.

The 2 DOF quarter-car model is able to describe the wheel-hop mode. This mode has an important contribution to the dynamic behavior of the vehicle model as can be seen in Figure 2.2. Here the absolute values of the frequency response functions from vertical road velocity to vertical sprung mass displacement are plotted. This shows that the 2 DOF model is a more complete representation of the vertical dynamics of a passenger car than the 1 DOF model. Also, by using the 2 DOF quarter-car model it becomes visible that a trade-off has to be made between driver comfort and road holding. Here the driver comfort depends on $\ddot{z}_s$, and the road holding on $\ddot{z}_u$. This trade-off should be made during the design of the (semi-)active suspensions controller by the selection of the corresponding performance criteria.

Furthermore it is worth mentioning that the 2 DOF quarter-car model with neglected vertical tire damping has an invariant point at the wheel-hop frequency $\omega_{wh} = \sqrt{k_t/m_u}$ in the transfer function from the vertical road displacement to the sprung mass acceleration. Similarly there is an invariant point at the rattle-space frequency $\omega_{rs} = \sqrt{k_t/(m_s + m_u)}$ in the transfer function from the vertical road displacement to the suspension deflection [27]. At these points the magnitude of the involved frequency response can not be affected by active controllers placed between the sprung and unsprung mass [57].

In [32] three quarter-car models, specifically designed for the analysis of the primary suspension of a truck, are proposed. According to the author the differences between front and rear axles and various suspension types, a minimum of three quarter-truck models is required. One for a typical front axle with leaf-spring suspension, one for a drive axle with leaf-spring suspension and one for a drive axle
with air-suspension. The secondary, cabin suspension is, however, not incorporated in these models.

In [22] and [59] a (semi-)active cabin suspension of a commercial vehicle is modeled, but no axle suspension taken into account. This means the cabin suspension is the only, and thus primary, suspension system in these models.

A suitable vehicle model, including a secondary suspension system, for the description of the vehicle dynamics in the 1 dimensional heave direction is not found in the literature. It might be interesting to analyse the dynamic behavior of an extended quarter-car model. A quarter-car model can be made containing at least three masses (front axle, chassis and cabin), or possibly four (including the engine mass). As this model can describe the primary and secondary truck suspension, it might be able to give a better representation of the vehicle dynamics in vertical direction, compared to the quarter-car models found in the literature so far.

### 2.2.2 Half-car models

To be able to investigate the vehicles pitch dynamics in combination with the heave dynamics, a 2D pitch-plane half-car model is necessary. In [45], it is shown that a half-car model can be decoupled into two quarter-car models under certain conditions. These conditions involve the geometry of the car and the ratio between the weight assigned to the desired reduction of heave and pitch accelerations respectively. For a 2D half-truck model this decoupling is not so self-evident due to the cabin which is positioned above the front axle and the possible presence of a trailer. It might be interesting to investigate whether such a decoupling is possible for a truck model and, if so, under which conditions. This could help to simplify the problem of the design of an optimal cab suspension in pitch and heave direction, by dividing it in 2 1D subproblems.

In [63] a half-truck model is described by 4 DOF and 6 DOF half-car models which do not include the cabin suspension. In [64] a half-truck model which does include the cabin suspension is used. This 6 DOF model lumps the mass and inertia of the trailer together with the mass and inertia of the chassis. A 13 DOF 2D half-truck model including the cabin suspension, engine suspension and separate description of the trailer and trailer-axle motion can be found in [10]. Figure 2.3 shows a schematic picture of this half-truck model created by Huisman. Either of the two models with cabin suspension mentioned above are potentially interesting for the analysis of a truck cabin suspension in heave and pitch direction.

Note that a 2D half-car models also exist in the roll plane, [71]. These models can be used to investigate vehicle heave and roll dynamics. Although, it is shown in [45] that the roll degree of freedom can be decoupled from pitch and heave, no 1D roll models are found in literature on vehicle suspension systems regarding the improvement of driver comfort. In general, this degree of freedom is mainly investigated in research addressing the roll-over stability of vehicles or in a combination with heave and pitch in comfort oriented research on full-vehicle models.
2.2.3 Full-vehicle models

The most complex form of vehicle models is the full-vehicle model. This 3D model can be used to assess the dynamic behavior of a vehicle suspension in heave, pitch and roll direction. In [45] an example of such a full-car model can be found. It is shown that a full-car model can be decoupled into two quarter-car models (under the conditions mentioned in Section 2.2.2) and a model describing the dynamics of the vehicle in roll direction. An example in which this decoupling, which remains a crude approximation of reality, is not used can be found in [98]. A full-vehicle model is used here to design optimal passive and (semi-)active suspensions.

A full-truck model of a three-axle platform truck is proposed in [92]. The cabin suspension consists of four spring/damper combinations positioned in vertical direction on the corners of the cabin. In [26] a modular simulation model for a tractor semi-trailer system is developed and validated. The more detailed cabin suspension in this model consists of four spring/damper combinations positioned in vertical direction on the corners of the cabin in addition to a roll bar and two dampers placed in lateral direction at the rear of the cabin. For controller design this model is too complex, leading to large simulation times. It can be useful, however, for controller evaluation. This model one of the few models (if not the only) that is extensively validated with measurements. The validation is done in the time domain. The model is capable of simulating complex manoeuvres like braking during cornering.

Finally, it is important to mention the difference between multi-body models and finite element method (FEM) models (and combinations thereof). The first class of vehicle models uses rigid bodies that represent the most important masses of the vehicle, connected by elements like springs, dampers and various joints. The latter class describes the vehicle using flexible bodies. A comparison between a vehicle model using only multi-body technique and a model using a combination of multi-body and FEM techniques can be found in [36].

2.3 Passive suspension component models

In the previous section several vehicle models are discussed. When a suitable vehicle model is selected, the next step is to integrate various passive or semi-active suspension elements in these models. In this thesis the performance of different passive suspension concepts is investigated. To this end these concepts need to be applied in the selected vehicle model, replacing the standard parallel spring/damper secondary suspension system. In this section the principle and modeling of two passive suspension elements are discussed. In literature is claimed that these elements, the FSD damper and the inerter, can increase driver comfort considerably when applied in the primary suspension of a passenger car.
Section 2.3.1 discusses the FSD damper and Section 2.3.2 the inerter.

### 2.3.1 FSD damper

A promising suspension concept found in literature, which may possibly further improve the performance of the cabin suspension system, is the so-called Frequency Selective Damping (FSD). This concept is developed, for passenger cars, by Koni [88]. In the FSD concept the frequency of the damper movement is used as a reference for the damper force. This allows for a different damping characteristic for high frequency input (corresponding to vehicle ride) and low frequency input (corresponding to vehicle handling). In practice, this means that the damper force is largest for low-frequency damper motion and decreases when increasing the frequency of the damper motion. When regarding a cabin suspension system, the idea is that the transmission of high frequency vibrations from the truck chassis to the cabin will be reduced due to the low damping force at these frequencies. Meanwhile the higher damper force for low frequency vibrations would cause the cabin to follow the low frequency motion of the chassis and keep the cabin suspension deflection within limits.

Following the research presented in [28], the damping force delivered by an FSD damper can be written as

\[
F_{\text{FSD}} = D_{\text{FSD}} \dot{x}_p \cdot [0.9(1 - e^{-\Gamma \tau}) + 0.1],
\]

Equation (2.3)

where \( \tau \) is the time that the damper piston is traveling in one direction. For example, when the piston would make a pure sine motion with frequency \( f \), \( \tau \) could be written as \( \tau = \frac{1}{2f} \). \( D_{\text{FSD}} \) is a damping constant, when multiplied with the piston velocity \( \dot{x}_p \) it determines the damping force delivered for low-frequency piston motion, i.e. when \( \tau \to \infty \). For high-frequency piston motion, \( \tau \to 0 \), and the delivered damping force becomes \( F_{\text{FSD}} = 0.1D_{\text{FSD}} \dot{x}_p \). The variable \( \Gamma \) determines the shape of the exponential function and can therefore be used to adjust the frequency dependent behavior of the FSD damper. In [88], Koni claims remarkable improvement in both ride and handling from fitting the FSD damper to various passenger cars. It is therefore worthwhile to investigate the application of an FSD damper in a truck cabin suspension system.

### 2.3.2 Inerter

In [81] a mechanical element called the **inerter** was introduced, generating a force as a function of a relative acceleration. In a mechanical system (for example a quarter-car model as is mentioned in Section 2.2.1) the absolute acceleration \( \ddot{x} \) with respect to the inertial frame of a center of mass, represented by a mass element, requires a force \( F \) which is proportional this absolute acceleration: \( F = m \ddot{x} \). Here \( m \) is the mass in kilograms. This differs from other mechanical elements like the spring and the damper for which the relative displacement or, respectively, velocity of the two ends of the element requires a force: \( F = k(x_1 - x_2) \) in case of a spring with stiffness \( k \) or \( F = d(\dot{x}_1 - \dot{x}_2) \) for
a damper with damping constant $d$. The two ends of the spring or damper are in literature sometimes referred to as terminals. Therefore, springs and dampers are two-terminal elements, while the mass element only has one terminal; the position of the center of mass.

The (ideal) inerter is a two-terminal mechanical device with the property that the equal and opposite force $F$ applied at the terminals is proportional to the relative acceleration between the nodes, i.e.

$$
F = b(\ddot{x}_2 - \ddot{x}_1)
$$

where $x_1$, $x_2$ are the displacements of the two terminals and $b$ is a constant of proportionality called the \textit{inertance} which has the unit of kilograms. The stored energy in the inerter is equal to $\frac{1}{2}b(\ddot{x}_2 - \ddot{x}_1)^2$. A variety of different physical realizations of an inerter are possible (see [80]). A simple approach is to take a plunger sliding in a cylinder which drives a flywheel through a rack, pinion and gears (see Figure 2.4).

In [82] different configurations, or networks, for a suspension strut containing springs, dampers and inerters are analyzed. The parameters stiffness $k$, damping $c$ and inertance $b$ are optimized for the various configurations with respect to different cost functions. When optimizing for vehicle comfort, by minimizing the RMS acceleration of the sprung mass, the configuration shown in Figure 2.5 is shown to be preferable for a primary vehicle suspension. In this figure the inerter is represented by the rectangular block. This configuration consists of a spring in parallel with a damper and inerter which are placed in series. To prevent drift of the damper and/or inerter to the limit of travel in the course of operation, a pair of springs is added. These springs, with stiffness $k_1$, are called centering springs. The centering springs may be preloaded against each other. In Figure 2.5 $x_b$, $x_c$ and $x_t$ are the positions of the bottom of the suspension, the center between the damper and inerter, and the top of the suspension respectively. The force delivered by this suspension as a result of the motion at $x_b$ and $x_t$ is given by

$$
F_t = k(x_b - x_t) + \left(\frac{1}{k_1(x_b - x_c) + b(\ddot{x}_b - \ddot{x}_c)} + \frac{1}{k_1(x_c - x_t) + d(\ddot{x}_c - \ddot{x}_t)}\right)^{-1}.
$$

Since

$$
k_1(x_c - x_t) + d(\ddot{x}_c - \ddot{x}_t) = k_1(x_b - x_c) + b(\ddot{x}_b - \ddot{x}_c),
$$

$x_c$ is given by

$$
b\ddot{x}_c + d\dddot{x}_c + 2k_1x_c = k_1(x_t + x_b) + d\dddot{x}_t + b\dddot{x}_b.
$$

Therefore the signals $x_b$, $\dddot{x}_b$, $x_t$ and $\dddot{x}_t$ are required when calculating the suspension force $F_t$ using (2.5) and (2.7).

The optimized configuration of Figure 2.5 can cause a 10-17% reduction of the RMS acceleration of the sprung mass when applied in a quarter-car suspension model. When optimizing a full-car suspension model the configuration can lead to an improvement in driver comfort up to 9.26%. It is therefore worthwhile to investigate the application of inerters in a truck cabin suspension.

### 2.4 Semi-active damper models

As is the case for the passive suspension elements, models describing the semi-active elements are necessary to integrate these elements in the selected vehicle model. As mentioned in the introduction, this research focusses on variable rate dampers as semi-active suspension elements. It is necessary to select models which describe the characteristics of an ER/MR and a variable orifice damper sufficiently accurate. Meanwhile, also the complexity of the models has to be taken into account, since the simulation time must stay within reasonable limits. Models with different levels of complexity, accuracy and their own limitations are addressed in this section. In Section 2.4.1 some basic methods to model semi-active suspension components are discussed. These models can be used in a simplified analysis of different control strategies for semi-active suspension systems. They are, however, unable to capture the exact (nonlinear) behavior of above mentioned variable rate damping components. It is likely that this behavior influences the performance of a semi-active suspension system subject to a certain
control strategy. A semi-active damper can, for example, show hysteretic effects in its force-velocity characteristics. This means it might add energy to the total system at some instances. Furthermore the basic models cannot describe different damping characteristics for bump and rebound. More detailed models are investigated to be able to take the nonlinearities of the different variable rate dampers into account. These models of variable orifice dampers and ER/MR dampers are discussed in Sections 2.4.2 and 2.4.3 respectively.

2.4.1 Simple semi-active damper models

A variable rate damper is characterized by the fact that it can only dissipate energy in a suspension system. It cannot add energy to the system. Therefore an idealized variable rate damper can be modeled as a suspension component which is able to track any reference force signal infinitely fast, regarding only the so-called passivity constraint

\[ F_d(t) = \begin{cases} F_{ref}(t) & \text{if } \dot{x}(\dot{x} - \dot{x}_0) \geq 0 \\ 0 & \text{if } \dot{x}(\dot{x} - \dot{x}_0) < 0 \end{cases} \]

Here \( F_d(t) \) is the damper force supplied by the variable rate damper as a function of time. \( F_{ref}(t) \) is the desired reference force and \( x, x_0 \) are, in case of a semi-active cabin suspension system, the vertical displacement of the cabin and the truck's chassis respectively. This implementation assumes that the variable rate damper can supply any energy dissipating force at any relative velocity \( \dot{x} - \dot{x}_0 \), infinitely fast. This will not be the case in practice. In ER/MR dampers the characteristics of the damping fluid has to be altered using an electric or magnetic field. In a variable orifice damper solenoid valves have to be opened or closed to change the damping characteristics. These mechanisms all have a certain response time. As a result the changes in the damper characteristics can be made with a limited bandwidth.

A more realistic model is given in [74]. Herein the damper force at a certain relative velocity is limited to a minimal and maximal damping coefficient. The bandwidth of the damper is also taken into account. In this case the dampers dynamics are not infinitely fast, but are described by a first order filter. This results in the following description for the damper force:

\[ \begin{align*} F_d(t) &= c_d(t)(\dot{x} - \dot{x}_0) \\ \dot{c}_d(t) &= -\beta c_d(t) + \beta c_{ref}(t), \quad 0 \leq c_{min} \leq c_d(t) \leq c_{max} \forall t \end{align*} \]

Here \( c_d(t) \) is the actual damping coefficient as a function of time and \( c_{ref}(t) \) is the desired damping coefficient as calculated by some control strategy. The dampers bandwidth is controlled by \( \beta \) and is set at 10 Hz. Furthermore \( c_{min} \) and \( c_{max} \) are the lower and upper bound of the damping coefficient respectively. This relatively simple model can be used to investigate the influence of a dampers bandwidth and (linear) bounds on the damping force, on the performance of a certain control strategy. However, it does not account for non-linearities in the force-velocity characteristics of the damper such as hysteresis, saturation and difference between bump and rebound characteristics. The model also does not give any information on the power consumption of the variable rate damper. More detailed models are required to take these effects into account.

2.4.2 Variable orifice damper models

Since the 1980s many research has been done on the use of controllable dampers in semi-active suspension systems. One of the first variable rate dampers used in this area was the variable orifice damper. The simplest implementation of this type of damper is the two-state variable orifice damper. This is basically a passive damper to which a bypass valve is added in parallel to the conventional damper valves, as depicted in Figure 2.6. The bypass valve is operated through a solenoid. When it is opened the oil flow inside the damper is diverted past the damper valves and a low damping characteristic is achieved. Alternatively, if the bypass valve is closed the oil flow goes through the damper valves and a high damping characteristic is achieved.
In a continuously variable damper (CVD) the exact position of the solenoid bypass valve is controlled which means that any intermediate damping characteristic between the highest and lowest damping characteristic can be achieved, of which an example is shown in Figure 2.7. The time it takes for the damper to change its damping characteristic is an important design parameter and has a major influence on the performance improvements that can be achieved. This response time is determined to a large extent by the transient response of the bypass valve [20].

The damper force in a variable orifice damper is dependent on the size of the valve opening, which is related to the input current, and the relative velocity of the two ends of the damper. The relation between the relative velocity, the input current and the damper force can be found and modeled in two ways.

The first way is to experimentally determine the damper force at different constant values for the input current and relative velocity. In [39] and [69], for example, these results are used to create a lookup table to model the damper force characteristics. This can also be done by using one or more
polynomial functions, as is done in [9] and [42]. In these relatively simple models, the transient dynamics of the bypass valves are often modeled by applying a second order filter on the damper force calculated by the lookup tables or polynomials:

\[ F_d = \frac{\omega_0^2}{s^2 + 2\zeta\omega_0 s + \omega_0^2}. \]  

(2.10)

Here \( F_d \) is the actual damper force and \( F_s \) is the steady-state damper force, calculated through the lookup table or the polynomial description of the damper force characteristic. Furthermore \( \omega_0 \) is the natural frequency of the filter and \( \zeta \) is its damping ratio. The values of the parameters of this filter vary amongst different references.

In [9] it is recognized that different transient responses exist for turning the damper from on to off setting and vice versa. Nevertheless a second order filter is used to describe the dynamics, with \( \omega_0 = 200\pi \) and \( \zeta = 0.3 \). In [69] the values \( \omega_0 = 100\pi \) and \( \zeta = 0.6 \) are used. For the time it takes for the force to reach steady-state position after a step input various values are found, e.g. 12 ms [61] and 15 ms for off-on switching and 6 ms [53], [62] and 50 ms [39] for on-off switching.

The second way is an analytic approach. Here a mathematical expression for the entire hydraulic system including the cylinder and valve dynamics is investigated. However, the mechanism of a variable orifice damper is very complicated. The damping force characteristics during the expansion and compression strokes are different because the volumes of the compression and rebound chambers are different and some valves allow the fluid to flow only in one direction. Also, the properties of the oil and gas mixture may vary with temperature and in time.

Furthermore it is difficult to measure the parameter values for the analytical models. A relatively simple example of the analytical approach can be found in [61]. A highly simplified model of the CVD as depicted in Figure 2.6 is used here. Constant values for the density of the damper fluid and bulk moduli are assumed and deformation in the damper is disregarded. The damper model contains only one valve (the bypass valve), for which no dynamics are modeled. This leads to the following expression for the damper force \( F_d \):

\[
\begin{cases}
F_d = A_p \Delta P \\
\Delta \dot{P} = \frac{V_1 + V_2}{V_1 V_2} \beta A_p v_{rel} - \frac{V_1 + V_2}{V_1 V_2} \beta C_d A_v \text{sgn}(\Delta P) \sqrt{\frac{2|\Delta P|}{\rho}},
\end{cases}
\]  

(2.11)

where \( A_p \) is the piston area and \( A_v \) is the bypass valve orifice area, \( \Delta P \) is the pressure difference between the rebound chamber, with volume \( V_1 \) and the compression chamber with volume \( V_2 \). Furthermore \( C_d \) is defined as the valve loss coefficient, \( v_{rel} \) is the piston velocity, \( \rho \) is the density of the damper fluid and \( \beta \) is the bulk modulus of the hydraulic oil. In [40] and [53] a much more detailed description of a CVD is used. This leads to highly complex models which are difficult to parameterize and comprehend.

The amount of research done on semi-active truck cabin suspensions is very limited. It is therefore difficult to find a parameterized model of a CVD suitable for this kind of suspension system. Therefore, it seems preferable to use relatively simple models including lookup tables or polynomial descriptions of the damper force characteristics. This can be combined with a second order filter to describe the valve dynamics. These models can be adjusted or parameterized easier, which is favorable when measurement data of a suitable damper for a semi-active truck cabin suspension system is available.

### 2.4.3 ER and MR damper models

The application of electrorheological (ER) and magnetorheological (MR) dampers as semi-active actuators in vehicle suspensions became a popular concept in the 1990’s. These dampers make use of ER or MR damping fluids which consist of, respectively, micron-sized polarisable or magnetizable solid particles dissolved in a non-conducting liquid like mineral or silicone oil. Under the influence of an electric or magnetic field these particles can be arranged in a chain-like structure. This results
Table 2.1: Typical properties of some electro- and magnetorheological fluids [13].

<table>
<thead>
<tr>
<th>Property</th>
<th>ER fluid</th>
<th>MR fluid</th>
</tr>
</thead>
<tbody>
<tr>
<td>response time</td>
<td>milliseconds</td>
<td>milliseconds</td>
</tr>
<tr>
<td>plastic viscosity $\eta$ (at 25 °C)</td>
<td>0.2 to 0.3 Pa·s</td>
<td>0.2 to 0.3 Pa·s</td>
</tr>
<tr>
<td>operable temperature range</td>
<td>+10 to +90 °C (ionic, DC)</td>
<td>-40 to +150 °C</td>
</tr>
<tr>
<td></td>
<td>-25 to +125 °C (non-ionic, AC)</td>
<td></td>
</tr>
<tr>
<td>power supply (typical)</td>
<td>2 to 5 kV</td>
<td>2 to 25 V</td>
</tr>
<tr>
<td></td>
<td>1 to 10 mA</td>
<td>1 to 2 A</td>
</tr>
<tr>
<td></td>
<td>(2 to 50 watts)</td>
<td>(2 to 50 watts)</td>
</tr>
<tr>
<td>maximum yield stress $\tau_y$</td>
<td>2 to 5 kPa</td>
<td>50 to 100 kPa</td>
</tr>
<tr>
<td></td>
<td>(at 3 to 5 kV/mm)</td>
<td>(at 150 to 250 kA/m)</td>
</tr>
<tr>
<td>maximum field</td>
<td>ca. 4 kV/mm</td>
<td>ca. 250 kA/m</td>
</tr>
<tr>
<td>$\eta/\tau_y^2$</td>
<td>$10^{-7}$ to $10^{-8}$ s/Pa</td>
<td>$10^{-10}$ to $10^{-11}$ s/Pa</td>
</tr>
<tr>
<td>density</td>
<td>1 to 2 g/cm$^3$</td>
<td>3 to 4 g/cm$^3$</td>
</tr>
</tbody>
</table>

in large, reversible changes in the damping fluids flow properties such as the viscosity. The response time of the ER and MR fluid properties to changes in the electric or magnetic field respectively is in the order of milliseconds. This means that the damping capabilities of the ER and MR dampers can be adjusted very quickly using relatively low power. Differences of some properties of ER and MR fluids are given in Table 2.1, but both ER and MR dampers have the same non-linearities such as hysteresis and saturation which means their modeling techniques are similar [13], [83].

To date, different models describing the behavior of ER or MR dampers have been proposed. These models can be divided in two classes. The semi-physical, or parametric, models, and the black-box, or non-parametric, models. The parametric modeling technique characterizes the device as a collection of linear and/or nonlinear springs, dampers, and other physical elements. Examples of parametric models are the (extended) Bingham, three element, BingMax and (augmented) non-linear viscoelastic-plastic models described in [13]. Other semi-physical models are the hysteretic Bingham and the hysteretic biviscous models in [44] and some hydraulic models described in, for example, [44] and [56]. The nonparametric modeling technique employs analytical expressions to describe the characteristics of the modeled devices based on both measurement data analysis and device working principles. Examples of non-parametric models are Chebyshev polynomial fit or neural networks models [13], the black-box model proposed in [83], a polynomial model as in [16] and the so-called Nonlinear-ARX model proposed in [73].

According to [73] the state-of-the-art semi-physical ER or MR damper model is the enhanced Bouc-Wen model as introduced in [50]. A schematic picture of the model is given in Figure 2.8. The damper-force $F$ is given by

$$F = c_1 \dot{y} + k_1 (x - x_0), \quad (2.12)$$

where $\dot{y}$ can be written as

$$\dot{y} = \frac{1}{c_0 + c_1} \alpha z + c_0 x + k_0 (x - y) \quad (2.13)$$

and the evolutionary variable $z$ is governed by

$$\dot{z} = -\gamma |\dot{x} - \dot{y}| z |z|^{n-1} - \beta |\dot{x} - \dot{y}| |z|^n + A (\dot{x} - \dot{y}). \quad (2.14)$$

To include the effect of the voltage applied to the damper to vary its damping characteristics, the parameters $\alpha$, $c_0$ and $c_1$ vary linearly with the applied voltage $u$ according to

$$\alpha (u) = \alpha_a + \alpha_b u, \quad c_0 (u) = c_{0a} + c_{0b} u \quad \text{and} \quad c_1 (u) = c_{1a} + c_{1b} u, \quad (2.15)$$

where the dynamics involved in the damper fluid reaching rheological equilibrium are accounted for through the first order filter

$$\dot{u} = -\eta (u - v), \quad (2.16)$$
with bandwidth \( \eta \) and \( v \) being the voltage applied to the current driver. In the literature \( \eta = 190 \text{[s}^{-1}] \) is often used.

The parameters \( \beta \), \( \gamma \), and \( A \) in the Bouc-Wen model are used to control the linearity in the unloading and the smoothness of the transition from the pre-yield to the post-yield region. The stiffness of the dampers accumulator (see Figure 2.9) is represented by \( k_1 \). The accumulator is a pressurized volume of gas that is physically separated from the damper fluid by a floating piston or bladder. The accumulator serves two purposes. The first is to provide a volume for the damper fluid to occupy when the shaft is inserted into the damper cylinder. The second is to provide a pressure offset so that the low-pressure side of the dampers piston is not reduced enough to cause cavitations of the damper fluid. The viscous damping observed at larger velocities is represented by \( c_0 \). A linear damper, represented by \( c_1 \), is included in the model to produce the roll-off at low velocities, \( k_0 \) is used to control the stiffness at larger velocities, and \( x_0 \) is the initial displacement of spring \( k_1 \). This results in a model with a total of 14 parameters \( (c_{0a}, c_{0b}, k_0, c_{1a}, c_{1b}, k_1, x_0, \alpha_a, \alpha_b, \gamma, \beta, A, n \text{ and } \eta) \) which have to be determined for the damper. This parametrization is not an easy task, as is shown in [77], where it is found that a 3-stage identification procedure is necessary to obtain a satisfactory parametrization.

As is the case for the previously mentioned continuously variable dampers, parameter sets for (Bouc-Wen) models of ER or MR dampers which are suitable for the application in the cabin suspension of a truck are hard to find, due to the limited amount of research in this area. It is therefore worthwhile to consider a simpler model which is easier to parameterize, but is still able to describe the dampers behavior with an acceptable accuracy.

In [44] it is shown that the so-called hysteretic Bingham plastic model shows similar values of the relative RMS error as the Bouc-Wen model, for a certain force time history. The damper force of the hysteretic Bingham plastic model is given by

\[ F = C_{po} \dot{x} + Kx + F_y \tanh ((\dot{x} + \lambda_1 x)\lambda_2) , \]  

(2.17)

where \( C_{po} \) is the post-yield damping constant, \( F_y \) is the yield force, \( K \) is the stiffness due to gas pressure, \( x \) is the piston position and \( \lambda_1 \) and \( \lambda_2 \) are parameters that account for the width and slope of the pre-yield hysteresis loop, respectively.

With parameters which are only dependent on the input voltage, the hysteretic Bingham plastic model, unlike the Bouc-Wen model, cannot describe the change in the shape of the hysteresis loop as a function of the frequency of the piston velocity input signal. Experiments however show that the shape of the hysteresis loop is frequency dependent [16]. A relatively simple model which does have a frequency-dependent hysteresis loop is the black-box model proposed by Song et al. in [83]. This model describes the hysteresis loop by means of a first order filter. This means that the shape of the hysteresis loop is in fact dependent on the frequency of the piston velocity input signal. In this model
the damper force $F$ is described by

$$A(I) = \sum_{i=0}^{n} a_i I^i,$$

(2.18)

$$S_b(V) = \tanh[(b_1 I + b_0)V],$$

(2.19)

$$F_s = A(I)S_b(V),$$

(2.20)

$$\dot{x} = -(h_0 + h_1 I + h_2 I^2)x + h_3 F_s$$

(2.21a)

$$F_h = (h_0 + h_1 I + h_2 I^2)x + h_4 F_s$$

(2.21b)

and

$$F = F_h + F_{bias}.$$  

(2.22)

Here $A(I)$ is a polynomial with coefficients $a_0$ up to $a_n$ which describes the maximum damper force as a function of the input current $I$. In this case the polynomial is of the order 4. $S_b$ is a shape function with coefficients $b_0$ and $b_1$, dependent on input current $I$ and piston velocity $V$. The first order filter with coefficients $h_0$ up to $h_4$ is described by (2.21) and (2.22) gives the damper force $F$ with the addition of a constant offset-term $F_{bias}$.

Figure 2.10 shows the force-velocity diagram of a semi-active damper, represented by the enhanced Bouc-Wen model with parameters found in literature. In this figure this Bouc-Wen model is compared with the hysteretic Bingham plastic model and the Song black-box model, which are parameterized in order to fit the force-velocity characteristics of the Bouc-Wen model. This parametrization is performed for a sine-shaped piston velocity signal with a fixed frequency of 2 [Hz]. It can be seen that the hysteretic Bingham plastic model can well approximate the Bouc-Wen model when its parameters are optimized for a fixed frequency. The force-velocity characteristic of the black-box model by Song deviates more from the Bouc-Wen model characteristic. More on the parametrization of the hysteretic Bingham plastic model and the Song black-box model and a more elaborate comparison between the three models can be found in Appendix A. Note that the values on the axis of Figure 2.10 are small compared to the values in Figure 2.7. This is due to the fact that the only parameter set available for a Bouc-Wen damper model does not represent a damper with characteristics suitable for a truck cabin suspension.

The results displayed in this appendix show that it is difficult to describe the frequency dependent hysteresis in an ER or MR damper correctly using a relatively simple model. Furthermore Figure 2.10
shows that, at least at 2 [Hz], the hysteretic effect is only visible for relatively small piston velocities. It is therefore worthwhile to investigate the importance of the contribution of the hysteresis loop to the overall behavior of a semi-active truck cabin suspension system. In case the hysteresis loop is of low importance the hysteretic Bingham plastic model appears to be a good choice since it is relatively simple. In this case even the standard Bingham plastic model, without any description of the hysteresis loop, can be considered. When it is found that the hysteretic behavior of the ER or MR damper has an important contribution to the overall behavior of the suspension system, it can be considered to apply some scaling on a parameterized Bouc-Wen model. This could be done to try and produce a damper force characteristic which is representative for an ER or MR damper suitable for a truck cabin suspension system.

2.5 Suspension optimization

The variables of the different investigated suspension concepts should be optimized to the same objective function and constraints, in order to make a fair comparison. In Section 2.5.1, the optimization criteria for vehicle suspensions most commonly used in literature are discussed. When the optimization criteria are selected, a suitable optimization method has to be chosen to solve the optimization problem. In this case it is important that the optimization method can handle nonlinear systems. Different optimization method used for the optimization of vehicle suspensions are discussed in Section 2.5.2.

2.5.1 Optimization criteria

The goal of the secondary suspension system of a heavy road vehicle, i.e. the cabin suspension is to provide the driver with a comfortable ride without requiring soft primary (axle) suspensions. The main optimization criteria for a passive cabin suspension are driver comfort and suspension working space. For a (semi-)active system energy consumption can be added. Criteria like road holding and infrastructure damage can be disregarded, [17].

For the assessment of driver comfort the ISO 2631 standard [1] is widely used. Herein, the acceleration signal at the position of the driver is weighted per frequency band for human sensitivity. The root-mean-square (RMS) value of this weighted acceleration signal is called the ride index, which is a measure for the driver comfort. This RMS signal is calculated according to

\[
a_w = \left[ \frac{1}{T} \int_0^T a_w^2(t) \, dt \right]^{\frac{1}{2}},
\]

where \(a_w(t)\) is the instantaneous frequency-weighted acceleration and \(T\) is the duration of the measurement. When considering vibrations in more than one direction, the values of the different weighted RMS acceleration should be combined as follows:

\[
a_v = (k_x^2 a_{wx}^2 + k_y^2 a_{wy}^2 + k_z^2 a_{wz}^2)^{\frac{1}{2}}.
\]

Here \(a_{wx}, a_{wy}, a_{wz}\) are the weighted RMS accelerations with respect to the orthogonal axes \(x, y\) and \(z\) respectively and \(k_x, k_y, k_z\) are multiplying factors. \(a_v\) is called the vibration total value.

It is also possible to use the fourth power vibration dose value (VDV) as a measure for driver comfort, which is more sensitive to peaks in the acceleration spectrum. The VDV can be used in case the basic RMS evaluation method is unsuitable for describing the severity of the vibration in relation to its effects on human beings. Whether or not this is the case can be indicated by the so-called crest factor. The crest factor is defined as the modulus of the ratio of the maximum instantaneous peak value of the frequency-weighted acceleration signal to its RMS value. The basic RMS method is considered sufficient for vibrations with a crest factor below or equal to 9.
The VDV, which is used for example in [37] and [84], is given by [1]:

\[
VDV = \left[ \int_0^T |a_w(t)|^4 \, dt \right]^{\frac{1}{4}}. \tag{2.25}
\]

In any of the above mentioned cases the acceleration of the driver (and therefore the cabin) should be minimized in order to achieve optimal comfort. In practice minimization of the cabin acceleration means minimization of the suspension stiffness and damping, which leads to large suspension deflections [49].

To avoid packaging problems, the suspension deflection should also be a criterion in the optimization process. This can be done by choosing a maximum value for the suspension working space as a constraint in the optimization problem [11], [37]. Another possibility is to take the suspension deflection into account in the objective function to be minimized, using a weighting factor to indicate the relative importance of keeping respectively the cabin accelerations and the suspension deflection small [6], [30].

### 2.5.2 Optimization methods

To solve the optimization problem different strategies can be used. In [11], transfer functions from the road input to the motion of several points of interest on a tractor semi-trailer combination are derived from a finite element model. These transfer functions are then used to calculate, among other things, the ride index, the RMS value of the relative displacement and the RMS value of the dynamic loads at the points of interest. These quantities are used in an optimization problem where the ride index is the objective function to be minimized. A sequential quadratic programming (SQP) algorithm is used to solve the optimization problem. More information on the SQP method can be found in [66].

Another approach is to treat the optimization problem as a structure-constrained optimal control problem, as is done in [19], [6] and [98]. In these references, it is shown that structure-constrained optimal LQ control is a form of structured optimal $\mathcal{H}_2$ control. These two approaches are equivalent in the time and frequency domain respectively. This is shown in Appendix B.

Furthermore it is shown that, for linear systems, the RMS value of the output power of $y(t)$ is equal to $||F_l(P,F)||_2$. This is the 2-norm of the lower linear fractional transformation of the generalized plant (in this case the vehicle) transfer matrix $P$ and the transfer matrix of the controller (here the suspension) $F$. So, this approach also leads to the minimization of the RMS value of the criterion signal in response to white noise, which is an appropriate measure of the performance of an automotive suspension in case the input signal $w(t)$ represents the vertical velocity of the road disturbance. When using the optimal LQG/$\mathcal{H}_2$ approach to design a passive suspension system, the structure of $F(s)$ should be constrained since the control forces generated by this system cannot depend on all the states of the system, but only on the relative displacement and velocity between the sprung and unsprung mass at each wheel. In an unconstrained optimal LQG/$\mathcal{H}_2$ control problem, the feedback law

\[
u^* = F^* x
\]

that minimizes (B.7) is given by

\[
F^* = -R^{-1} B^T S.
\]

Here $u^*$ is the force vector delivered by the suspension system and $x^*$ is the state vector of the vehicle model. $S$ satisfies the Riccati equation

\[
SA + A^T S - SBR^{-1} B^T S + C^T QC = 0.
\]

The structure of $S$ is also constrained for the reason that the control forces generated by this system can only depend on the relative displacement and velocity between the sprung and unsprung mass at each wheel. This means that the Riccati equation can not be solved analytically anymore. The solution for the optimal control has to be found by using an iterative algorithm. In [6], [19] and [98] different approaches for describing and solving the constrained optimal control problem are discussed.
The approaches to design an optimal passive suspension system described so far all regard LTI systems. When using vehicle models or suspension components with nonlinear characteristics these approaches are unsuitable. Gonçalves [37] considers a flexible multibody model of a car with a flexible chassis and Georgiou et al. [30] consider a quarter-car model with nonlinear spring and damper characteristics including the possibility to describe the situation of the tire lifting off of the road. Both require the sprung mass acceleration in the objective function of the optimization problem. This acceleration signal (among other necessary signals) is calculated in the time domain by integrating the nonlinear equations of motion of the vehicle models. The input signal for these equations is a description of the road irregularities experienced by the vehicle for a certain period of time.

In [37] the VDV from (2.25) is used as an objective function and the maximum suspension travel is given by an inequality constraint in the optimization problem which is solved by using a modified Method of Feasible Directions (MFD). More information on MFD algorithms can be found in [66].

In [30] the objective function is the sum of three separate objective functions, namely the mean squared values of absolute acceleration of the sprung mass, suspension travel and dynamic load on the tires. No weighting matrices are used here, since a multi-objective optimization methodology is used. This methodology calculates an infinite set of solutions forming a characteristic front in the objective space. These solutions are optimal in the sense that they minimize all the objective functions simultaneously. Meaning that any change in these solutions cannot improve any objective without causing a degradation in at least one other objective function. In practice this means that a surface in the objective space is found which covers all solutions possible for any combination of weighting of the three separate objective functions. This can be used to find e.g. the minimum achievable absolute sprung mass acceleration given selected values for the maximum suspension travel and dynamic tire load, including the corresponding suspension parameters.

As mentioned in the introduction, van den Heuvel [93] and Tamis [87] investigated the optimization of nonlinear suspension components in the early 1990’s. For the optimization method used in both these references, approximating mathematical models are assumed for all responses that determine the objective function and the constraints. The parameters of these models are estimated from the results of a number of simulations done with the nonlinear vehicle model, using different values for the design variables. The objective function and constraint values as a function of the design variables are found by linear interpolation between the results of the different simulations. An SQP algorithm is then used to solve the optimization problem for the approximating models. However, it must be mentioned that this approach was mainly selected because of the lack of computational power at the time which was necessary to calculate the responses of the nonlinear vehicle model at every iteration of the SQP algorithm.

It can be concluded that the time domain optimization of the suspension system seems favorable. Unlike the frequency-domain methods, this method allows the use of nonlinear components in the vehicle model. The use of nonlinear models can cause non-convex optimization problems. However, this problem is not explicitly addressed in literature found on the optimization of nonlinear vehicle suspension systems. A general solution to minimize the chance of finding a local minimum in the objective function, instead of the global minimum, is the use of several different initial values for the design variables.

### 2.6 Summary

In this chapter the results of a literature study on vehicle models, suspension component models and suitable optimization methods for truck cabin suspension systems are presented. Vehicle models which describe the dynamics of the total system are required for the optimization and evaluation of a passive or semi-active truck cabin suspension. Two potential models suitable for these tasks in 2D pitch and heave direction are found in the literature. Also a validated full-vehicle model of a truck, necessary for the eventual validation of the designed passive or semi-active suspension, is available. Quarter-car models found in literature so far only contain one (primary) suspension system. A logical first step is to try and design an optimal cab suspension system looking at a single direction (heave, 1D) only. To be able to do this, the possibility of the use of an extended quarter-car model which includes
a secondary (cabin) suspension for suspension design in 1D (heave) direction should be investigated. This is done in Chapter 4.

Two passive suspension components have been discussed, namely the FSD damper and the inerter. These components have proven to be able to increase driver comfort when applied in primary vehicle suspensions. Their potential performance in a secondary vehicle suspension system should therefore be evaluated, as is done in Chapter 4.

Different possibilities of modeling the semi-active suspension components are investigated. The focus is thereby on variable orifice dampers and ER/MR dampers. Both can be approximated by basic (piece-wise) linear models which describe the actuator limitations: force limitation, passivity constraint and limited bandwidth.

For the description of the nonlinear behavior of the semi-active actuators however, more complex models are needed. The use of a static force-velocity-current mapping in combination with a second order filter seems an acceptable option for modeling a variable orifice damper.

The Bouc-Wen model is the state-of-the art model for the description of ER and MR dampers, which contain hysteretic effects. This model is however rather complex. An investigation of the influence of the hysteretic effects in a truck cabin suspension could be useful. Although the Bouc-Wen model is the state-of-the-art, possibly less complex models, e.g. the (hysteretic) Bingham plastic model, will also suffice. For the description of the damper dynamics when using the Bouc-Wen model a first order filter seems widely accepted.

For both the variable orifice dampers and the ER/MR dampers representative parameters sets for semi-active dampers in a truck cabin suspension are not available. It may be worthwhile to take a discerning look at what would be an appropriate choice for these parameter sets for the different models. Also the parameters of the first- and second-order filters describing the dampers dynamics vary among different papers. Therefore the influence of the bandwidth of the semi-active dampers on the suspensions system performance should be investigated. Such an investigation is performed in Appendix C.

When regarding the optimization of the variables of a cab suspension system, several performance criteria can be used. The main criteria here are found to be the driver comfort and suspension working space. In case of the semi-active suspension system, energy consumption can be added to these two. Several methods for solving the optimization problem are investigated. An optimization in the time domain domain is preferred while it allows for nonlinear suspension elements to be used. The semi-active suspension is also nonlinear which means the optimization and evaluation of this system should also be done in the time domain.
Chapter 3

Control strategies for semi-active suspension systems, a literature review

3.1 Introduction

In the previous chapter the available literature on modeling and optimization techniques is studied to be able to compare the performance of different passive and semi-active cabin suspension systems. An important part of Chapter 2 is dedicated to the modeling of semi-active suspension components, in particular variable rate dampers. The characteristics of such a variable rate damper play an essential role in the performance of a semi-active suspension system including this component. However, the achievable performance is also determined for the largest part by the applied semi-active control strategy.

The control strategy prescribes the desired change in the damping characteristic, as a result of the environmental disturbances on the vehicle. Examples of these disturbances are the vibrations caused by the unevenness of the road profile, manoeuvres like steering, braking and accelerating, load changes, aerodynamic forces, etc.

Over the past decades extensive research is done on the control of (semi-)active suspension systems. One of the first approaches on the optimal control of an active vehicle suspension system is described by Bender et al. in 1967, [8]. This approach of using LQ-optimal control on a single degree of freedom suspension system lead to the well known skyhook damper control strategy. In 1974, Karnopp discussed the approximation of this control strategy for a semi-active suspension system, using a damper with a variable force characteristic instead of a fully active actuator [51]. The application of optimal control strategies in semi-active suspension systems is called clipped optimal control. This clipped optimal control method is also used for other optimal active control strategies besides LQ-optimal control, e.g. $\mathcal{H}_\infty$ control [72].

In 1994 Tseng and Hedrick showed that a clipped optimal active controller is not always the optimal controller for a semi-active suspension system, [89]. An optimal controller can only be found when taking the constraints of the semi-active system into account during the phase of the controller design, not afterwards.

An overview in literature on the available control algorithms for semi-active truck cabin suspension systems is missing. It must be mentioned that only a small part of the research on control of (semi-)active suspensions, considers a secondary semi-active suspension system such as a truck cabin suspension. Nevertheless, inspiration for the control of such a suspension system can be found in research done on, for example, 1 DOF and 2 DOF quarter-car, half-car and full-car suspension systems on which an extensive amount of literature is available.

In this chapter the results are presented of a literature study on control algorithms for semi-
active vehicle suspensions. Because of the conclusions that can be drawn from the work of Tseng and Hedrick, the focus in this chapter lies on control strategies which are designed while taking into account the constraints of a semi-active suspension. Four of these strategies found in literature are discussed in detail. These strategies are semi-active LQ-optimal control, leading to the so-called Acceleration Driven Damper (ADD) control method, $\mathcal{H}_\infty/LPV$ control, Model Predictive Control (MPC) and Lyapunov control. Additionally various other methods to control semi-active suspension systems are briefly mentioned.

This chapter is organized as follows. In Section 3.2 the semi-active LQ-control strategy ADD is discussed including its predecessors like the (clipped) skyhook control method. In Section 3.3 the use of Linear Parameter Varying (LPV) theory to describe the semi-active suspensions nonlinear constraints is discussed. This method can be used to design a semi-active controller when using it in combination with a linear control strategy, in this case $\mathcal{H}_\infty$ control. The $\mathcal{H}_\infty$ control method for a quarter-car suspension system will also be discussed in this section. Section 3.4 discusses the use of MPC to design control strategies which perform better than clipped-optimal strategies. In Section 3.5 the use of Lyapunovs direct approach to stability analysis in the design of a semi-active controller is discussed. In Section 3.6 some other control strategies for semi-active suspension systems found in literature are mentioned.

3.2 Semi-active LQ-optimal control

The idea of using linear optimal control theory with quadratic performance criteria for the design of an active vehicle suspension is first described in 1967, [7], [8]. This so-called LQ-optimal control theory is applicable to linear time-dependent systems which can be written in state-space form as

$$\dot{x}(t) = A(t)x(t) + B(t)u(t),$$

where $x(t)$ is the state-vector as a function of time $t$, $u(t)$ is the input vector as a function of $t$ and $A(t)$ and $B(t)$ are the time-dependent system matrix and input matrix respectively. The LQ-optimal control theory defines a criteria to formulate the evaluation of the desired control performance and control effort in advance. The optimal control law is found by minimizing this criteria [54]. Such a quadratic criteria is typically given by

$$J = \int_{t_0}^{t_e} \left[ x^T(t)Q(t)x(t) + u^T(t)R(t)u(t) \right] dt + x^T(t_e)P_e x(t_e),$$

where $t_0$ is the begin value and $t_e$ is the end value of the time-interval on which the system has to be controlled. Furthermore $Q(t)$, $R(t)$ and $P_e$ are weighting matrices. In [7], [8] a 1 DOF suspension system with mass $m$ and suspension force $F_c$ as shown in Figure 3.1 is investigated. In this case the vertical acceleration of the sprung mass $\ddot{z}$ should be minimized for optimal driver comfort, but also the suspension stroke $z - z_0$ should be controlled due to space limitations. The equations of motion of the system in Figure 3.1 can be given by

$$\begin{cases} \dot{z}_1 = z_2 - w \\ \dot{z}_2 = u \end{cases},$$

where $z_1 = z - z_0$ is the stroke, $z_2 = \dot{z}$ is the vertical velocity of the suspended mass, $u = \ddot{z}$ is the vertical acceleration of the suspended mass, which is regarded as the control input, and $w = \dot{z}_0$ is the vertical ground input velocity due to road roughness effects. When the control interval is chosen very large, which means that $t_e \to \infty$ in (3.2), the optimal control law for this system is independent of end weighting matrix $P_e$. As is mentioned in Section 2.5.2 the optimal control law, in case of an LTI system has the form

$$u = Fx,$$

with

$$F = -R^{-1}R^T S.$$
where \( S = S^T > 0 \) satisfies the algebraic Riccati equation
\[
SA + A^T S - SBR^{-1}R^T S + Q = 0. \tag{3.6}
\]
Assuming that the vertical velocity of the road irregularities \( \dot{z}_0(t) \) is a Gaussian distributed white-noise signal with a mean value of zero, the cost function for the LQ optimal control problem can be written as
\[
J = E(\ddot{z}^2 + ru^2), \tag{3.7}
\]
with weighting-constant \( r \). Here \( E(\cdot) \) stands for the “expected value”, representing steady-state mean square values. According to [45], the optimal control acceleration \( u_{LQ} \) delivered by the active suspension system which minimizes this cost function is now given by
\[
u_{LQ} = -r^{-1/2}z_1 - \sqrt{2r^{-1/4}}z_2. \tag{3.8}
\]
This means that the control force generated by the suspensions actuator should be \( F_c = u_{LQ}m \). Part of this feedback law can be obtained by using a passive spring element. However, in the case of a vehicle suspension it is not possible to connect a damper from the isolated mass to an inertial reference so that the damper force is proportional to the absolute mass velocity \( z_2 = \dot{z} \). This configuration, depicted in Figure 3.2, is called the skyhook damper scheme.

Application of the skyhook control law in combination with a semi-active damper element is first described in [51] in 1974. A semi-active damper can only dissipate energy from the system, this is called the passivity constraint. Consequently the damper can only produce a force with the opposite sign of the relative velocity \( (\dot{z} - \dot{z}_0) \). So, the requested control force \( F_c = u_{LQ}m \) cannot always be delivered. At times the damper can only produce a force with a sign opposite to \( F_c \). In this case the closest approximation to the requested force is to produce no force at all. Therefore, in the semi-active case, the skyhook part in the control law of (3.8) can be replaced with:
\[
F_d = \begin{cases} \sqrt{2r^{-1/4}}z_2 & \text{if } \dot{z}(\dot{z} - \dot{z}_0) \geq 0 \\ 0 & \text{if } \dot{z}(\dot{z} - \dot{z}_0) < 0 \end{cases}. \tag{3.9}
\]

In 1975 Margolis et al., [60], introduced a variant on the semi-active skyhook control law, the so-called on-off semi-active strategy. Here the passivity constraint mentioned before is also taken into account, but instead of applying the LQ-optimal skyhook force \( u_{LQ}m \) when \( \dot{z}(\dot{z} - \dot{z}_0) \geq 0 \) the damper is simply turned on in this case. In case \( \dot{z}(\dot{z} - \dot{z}_0) < 0 \) the damper is switched off in which case it produces a low or near-zero force. The resulting control law for this semi-active damper is
\[
F_d = \begin{cases} c_{\max}(\dot{z} - \dot{z}_0) & \text{if } \dot{z}(\dot{z} - \dot{z}_0) \geq 0 \\ c_{\min}(\dot{z} - \dot{z}_0) & \text{if } \dot{z}(\dot{z} - \dot{z}_0) < 0 \end{cases}. \tag{3.10}
\]
where $c_{\text{max}}$ and $c_{\text{min}}$ are the dampers maximum and minimum damping constants respectively. Muijderman investigates a semi-active suspension system using preview in [62]. Herein the on-off strategies Direct Calculation Control (DCC) and Branch-and-Bound Control (BCC) are compared to two continuous strategies in the form of two Sequential Quadratic Programming Controllers with different objective functions. For the investigated road inputs, the obtainable performance of the on-off and the continuous strategies are similar.

Many variations on the two control laws mentioned above have been investigated to date. For example, the possibility of limiting the damper maximum force in the semi-active strategy of (3.9) results in so-called clipped semi-active control [45]. Furthermore application of the skyhook control law in 2 DOF quarter-car models as shown in Figure 3.3 is examined. Along with the application in 2 DOF quarter-car models came the investigation on the control of the unsprung mass acceleration in, for example, [9], [35], [90] and [91]. These investigations are mainly performed from a road-holding point of view. The resulting control laws, which are often a combination of the skyhook law and the so-called groundhook control law. The groundhook control law imitates a damper placed between the unsprung mass, representing the wheel, and the road surface. These combinations are therefore assumed to be of less importance when regarding a secondary suspension system like the truck cabin suspension. Other variations on the semi-active skyhook control law of (3.9) and the on-off strategy are, for example, the No Jerk semi-active skyhook strategy in [4], the modified skyhook strategy in [78], the road adaptive skyhook control in [42] and the linear skyhook strategy which can be found in [74]. In the No Jerk strategy the damping coefficient $c$ is given by

$$c = \begin{cases} K|\ddot{z}(\dot{z} - \dot{z}_0)| & \text{if } \dot{z}(\dot{z} - \dot{z}_0) \geq 0 \\ 0 & \text{if } \dot{z}(\dot{z} - \dot{z}_0) < 0 \end{cases},$$

(3.11)

where $K$ is a constant gain. This means that the damping coefficient is 0 at $\dot{z}(\dot{z} - \dot{z}_0) = 0$ and a smooth increase or decrease of the damper force is realized when the function $\dot{z}(\dot{z} - \dot{z}_0)$ crosses 0.

The damping coefficient in [78] is given by

$$c = \begin{cases} c_{\text{max}} & \text{if } \dddot{z}(\dot{z} - \dot{z}_0) \geq 0 \\ 0 & \text{if } \dddot{z}(\dot{z} - \dot{z}_0) < 0 \end{cases}.$$  

(3.12)

Here the jerk of the sprung mass $\dddot{z}$ is used where the absolute velocity of the sprung mass is normally used in the semi-active skyhook algorithm. According to [78], $\dot{z}$ can theoretically be obtained by integrating the acceleration of the sprung mass and passing the result through a high-pass filter to remove the direct current offset. However, in practice, this is difficult to do because the acceleration offset is not constant and the initial condition of the integral is hard to be determined. To avoid the use of a complex observer, the jerk of the sprung mass is used instead. This signal can be obtained by differentiating the filtered acceleration of the sprung mass, a signal that can be measured in practice. Note that for a sinusoidal input, the phase difference between the jerk $\dddot{z}$ and absolute velocity $\dot{z}$ is $\pi$ in this case, which is different from the skyhook techniques normally used.
For the road adaptive strategy, the desired force produced by the semi-active damper is calculated by
\[
\begin{align*}
   u &= -c_{sky}\ddot{z} - c_v(\dot{z} - \dot{z}_0), \\
   &\text{with } c_{sky} = c_{sky}(\dot{z}_r) \text{ and } c_v = c_v(\dot{z}_r),
\end{align*}
\]  
(3.13)
where \(c_{sky}(\dot{z}_r)\) is the skyhook gain and \(c_v(\dot{z}_r)\) is the variable damper gain, which are both dependent on the road estimate \(\dot{z}_r\). The actual force produced by the semi-active damper \(f_s\) is limited according to
\[
\begin{align*}
   f_s &= \begin{cases}
            f_s^* & \text{if } f_s^* \leq u \\
            u & \text{if } f_{ss}^- < u < f_s^*, \\
            f_{ss}^+ & \text{if } f_{ss}^+ \geq u
          \end{cases}
\end{align*}
\]  
(3.14)
where \(f_s^*\) and \(f_{ss}^\pm\) denote the maximum and minimum damping forces available at a given relative velocity, respectively. The road estimate \(\dot{z}_r\) is determined from the measured vertical sprung mass acceleration \(\ddot{z}\) by means of an estimated transfer function from \(z_r\) to \(\ddot{z}\).

Other examples of LQ-optimal control strategies can be found in, for example, [45], where performance indices including the jerk of the sprung mass and performance indexes for half-car and full-car systems can be found.

The aforementioned semi-active controllers are empirical approximations of optimal controllers for active suspension systems. A method which is optimal with respect to a semi-active 2 DOF quarter-car suspension is presented in 2005 by Savaresi et al., [73]. A finite-time optimal control problem over the time-interval \([t_0, t_f]\) is investigated, with the performance index
\[
J = \int_{t_0}^{t_f} \dddot{z}(t)^2 \, dt,
\]  
(3.15)
where \(\dddot{z}\) is the vertical acceleration of the sprung mass of a 2 DOF quarter-car model with a semi-active suspension system. The control input of the system is the reference value for the damping coefficient \(c_{in}(t)\) subject to the constraint \(c_{in} \in [0, c_{max}]\). In this system the vertical sprung mass acceleration \(\dddot{z}\) and the suspension stroke \(z - z_0\) can be measured every \(\Delta T = t_f - t_0\) seconds. Hence the controller found by solving this finite-time LQ-optimal problem selects the optimal reference value for the suspensions semi-active damper at each sampling period \([k\Delta T, k\Delta T + \Delta T]\), \(k \in \mathbb{Z}\). This results in the following control law:
\[
c_{in}(t_0, t_0 + \Delta T) = \begin{cases} 
   c_{max} & \text{if } \dddot{z}(t_0)(\ddot{z}(t_0) - \ddot{z}_0(t_0)) > 0 \\
   0 & \text{if } \dddot{z}(t_0)(\ddot{z}(t_0) - \ddot{z}_0(t_0)) \leq 0
\end{cases}
\]  
(3.16)
This method is called Acceleration Driven Damper (ADD) and is optimal with respect to (3.15) for the control of the semi-active suspension system. A drawback of this method is that the performance index is a function of the vertical sprung mass acceleration only and the suspension stroke is not taken into account. The stiffness of the suspension is selected here on beforehand. Together with the maximum damping coefficient \(c_{max}\) in (3.16) this determines the maximum suspension stroke of the system, subject to a certain road input.

It is worthwhile to consider the LQ-optimization of a semi-active suspension system of which the choice of the (passive) stiffness is still open. The optimal value for this stiffness would then depend on the chosen performance index. In this case the performance index should penalize both vertical sprung mass acceleration and suspension stroke. In [73], it can also be seen that the skyhook approach performs better than the ADD at low frequencies. Therefore a combination between the control law of (3.16) and the 2-state skyhook control law of (3.10) is further developed in [75]. The resulting control law is
\[
c_{in}(t) = \begin{cases} 
   c_{max} & \text{if } [(\dddot{z}^2 - \alpha^2\dddot{z}^2) \leq 0 \wedge \dddot{z}(\ddot{z} - \ddot{z}_0) > 0] \lor [(\dddot{z}^2 - \alpha^2\dddot{z}^2) \leq 0 \wedge \dddot{z}(\ddot{z} - \ddot{z}_0) > 0] \\
   c_{min} & \text{if } [(\dddot{z}^2 - \alpha^2\dddot{z}^2) \leq 0 \wedge \dddot{z}(\ddot{z} - \ddot{z}_0) \leq 0] \lor [(\dddot{z}^2 - \alpha^2\dddot{z}^2) > 0 \wedge \dddot{z}(\ddot{z} - \ddot{z}_0) \leq 0]
\end{cases}
\]  
(3.17)
where $\alpha$ is the cross-over frequency for the two strategies. When the term $(\ddot{z}^2 - \alpha^2 \dot{z}^2) \leq 0$ the skyhook strategy is selected and when $(\ddot{z}^2 - \alpha^2 \dot{z}^2) > 0$ the ADD strategy is selected. In [76] the strategy of (3.17) is simplified to

$$c_{in}(t) = \begin{cases} c_{max} & \text{if } (\ddot{z}^2 - \alpha^2 \dot{z}^2) \leq 0 \\ c_{min} & \text{if } (\ddot{z}^2 - \alpha^2 \dot{z}^2) > 0 \end{cases}.$$  

(3.18)

The performance is hereby slightly reduced but is still claimed to be better than the skyhook or ADD strategies. An advantage is that only one sensor is necessary since the unsprung mass velocity $\dot{z}_0$ is not present in the control algorithm anymore. This sensor is typically an accelerometer on the sprung mass, which also provides an estimation of the body vertical speed by low-pass filtering the acceleration with a low-frequency first-order linear filter.

In this section it is shown that using linear optimal control theory with quadratic performance criteria for the control of an active vehicle suspension system has lead to the well-known skyhook control algorithm. For a semi-active suspension system this strategy can be approximated by the semi-active skyhook algorithm. This algorithm is frequently used in literature, which makes it worthwhile to investigate the performance when applied in a truck cabin suspension. Several variations of this semi-active skyhook strategy exist. However, all of them remain empirical approximations of a control strategy which is designed for an active suspension system. The ADD algorithm is designed using optimal control with a quadratic performance criterion, based on the characteristics of a semi-active vehicle suspension. This makes it worthwhile to investigate the performance of the ADD algorithm, or variations thereof, when applied in a truck cabin suspension.

### 3.3 $\mathcal{H}_\infty$/LPV-control

In Chapter 2 the equivalence of LQ optimal and $\mathcal{H}_2$ control is already mentioned. The $\mathcal{H}_2$ control theory minimized the norm $|| \cdot ||_2$ of the transfer from the external disturbances to the controlled outputs of the system. The $\mathcal{H}_\infty$ control theory seeks to minimize the norm $|| \cdot ||_\infty$ of this same transfer, as is further explained in this section. This $\mathcal{H}_\infty$ theory can also be used to control (linear) semi-active skyhook control algorithm. For a semi-active suspension system this strategy can be approximated by the semi-active skyhook algorithm. This algorithm is frequently used in literature, which makes it worthwhile to investigate the performance when applied in a truck cabin suspension. Several variations of this semi-active skyhook strategy exist. However, all of them remain empirical approximations of a control strategy which is designed for an active suspension system. The ADD algorithm is designed using optimal control with a quadratic performance criterion, based on the characteristics of a semi-active vehicle suspension. This makes it worthwhile to investigate the performance of the ADD algorithm, or variations thereof, when applied in a truck cabin suspension.

The application of $\mathcal{H}_\infty$-control in (linear) active vehicle suspension systems is being investigated since the 1990’s [45]. To date a large amount of publications has appeared on this subject. Research has been done on $\mathcal{H}_\infty$-control of active suspension systems using quarter-car [24], half-car [15] pitch-plane or roll-plane [71] and full-car models [95]. This research focuses on the primary vehicle suspension, pursuing the reduction of both sprung and unsprung mass acceleration for optimal driver comfort and vehicle handling.

In 2003 Sammier et al. applied an $\mathcal{H}_\infty$ synthesized controller to a (non-linear) 2 degree-of-freedom semi-active quarter-car suspension model [72]. The $\mathcal{H}_\infty$-control problem is formulated using the general control configuration in Figure 3.4 (I) where $\mathcal{P}$ is a linear system given by

$$\begin{pmatrix} e \\ y \end{pmatrix} = \begin{pmatrix} P_{11}(s) & P_{12}(s) \\ P_{21}(s) & P_{22}(s) \end{pmatrix} \begin{pmatrix} w \\ u \end{pmatrix},$$

(3.19)

$w$ is the exogenous input vector, $u$ is the control input vector, $e$ is the controlled output vector and $y$ is the measurement vector. The control design consists of finding a controller $K(s)$ for a generalized plant $\mathcal{P}$. This $K(s)$ should, based on the information given by $y$, produce a control signal $u (= K(s) y)$ which ensures internal stability of the closed-loop system and counteracts the influence of $w$ on $e$. This means that the closed-loop transfer norm from the exogenous inputs $w$ to the controlled outputs $e$ should be minimized. Given $\gamma$, a prespecified maximum attenuation level, a $\mathcal{H}_\infty$ suboptimal control problem is to design a controller that internally stabilizes the closed-loop system and ensures:

$$||T_{ew}(s)||_\infty = \max \sigma(T_{ew}(j\omega)) \leq \gamma,$$

(3.20)

where

$$T_{ew}(s) = P_{12}(s)K(s)(I - P_{22}(s)K(s))^{-1}P_{21}(s) + P_{11}(s)$$

28
is the closed-loop transfer matrix from $w$ to $e$ and $\sigma(T_{ew}(j\omega))$ is the maximal singular value of $T_{ew}(j\omega)$. In general, some weights are considered on the controlled outputs (including the actuator force). They represent the performance specifications in the frequency-domain. $P$ thus includes the plant model $G$ and the considered input and output weights ($W_i, W_o$) as shown in Figure 3.4 (II). In this case, the $H_\infty$ control problem is referred to as a mixed sensitivity problem with $W_i$ and $W_o$ thus appearing in (3.20) as weights on the sensitivity functions. Methods to solve such a mixed sensitivity problem can be found in [79]. In [72] the plant $G(s)$ is the frequency-domain description of the active 2 DOF quarter-car suspension system

$$
\begin{align*}
    m_s \dddot{z}_s + k_s (z_s - z_u) + d_s (\dot{z}_s - \dot{z}_u) &= u \\
    m_u \ddot{z}_u - k_s (z_s - z_u) - c (\dot{z}_s - \dot{z}_u) + k_t (z_u - z_r) &= -u 
\end{align*}
$$

which is equivalent to the system in Figure 2.1b, corresponding with (2.2), where active force $F_c$ is the controlled input $u$. The plant $G(s)$ can be split into

$$
\begin{pmatrix}
    \dot{z} \\
    z_s - z_u \\
    \ddot{z}_s 
\end{pmatrix} =
\begin{pmatrix}
    G_{11} & G_{12} \\
    G_{21} & G_{22} \\
\end{pmatrix}
\begin{pmatrix}
    z_r \\
    u 
\end{pmatrix}
$$

where $z = [z_s, z_u, \ddot{z}_s]^T$. Using this together with the weighting matrices $W_z(s)$, containing the performance requirements, $W_u(s)$ containing the actuators bandwidth limitations and $W_n(s)$ normalizing the measurement noise $n$, the control scheme for the active 2 DOF quarter-car suspension system is shown in Figure 3.5. For the previously mentioned input weight holds $W_i = W_u(s)$ and for the output weight holds $W_o = \text{diag}(W_n, W_z)$. The controller $K(s)$ can now be synthesized by solving (3.20). The resulting controller, synthesized for a linear active suspension system $G$, can be implemented in a non-linear semi-active 2 DOF quarter-car suspension system, taking into account the boundaries on the actuator force and the passivity constraint. This results in a so-called clipped $H_\infty$-control. However, clipped approaches do not ensure internal stability and reduce the achievable performance [68].

In 2007, Poussot-Vassal et al. proposed a $H_\infty$-based semi-active suspension control strategy that a priori satisfies the limitations of a semi-active suspension (passivity constraint and force bounds) using linear parameter varying (LPV) theory [67]. LPV theory allows to model nonlinearities or to make the controllers performance requirements variable through the linear introduction of parameters. Since a decade, LPV modeling is increasingly being used and allows the extension of classical linear robust control methodology to a larger class of systems, keeping the use of linear tools [68].

For a semi-active 2 DOF quarter-car suspension system the generalized control configuration is shown in Figure 3.6. The generalized plant $P$ can be written as

$$
P(\rho) : \begin{pmatrix} \dot{x} \\ z \\ y \end{pmatrix} = \begin{pmatrix} A(\rho) & B_1 & B \\ C_1 & D & E \\ C & Q \end{pmatrix} \begin{pmatrix} x \\ w \end{pmatrix} \quad (3.23)$$
where the state variables vector $x = [x_{\text{quarter}}, x_{\text{weighting}}]^T$ includes the state variables vector of the weighting functions containing the performance specifications as well as the state variables vector of the quarter-car model described by

$$\begin{cases} m_s \ddot{z}_s + k(z_s - z_u) = F_{\text{SA}} \\ m_u \ddot{z}_u - k(z_s - z_u) + k_t(z_u - z_r) = -F_{\text{SA}} \end{cases},$$

(3.24)

where $F_{\text{SA}}$ is the control force which is delivered by the semi-active damper. This force can be written as

$$F_s = C \ddot{z}_{\text{def}} + u.$$  

(3.25)

Furthermore, $m_s$ and $m_u$ are the sprung and unsprung masses respectively, $z_{\text{def}} = (z_s - z_u)$, where $z_s$ and $z_u$ are the vertical displacement of the sprung and unsprung mass respectively and $k$ and $k_t$ are the stiffness of the suspension and the tire respectively. In Figure 3.6, $z = [z_1, z_2, z_3, z_4]^T$.
are the weighed performance outputs to minimize, \( y \) are the measured variables and \( w = \dot{z}_r \) is the road disturbance input.

In (3.25) \( C \) a damping constant, describing the passive part of the delivered damper force. The control input \( u \) is the force that is additionally delivered by the damper to achieve the varying performance. Different choices for \( C \) can be found in the literature. In [67] \( C \) describes the (linearized) force delivered by the semi-active damper in case no current is applied to the damper. In [5] \( C \) is the average damping rate of the damper.

The matrix \( A \) in (3.23) is a function of \( \rho \), which is the varying parameter used to schedule the controller. This parameter controls whether or not the semi-active actuator can apply a force to the vehicle system. Matrix \( A \) is a function of \( \rho \) because the weighting functions \( W_i(\rho), i = 1, 2, 3, 4 \) are a function of \( \rho \). So by varying \( \rho \) the performance specifications of the controller to be synthesized can be changed. Therefore \( \rho \) should be chosen in such a way that the weighting functions have low performance specifications and penalize the control signal \( u = K(\rho)y \), in case the reference force for the semi-active actuator exceeds the constraints of this actuator. In this case the control signal will be practically zero and the semi-active actuator will act as a passive damper with damping constant \( C \).

This idea is analogous to the idea of the clipped optimal control mentioned in Section 3.2, where the semi-active damping device is set to off in case the active optimal skyhook strategy demands a force that cannot be delivered by the semi-active damper.

A controller \( K(\rho) \) for the system (3.23) can be found by solving the \( H_\infty \) control problem, of which the solution is given by the Bounded Real Lemma extended to LPV systems [68]. This consists of an infinite set of linear matrix inequalities. The polytopic approach is used to solve the Bounded Real Lemma for \( \rho = \rho_{\text{min}} \) and \( \rho = \rho_{\text{max}} \), resulting in the controller

\[
K(\rho) = \rho \cdot K_{\rho_{\text{min}}} + (1 - \rho) \cdot K_{\rho_{\text{max}}} ,
\]

where \( K_{\rho_{\text{min}}} \) is the \( H_\infty \) optimal controller when \( \rho = \rho_{\text{min}} \) and \( K_{\rho_{\text{max}}} \) is the \( H_\infty \) optimal controller when \( \rho = \rho_{\text{max}} \). More details on the Bounded Real Lemma and the polytopic approach can be found in [5], [68].

As mentioned before, \( \rho \) should be dependent on the fact whether or not the reference force \( Cz_{\text{ref}} + u \) exceeds the constraints of the used semi-active actuator. These constraints can be given by any (non-linear) actuator model. When the semi-active actuator exhibits, for example, hysteretic behavior, this

![Figure 3.7: Implementation scheme of the \( H_\infty / LPV \) control strategy [68]](image-url)
can be taken into account regarding the reference force. This is an advantage of the LPV approach in general. Different examples of a formulae for $\rho$ can be found [68], [5] and [67]. The implementation scheme of the $\mathcal{H}_\infty$/LPV control strategy is shown in Figure 3.7. Note that stability properties for LPV systems are addressed in [85].

In [5] the $\mathcal{H}_\infty$/LPV approach is compared to the ADD control algorithm of (3.16). The ADD approach shows a slightly better performance regarding the suppression of the sprung mass acceleration. The $\mathcal{H}_\infty$/LPV approach shows better performance in the suppression of the unsprung mass acceleration and rattle-space displacement. This not surprising since these objectives are not present in the performance specifications of the ADD approach. It would therefore be interesting to compare the semi-active LQ-optimal approach and the $\mathcal{H}_\infty$/LPV approach for similar performance objectives, related to the design of a semi-active cabin suspension. In this performance objective the emphasis will be more more on the reduction of the sprung mass acceleration and rattle-space displacement, while unsprung mass acceleration will be of significantly less importance. However, it can be a difficult task to derive the ADD control algorithm when the rattle-space displacement is added to the performance index in (3.15).

Finally it may be worthwhile to combine the LPV theory with other control design methods. Instead of $\mathcal{H}_\infty$, also pole placement, $\mathcal{H}_2$, etc. can be used.

### 3.4 Model predictive control

Model Predictive Control (MPC) is an optimization based control law. The optimal solution relies on a dynamic model of the process, minimizes a quadratic performance measure and satisfies given input and state constraints. This means that the constraints of the semi-active suspension system can be taken into account while calculating the optimal control input for the system. In [14] and [33] MPC is used to design a controller for a 2 DOF quarter-car semi-active suspension system. To do this, the continuous-time state-space description of the suspension system, which is generally given by

$$\dot{x}(t) = Ax(t) + Bu(t), \quad (3.27)$$

has to be transformed into a discrete time state-space description. By choosing an appropriate sampling interval $T_s$, and a discretization technique, the discrete time model may be written as

$$x(k+1) = Ax(k) + Bu(k). \quad (3.28)$$

Here $x$ is the vector with the systems state variables and $u$ is in this case the control force delivered by the semi-active actuator. Note that the disturbances from the road profile and other environmental disturbances are not taken into account in this description. The control force is subject to a number of constraints, such as the passivity constraint and constraints on the maximum force and dissipating power. Different methods to define these constraints can be found in [14] and [33]. The optimization problem consists of finding a control law for $u(k)$, subject to (3.28) and the constraints, which minimizes the objective function $J$. This objective function contains the performance criteria for the suspension system and can be written as

$$J(U, x_0) = \sum_{n=0}^{N-1} x_{k+n}^TQx_{k+n} + \sum_{n=0}^{N-1} u_{k+n}^TRu_{k+n}. \quad (3.29)$$

Here $Q$ and $R$ are weighting matrices and $U = [u_0, u_1, \ldots, u_{N-1}]^T$ is the vector of control inputs to be optimized. Furthermore $N$ is the so-called control horizon and the MPC control law is obtained by the following *receding horizon* strategy:

1. At time instant $k$, get $x_0 = x(k)$.
2. Solve the optimization problem

$$\min_U J(U, x_0),$$

s.t. (3.28) and the constraints. \quad (3.30)
3. Apply the first element of the solution sequence $U$ to the optimization problem as the actual control action $u(k) = u_0$.

4. Repeat the whole procedure at time $k + 1$.

Note that when the control horizon $N = 1$, the resulting control law will be equal to the clipped optimal LQ control law, since each time interval $T_s$ the LQ optimal control force $u_0$ will be solved according to

$$\min_{u_0} J = x_0^T Q x_0 + u_0^T R u_0,$$

subject to the semi-active systems constraints. When $N$ increases, the performance of the MPC strategy increases with respect to the clipped LQ optimal control strategy [33]. This means that a lower value for the objective function $J$ can be achieved.

The definitions of the constraints of the semi-active suspension as given in both [14] and [33] contain logic and/or binary variables. This means the optimization problem has to be solved using a Mixed Integer Programming routine. An on-line application of the procedure cannot be actually performed, since it requires the solution of the optimization problem (3.30) at each sampling time. This task cannot be performed on-line in the sampling periods required for the application in a semi-active suspension system, because of the required computation time. To overcome this problem [14] uses the Set Membership approach to nonlinear function estimation. Here it is stated that the MPC control input $u(t)$ is a nonlinear static function of $x(t)$, i.e.

$$u(k) = f(x(k)).$$

The function $f$ is not explicitly known, but the values of $f(x)$ may be known for a certain number of its arguments by performing off-line the MPC procedure starting from initial conditions $\hat{x}_i(k), \quad i = 1, \ldots, M$, so that:

$$\tilde{u}_i(k) = f(\hat{x}_i(k)), \quad k = 1, \ldots, M$$

From these known values of $\tilde{u}_i$ and $\hat{x}_i$, an approximation $\hat{f}$ of $f$ is derived. Therefore the following functions are defined:

$$f_u(x, \gamma) = \min_{i = 1, \ldots, M} \left( \tilde{u}_i + \gamma ||x - \hat{x}_i|| \right),$$

$$f_l(x, \gamma) = \max_{i = 1, \ldots, M} \left( \tilde{u}_i - \gamma ||x - \hat{x}_i|| \right).$$

Compute:

$$\gamma^* = \inf_{\gamma : f_u(x, \gamma) \geq \tilde{u}_i, i = 1, \ldots, M} \gamma.$$ (3.35)

The estimate of $f(x)$ is then given by

$$\hat{f}(x) = [f_u(x, \gamma^*) + f_l(x, \gamma^*)]/2$$

and the MPC control can be implemented on-line, by simply evaluating the function $u(k) = \hat{f}(x(k))$ at each sampling time. However, a large amount of initial values $x_i$ should be evaluated off-line for a sufficiently accurate result.

In [33] the online implementation problem is tackled by computing an explicit representation $u(k) = f(x(k))$ of the receding horizon control law as a collection of affine gains over (possibly overlapping) polyhedral partitions of the set of $n$ states $x \in \mathbb{R}^n$. The explicit controller is obtained by using the Hybrid Toolbox for MATLAB. When a control horizon of $N = 1$ is considered, the set of 4 states is divided in 8 regions. When $N = 2$ is considered there are 62 regions obtained, a few of which are overlapping.

The MPC strategy developed in [14] is compared with an on-off skyhook control strategy for a control horizon of $N = 10$. In [33] the MPC strategy is compared to the clipped optimal LQ strategy and a so-called Steepest Gradient Method for a control horizon up to $N = 40$. In both references the MPC strategies perform better than the other controllers.
A drawback of the MPC strategy in general is that accurate knowledge of the model parameters is necessary for good performance. When regarding a semi-active truck cabin suspension, it is not unlikely that the mass of the cabin changes over time due to, for example, a variation in passengers. This means that when the cabin mass changes, the state-space representation of the suspension system should be altered. All the different realizations require different optimization problems which should be solved off-line and for each scenario the controller should be able to select the right off-line set for the calculation of the control input.

### 3.5 Lyapunov control

For the semi-active suspension system, Lyapunov’s direct approach to stability analysis can also be used for the design of a feedback controller. The approach requires the use of a Lyapunov function, denoted \( V(x) \), which must be a positive definite function of the states of the system, \( x \). It is assumed that the origin is a stable equilibrium point. According to Lyapunov stability theory, if the rate of change of the Lyapunov function, \( \dot{V}(x) \), is negative definite, the origin is asymptotically stable in the sense of Lyapunov. Thus, in developing the control law, the goal is to choose control inputs for the semi-active suspension that will result in making \( \dot{V}(x) \) as negative as possible, to achieve the smallest possible excursions from the equilibrium point. An infinite number of Lyapunov functions may be selected, that may result in a variety of control laws.

The equations of motion of a 2 DOF quarter-car semi-active suspension system are given by (3.24) and can be written in state-space form as

\[
\dot{x} = Ax + BFs + Ez_r, \tag{3.37}
\]

where the state vector \( x = [z_s \ z_u \ s]^T \) and \( A, B \) and \( E \) are matrices of the appropriate size. A function typically used as a candidate Lyapunov function is

\[
V(x) = x^T P x, \tag{3.38}
\]

where \( P \) is a real, symmetric, positive definite matrix. The matrix \( P \) is found using the Lyapunov equation

\[
A^T P + PA = -Q \tag{3.39}
\]

for a positive definite matrix \( Q \). The time derivative of the Lyapunov function (3.38) is

\[
\dot{V}(x) = -\frac{1}{2} x^T Q x + x^T PBs + x^T Pz_r. \tag{3.40}
\]

The semi-active damper force is dependent on the relative velocity of the two ends of the damper and the applied input current \( u \), so \( Fs = Fs(\dot{z}_s - \dot{z}_u, u) \). Here the maximum dissipative force is produced when the input current \( u = u_{max} \). The only term in (3.40) which can be directly affected by a change in the control input is the middle term on the righthand side, which contains the semi-active damper force \( Fs \). Thus, the control law which will minimize \( \dot{V} \) is

\[
u = u_{max} H(-x^T PBs), \tag{3.41}
\]

where \( H(\cdot) \) is the Heaviside step function, shown in Figure 3.8 for \( H(1) [48] \). This algorithm is classified as a bang-bang controller due to the switch in the Heaviside step function. This is characteristic for a controller of a semi-active suspension system based on the Lyapunov stability theory. The control strategy should always minimize the chosen Lyapunov function, regardless of its definition. This means that whenever a control current is applied, this will always be the maximum control current. The challenge in the use of the Lyapunov algorithm is the selection of an appropriate matrix \( Q \), which determines the performance of the controller.

A candidate Lyapunov function given in [48] is the total vibratory energy in the system. The equations of motion of (3.24) can be written as

\[
M\ddot{z} + Kz = \begin{pmatrix} 1 \\ -1 \end{pmatrix} Fs + \begin{pmatrix} 0 \\ k_f \end{pmatrix}, \tag{3.42}
\]
where $M$ is the mass matrix, $K$ is the stiffness matrix and $z = [z_s \ z_u]^T$. The Lyapunov function is now given by

$$V(z) = \frac{1}{2} z^T K z + \frac{1}{2} \dot{z}^T M \dot{z} + k_i (\frac{1}{2} x_r^2 - x_2 x_r).$$

(3.43)

The derivative is

$$\dot{V}(z) = z^T K \dot{z} + k_i (z_r \dot{z}_r - \dot{z}_2 \dot{x}_r - \dot{z}_2 \dot{x}_r) + \dot{z}^T (-K z + \begin{pmatrix} 1 \\ -1 \end{pmatrix} F_s + \begin{pmatrix} 0 \\ k_i \end{pmatrix}).$$

(3.44)

and the control law is, analogous to (3.41),

$$u = u_{max} H(-z^T \begin{pmatrix} 1 \\ -1 \end{pmatrix} F_s).$$

(3.45)

This is a specific variant of the control law in (3.41).

In [52] an efficient semi-active on-off damping control law for vibration attenuation of a multidegree-of-freedom vibratory system has been developed, using Lagrange's equations and Lyapunov's direct method (see [65]). It minimizes the total vibratory energy of the structure, including the work done by external disturbances. The dissipative energy of the semi-active control device is maximized for specified vibrational response modes of the system by proper assignment of several weighting factors. The vibrational response mode can be determined at will, considering the vibrational characteristics of the system and the performance requirement.

Using Lyapunov theory control algorithms can be designed which take into account the constraints of the semi-active system on beforehand, as is the case for the previously discussed control strategies. For this reason it may be worthwhile to investigate the performance achievable with this control method.

### 3.6 Other control methods

Many other control strategies for semi-active suspension systems exist besides the strategies mentioned earlier in this chapter. Some of them are clipped strategies which attempt to approximate optimal active suspension controllers, e.g. the predictive semi-active control strategy described in [34] and the sliding mode strategy described in [23]. Other examples of semi-active control methods are the so-called sensitivity control [43], Convex Integrated Design method [29], the Rakheja-Sankar method [78] and neural networks, fuzzy logic or combinations thereof [12], [38].

Another control strategy which was investigated, among others, by Huisman [46] and Muijderman [62], [63] at the Eindhoven University of Technology is optimal control with preview. Here it is assumed that a part of the upcoming road profile is known. This means that the semi-active suspension control problem can be regarded as an optimization problem. The objective is to minimize the acceleration of the vehicles sprung mass over the period in which the road profile is known. Therefore
the optimal course of the semi-active dampers damping coefficient should be calculated, regarding the constraints of the semi-active suspension system. The information on the upcoming road profile should be obtained by using infrared sensors, radar or cameras. When using preview control only on the rear axle suspension the information can be obtained from sensors on the front axle. The application of this kind of sensors on a truck is regarded infeasible due to the expected high costs. Therefore semi-active suspension strategies using preview are not discussed in further detail in this report.

3.7 Summary

Many control strategies for semi-active suspension systems can be found in the literature. The majority are empirical approximations of optimal control strategies for active suspension systems. It is however shown that these so-called clipped control strategies are not optimal for semi-active suspension systems. The constraints inherent to the semi-active actuators should be taken into account during the controller design in order to come to a controller which is optimal with respect to the chosen performance objectives.

Four strategies which do so are discussed. The MPC strategy seems impractical for the use in a trucks cabin suspension system, as it is difficult to use under varying environmental influences. The semi-active mixed-ADD-skyhook and $LPV/H_{\infty}$ controllers are more interesting. Another potentially interesting controller is based on the Lyapunov stability theory. It may be worthwhile to investigate the performance of the various controllers found in the literature for similar performance criteria, which are relevant for truck cabin suspension systems.
Chapter 4

Optimization of passive suspension systems

4.1 Introduction

In order to position the performance of semi-active suspension systems in the right perspective, a relevant benchmark system has to be selected. For an fair comparison, both the semi-active suspension system and the benchmark system have to be optimized to the same criteria and under the same constraints.

The benchmark system should be able to give a sufficiently accurate description of the performance of the cab suspension of modern day trucks. In this thesis the focus lies on the suspension performance in the vertical (heave) direction. From the literature study presented in Chapter 2 it follows that vehicle heave dynamics are commonly described by the so-called 2 DOF quarter-car model. This model only contains one (primary) suspension and is therefore not suitable for the analysis of a truck cabin suspension. Therefore the use of an extended quarter-car model which includes a secondary suspension needs to be investigated.

In Chapter 2 of this thesis various passive suspension components that have been investigated over the past years, in order to improve vehicle ride quality are discussed. To properly judge the performance of a semi-active cabin suspension, the performance improvement that can be achieved using these passive suspension components in a cabin suspension should also be regarded. From Chapter 2 follows that frequency selective damping, stroke dependent damping and the inerter are concepts which are worthwhile to investigate in the context of a cabin suspension system. In literature, these concepts are all investigated when applied to a vehicle axle suspension. The benefits for a secondary suspension remains unclear.

In this chapter a model describing the dynamics of the truck cabin in heave direction is presented. Also an optimization criterium and constraints are chosen which are relevant for the design of a truck cabin suspension. The aforementioned passive suspension concepts are then evaluated and compared.

The model describing the cab dynamics is the so-called quarter-truck model. This model will be discussed and compared to the 2 DOF quarter-car model. The input of the quarter-truck model is the disturbance signal on the vertical tire deflection induced by the road. Since nonlinear suspension concepts are investigated, the model has to calculate the vehicle heave dynamics in the time-domain. A time-domain description for the input signal is therefore derived in this chapter.

The cabin suspension parameters of the linear quarter-truck model are optimized according to the chosen performance criteria. The performance of the optimized linear quarter-truck model serves as a benchmark for the other suspension systems, both passive and semi-active. The nonlinear passive suspension concepts are implemented in the cabin suspension of the quarter-truck model. They are optimized in the time domain for the chosen criteria, to be able to assess to potential improvement in driver comfort of these concepts.

This chapter is organized as follows. The quarter-truck model is discussed in Section 4.2 and
the time domain road disturbance signal is derived in Section 4.3. In Section 4.4 the choice of the optimization criterion and corresponding constraints is discussed. The benchmark linear passive quarter-truck model is optimized in Section 4.5. Sections 4.6 – 4.8 handle the optimization of the frequency selective damping, the stroke dependent damping and the inerter concepts respectively.

4.2 The quarter-truck model

As mentioned before, the 2 degree of freedom (DOF) quarter-car model is commonly used in literature to describe the heave dynamics of a passenger vehicle [45]. The linear passive 2 DOF quarter-car model is shown in Figure 4.1a. The equations of motion for this model are

$$\begin{cases} m_s \ddot{z}_s + \ddot{d}_s (\dot{z}_s - \dot{z}_u) + k_s (z_s - z_u) &= 0 \\ m_u \ddot{z}_u - \ddot{d}_s (\dot{z}_s - \dot{z}_u) - k_s (z_s - z_u) + k_t (z_u - z_r) &= 0 \end{cases}$$

(4.1)

where $m_s$ is the sprung mass and $m_u$ is the unsprung mass. The suspensions (constant) stiffness and damping coefficient are given by $k_s$ and $d_s$ respectively and $k_t$ is the vertical stiffness of the tire. The vertical displacement of the road, the sprung mass and the unsprung mass are $z_r$, $z_s$ and $z_u$ respectively.

This model does not include a secondary suspension system (nor an engine suspension), which makes it unsuitable for the description of truck cabin dynamics. Therefore the 4 DOF quarter-truck model is introduced here, as depicted in Figure 4.1b. In this Figure the subscripts $r$, $a$, $c$, $f$, $e$ and $t$ represent the road surface, front axle, engine, frame, cabin and front tire respectively. The parameters $m$, $k$ and $d$ represent masses, stiffness and damping coefficients and $z$ are the vertical displacements.

The equations of motion of this model are

$$\begin{cases} m_c \ddot{z}_c + k_c (z_c - z_f) + \ddot{d}_c (\dot{z}_c - \dot{z}_f) &= 0 \\ m_f \ddot{z}_f + k_a (z_f - z_u) + \ddot{d}_a (\dot{z}_f - \dot{z}_a) + k_c (z_f - z_c) + \ddot{d}_c (\dot{z}_f - \dot{z}_c) - k_e (z_c - z_f) - \ddot{d}_e (\dot{z}_c - \dot{z}_f) &= 0 \\ m_e \ddot{z}_e - k_e (z_f - z_c) - \ddot{d}_e (\dot{z}_f - \dot{z}_c) &= 0 \\ m_u \ddot{z}_u - k_u (z_f - z_u) - \ddot{d}_u (\dot{z}_f - \dot{z}_u) + k_t (z_u - z_r) &= 0 \end{cases}$$

(4.2)

Table 4.1 lists the parameters of the quarter-truck model. These values are identified using measurements of a truck driving over different road surfaces [25]. In Figure 4.2 typical shapes for both the two different transfers in both the quarter-car and the quarter-truck model are shown. Figure 4.2a show the transfer from the vertical road velocity $v_r$ to the sprung mass acceleration $a_s$ for the quarter-car model and to the vertical cabin acceleration $a_c$ for the quarter-truck model respectively. Figure 4.2b shows the transfer from the $v_r$ to the suspension deflection and cabin suspension deflection for
the quarter-car model and the quarter-truck model respectively. In section 2.5.1 these two criteria are mentioned as the main criteria in vehicle suspension optimization. For the quarter-car model the parameters $m_s = 300$ [kg], $m_u = 30$ [kg], $k_s = 20 \cdot 10^3$ [N/m], $d_s = 1 \cdot 10^3$ [Ns/m] and $k_t = 200 \cdot 10^3$ [N/m] are used.

Figure 4.2 clearly shows the differences between the two models. Around 10 [Hz] the anti-resonance caused by the suspended engine mass is visible for the quarter-truck model. However, the most important difference is the larger decrease of the transfer at higher frequencies for the quarter-truck model. The overall transfer of high frequency vibrations from the road surface to the suspended mass of interest is reduced. Besides that, the front axle resonance mode (also called wheel-hop) has less influence on the acceleration of the suspended mass, which determines the level of driver comfort.

In case of a nonlinear cabin suspension system, the force of this cabin suspension cannot be calculated in the linear quarter-truck model. To take this nonlinear force into account a term $F_c$ is introduced in the righthand-side of the first equation of (4.2) and a term $-F_c$ in the righthand-side of the second equation of (4.2). The force $F_c$ is now an input of the quarter-truck model and is calculated outside the linear quarter-truck model. A schematic representation hereof is given in Figure 4.3. When desired, in this case the linear cabin suspension damping coefficient can be set to $d_c = 0$ [Ns/m], as well as the linear stiffness $k_c$.

Analogous to the primary suspension case, for the cabin suspension the vertical cabin acceleration $\ddot{z}_c$ and the relative displacement between the cabin and the frame $z_c - z_f$ are related to the two main

![Figure 4.2: FRF absolute values for different transfers in the quarter-car and quarter-truck model.](image)

### Table 4.1: Quarter-truck parameter values.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m_c$</td>
<td>650</td>
<td>kg</td>
</tr>
<tr>
<td>$m_f$</td>
<td>643</td>
<td>kg</td>
</tr>
<tr>
<td>$m_e$</td>
<td>892.5</td>
<td>kg</td>
</tr>
<tr>
<td>$m_a$</td>
<td>350</td>
<td>kg</td>
</tr>
<tr>
<td>$d_c$</td>
<td>$13.3 \cdot 10^3$</td>
<td>Ns/m</td>
</tr>
<tr>
<td>$d_a$</td>
<td>$11 \cdot 10^3$</td>
<td>Ns/m</td>
</tr>
<tr>
<td>$d_e$</td>
<td>$8 \cdot 10^3$</td>
<td>Ns/m</td>
</tr>
<tr>
<td>$k_c$</td>
<td>$4 \cdot 10^4$</td>
<td>N/m</td>
</tr>
<tr>
<td>$k_a$</td>
<td>$30 \cdot 10^4$</td>
<td>N/m</td>
</tr>
<tr>
<td>$k_e$</td>
<td>$3.2 \cdot 10^6$</td>
<td>N/m</td>
</tr>
<tr>
<td>$k_t$</td>
<td>$1.2 \cdot 10^6$</td>
<td>N/m</td>
</tr>
</tbody>
</table>
criteria in the optimization of this suspension system. These signals are therefore two desired outputs of the quarter-truck model. A convenient choice for the input is, besides the nonlinear suspension force, the vertical velocity of the road surface \( \dot{z}_r \). This is a convenient choice because it can be described by a white-noise signal \([45]\). The description of this white-noise input signal will be addressed further in Section 4.3.

In order to use the quarter-truck model in the simulation required for the optimization and evaluation of the various suspension concepts, it is written in state-space form. This is done using the state-vector 
\[
x = \begin{bmatrix} z_c - z_f & z_f - z_a & z_a - z_r & \dot{z}_c & \dot{z}_f & \dot{z}_e & \dot{z}_a & \dot{z}_r \end{bmatrix}^T
\]

and output-vector 
\[
y = \begin{bmatrix} z_c - z_f & \ddot{z}_c \end{bmatrix}^T
\]

the quarter-truck model can be described by
\[
\begin{cases}
\dot{x} = Ax + Bu \\
y = Cx + Du
\end{cases}
\]  \hspace{1cm} (4.3)

with
\[
A = \begin{bmatrix}
0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & -1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
-k_c & 0 & 0 & 0 & -d_c & d_c & 0 & 0 \\
m_c & -k_c & m_f & 0 & m_f & -d_e & d_e + d_a & d_e \\
0 & m_c & m_f & 0 & 0 & 0 & 0 & 0 \\
0 & m_c & m_f & 0 & 0 & 0 & 0 & 0 \\
-k_e & 0 & -k_f & m_a & 0 & 0 & d_a & d_a \\
0 & m_a & m_f & 0 & 0 & 0 & 0 & -d_a \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & m_f \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix},
\]

\[
B = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix},
\]

\[
C = \begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix},
\]

\[
D = \begin{bmatrix}
0 & 0 & 0 & 0
\end{bmatrix}.
\]

This state-space description will be used further throughout this research for simulations in the optimization and evaluation of various suspension concepts.
4.3 Time domain description of the road disturbance signal

For the optimization and evaluation of nonlinear suspension concepts, the quarter-truck model requires an input signal in the time-domain. This signal should be representative for the vertical velocity at the contact point between the vehicle tire and the road surface, when driving with a certain forward velocity.

In the ISO 8608:1995(E) norm the following expression is proposed to represent the power spectral density (PSD) of the road profile \( s \):

\[
S_{zr}(n) = S_{zr}(n_0) \cdot \left( \frac{n}{n_0} \right)^{-2}, \quad \text{with } n_0 = 0.1 \text{ m}^{-1}.
\] (4.4)

Table 4.2 shows the values of the degree of road roughness \( S_{zr}(n_0) \), corresponding to different road classes. The spacial frequency \( n \) is defined by the relation

\[
n = \frac{1}{\lambda} = \frac{f}{V},
\] (4.5)

where \( \lambda \) is the road surface wavelength, \( f \) is the frequency and \( V \) is the vehicle forward velocity.

According to [98] the time domain displacement disturbance to the tire can now be represented by a white noise signal with a PSD of 1 \([\text{Hz}^{-1}]\), passing through a first-order filter given by

\[
G_{zr}(s) = \sqrt{S_{zr}} \cdot \frac{2\pi V n_0}{s}, \quad \text{with } s = j\omega.
\] (4.6)

Here \( j \) is the imaginary unit and \( \omega \) is the frequency in \([\text{rad/s}]\). This means that the road vertical velocity disturbance in the time domain can be represented by a white noise signal with a PSD of 1 \([\text{Hz}^{-1}]\), multiplied by the constant gain

\[
G_{wr} = \sqrt{S_{zr}} \cdot 2\pi V n_0.
\] (4.7)

Using a white noise signal as input for the quarter-truck model is convenient when applying a linear control theory such as LQ-optimal control to the system.

4.4 Optimization criterium

To be able to properly optimize the characteristics of a passive truck cabin suspension system, an optimization criterium has to be chosen which is representative for the problem at hand. The most important criteria for a passive cabin suspension are driver comfort and suspension working space, [17]. In general, two different approaches on taking these contradicting criteria into account can be found in the literature on this subject (see Section 2.5.1). It is desired to optimize the driver comfort,

Table 4.2: Classification of road roughness according to ISO 8608:1995(E).

<table>
<thead>
<tr>
<th>Road class</th>
<th>upper limit</th>
<th>geometric mean</th>
</tr>
</thead>
<tbody>
<tr>
<td>A (very good)</td>
<td>32</td>
<td>16</td>
</tr>
<tr>
<td>B (good)</td>
<td>128</td>
<td>64</td>
</tr>
<tr>
<td>C (average)</td>
<td>512</td>
<td>256</td>
</tr>
<tr>
<td>D (poor)</td>
<td>2048</td>
<td>1024</td>
</tr>
<tr>
<td>E (very poor)</td>
<td>8192</td>
<td>4096</td>
</tr>
<tr>
<td>F</td>
<td>32768</td>
<td>16384</td>
</tr>
<tr>
<td>G</td>
<td>131072</td>
<td>65536</td>
</tr>
<tr>
<td>H</td>
<td>-</td>
<td>262144</td>
</tr>
</tbody>
</table>
this should always be represented in the objective function of the optimization problem. To limit the
necessary suspension working space, this criterium can also be included in the objective function.
A weighting factor can be used to indicate the relative importance of the driver comfort and work-
ing space criteria. The second approach for taking into account limits on the working space, is to
add a constraint to the optimization problem which prescribes the maximum allowable suspension
deflection.

The ISO 2631 norm is widely used to represent driver comfort [1]. Herein, the acceleration signal
at the position of the driver is weighted per frequency band for human sensitivity. The RMS value of
this weighted acceleration signal is called the ride index, which is a measure for the driver comfort.
This RMS signal is calculated according to

\[
a_w = \left[ \frac{1}{T} \int_0^T a_w^2(t) \, dt \right]^{\frac{1}{2}},
\]

where \(a_w(t)\) is the instantaneous frequency-weighted acceleration and \(T\) is the duration of the mea-
surement. When considering vibrations in more than one direction, the values of the different weighted
RMS acceleration should be combined as follows:

\[
a_v = (k_x^2 a_{wx}^2 + k_y^2 a_{wy}^2 + k_z^2 a_{wz}^2)^{\frac{1}{2}}.
\]

Here \(a_{wx}, a_{wy}, a_{wz}\) are the weighted RMS accelerations with respect to the orthogonal axes \(x, y\) and \(z\)
respectively and \(k_x, k_y, k_z\) are weighting factors. \(a_v\) is called the vibration total value.

A fourth order approximation of the frequency weighting found in [97] is used in this research to
avoid numerical problems. The numerically optimized approximation of the weighting function for
vertical acceleration is given by

\[
W_k(s) = \frac{81.89s^3 + 796.6s^2 + 1937s + 0.1446}{s^4 + 80.00s^3 + 2264s^2 + 7172s + 24496}.
\]

In this research the vertical acceleration of the cabin is used to calculate the ride index.

When including the suspension working space in the objective function, this criterium is mostly
represented in literature by mean square value (MSV) or the RMS value of the suspension deflection
signal, analogous to the weighted acceleration [45], [6], [30]. The RMS suspension deflection is no
practical limitation in a truck cabin suspension system. However, for a stochastic signal with a normal
distribution and a mean value of \(\sigma\) the RMS value is equal to the standard deviation of the signal, \(\sigma\).
It is known that 99.7\% of the values drawn from a signal with normal distribution lies within three
standard deviations from the mean value of the signal. Therefore, there is a 99.7\% guarantee that the
absolute value of suspension deflection will not exceed three times the RMS value for linear systems
with a vertical road velocity input signal with a normal distribution and a mean value of \(\sigma\). In this way
the RMS suspension deflection is linked to the maximum suspension deflection, which is a practical
constraint of the suspension system. However, this does only hold for linear systems. For nonlinear
systems this relation between RMS suspension deflection and maximum suspension deflection does
not exist.

Since nonlinear suspension systems will also be investigated in this research, it is chosen to limit
the suspension deflection by adding constraints to the optimization problem. These constraints limit
the maximum absolute suspension deflection as a function of time to a value of 0.04 [m], which is a
common value in practice. This means that the cabin can travel 0.04 [m] upwards and 0.04 [m] down-
wards from its static position. This static position represents the position of the loaded cabin under
influence of gravity, in equilibrium with the force of the preloaded springs of the cab suspension. The
total working space of the cabin suspension is 0.08 [m]. The investigation of nonlinear suspension
systems requires in any case that all calculations in this optimization problem are performed in the
time domain. In this case the objective function to be minimized is given by the ride index (4.8).
The suspension system should perform optimally for a range of vertical road velocity input signals,
which relate to different combinations of vehicle forward velocity and road surface quality. To take this
into account, the objective function is chosen to be a combination of two values of the ride index $a_w$, calculated for two different road input signals. One of the road input signals can be qualified as good, regarding the combination of road roughness and vehicle forward velocity. This signal will be referred to as **Type I** from now on.

The Type I vertical road velocity disturbance signal $w_r(t)$ is selected according to the ISO 8608:1995(E) norm [2]. For the degree of road roughness $S_{zr}(n_0)$ a value of $S_{zr} = 16 \cdot 10^{-6}$ [m$^3$] is chosen, corresponding to road class A (see Table 4.2 in Section 4.3). For the forward vehicle velocity a value of $V = 22.2$ [m/s] ($= 80$ [km/h]) is selected.

The second road velocity disturbance signal represents a poor combination of forward vehicle velocity and road surface quality. This signal, $w_r(t)$, is from now on referred to as **Type II**. The Type II signal is selected such that the response of the identified quarter-truck model reaches the limits of the available cabin suspension working space. It is assumed that the cab suspension has a stroke of $\pm 0.04$ [m] measured from its equilibrium point. When keeping the vehicle forward velocity at $V = 22.2$ [m/s], these limits on the suspension working space are approached for $S_{zr} = 112 \cdot 10^{-6}$ [m$^3$].

The relative importance of the comfort indices calculated for the Type I and Type II road disturbance signals respectively is unknown. In other words, in general it is not known how large a part of the road driven by a truck can be considered as good and respectively poor road disturbance conditions. Therefore both conditions are present in the objective function with an equal weighting factor of 0.5. The objective function is given by

$$J = 0.5 \cdot \frac{a_w(w_{rI}(t), p)}{a_{wI}} + 0.5 \cdot \frac{a_w(w_{rII}(t), p)}{a_{wII}}, \quad (4.11)$$

where $p$ are the suspension system design variables and $a_{wI}$ and $a_{wII}$ are the comfort indices for the identified quarter-truck model calculated for the Type I and II road velocity disturbances respectively.

To keep the suspension deflection of the optimized system within the desired limits, proper constraint functions have to be selected. The identified, benchmark quarter-truck model will serve as a benchmark. Therefore, the suspension deflection of all the optimized suspension systems should be lower than, or at least equal to, the suspension deflection of the identified model under similar road input conditions. In practice, suspension working space limitations become important when driving on a road with a poor combination of vehicle forward velocity and road surface quality. Also driving over discrete obstacles can cause the suspensions to reach the limits of its working space.

The poor road input conditions used for the constraint function are described by the signal $w_{rII}(t)$, which is also used in the objective function. The traffic bump depicted in Figure 4.4 is chosen as discrete obstacle. When simulating driving over this obstacle with a forward vehicle velocity of $V = 8.33$ [m/s], the cabin suspension deflection of the identified quarter-truck model approaches the limit of $\pm 0.04$ [m]. When the suspension working space is taken into account in the objective function together with the driver comfort, the results of the optimization problem are often plotted for different values of the weighting factor that defines the relative importance of the two criteria (see e.g. [45]). In such figures one axis shows the RMS acceleration of the driver position, while the other axis represents the RMS suspension deflection. In these figures the relation between the allowable suspension deflection and the maximum achievable comfort can be seen. To show this relation for the optimization problem in this research, the problem is solved for different values of the maximum allowable suspension working space: $S_{max} = \pm 0.02, \pm 0.03, \pm 0.04, \pm 0.05, \pm 0.06$ [m]. The optimization problem is

![Figure 4.4: Discrete obstacle.](image-url)
now written as

\[
\begin{align*}
\min_{\mathbf{p}} J &= 0.5 \cdot a_{w,G}(w_G(t),\mathbf{p}) + 0.5 \cdot a_{w,P}(w_P(t),\mathbf{p}) \\
\text{s.t.} \quad &\max |z_c - z_f|_p \leq S_{max} \\
&\max |z_c - z_f|_{\text{bump}} \leq S_{max} \quad \text{for } S_{max} = 0.02, 0.03, 0.04, 0.05, 0.06
\end{align*}
\]

(4.12)

where \( |z_c - z_f|_p \) is the absolute deflection of the cabin suspension under Type II road input conditions and \( |z_c - z_f|_{\text{bump}} \) is the absolute deflection when driving over the discrete obstacle.

### 4.5 Optimal linear passive cabin suspension system

As mentioned before, the identified linear passive quarter-truck model will be used as a benchmark setting. A first step in finding the optimal passive cabin suspension system is to calculate the optimal suspension parameters for the benchmark system, regarding the optimization problem (4.12). In this case the design variables are the stiffness \( k_c \) and the damping \( d_c \) of the cabin suspension. When the values of these variables are unconstrained, the requirements on the working space limitations are met by a sufficiently large damping coefficient. The stiffness will go to zero. Nevertheless, the springs in the cabin suspension of the identified passive cabin suspension using linear springs and dampers can be considered close to optimal. The linear quarter-truck model with \( k = 4 \cdot 10^4 \text{ [N/m]} \) and \( d = 13.3 \cdot 10^3 \text{ [Ns/m]} \), which is almost the same as the value for the identified quarter-truck model. For all values of \( S_{max} \) the optimal suspension stiffness \( k_c \) lies on the lower limit, \( 4 \cdot 10^4 \text{ [N/m]} \). From this results can be concluded that the identified passive cabin suspension system, regarding the optimization problem

The constrained nonlinear optimization problem can now be written as (4.12) with the additional constraints

\[
4 \cdot 10^4 \leq k_1 \leq 12 \cdot 10^4 \\
3 \cdot 10^3 \leq d_1 \leq 36 \cdot 10^3
\]

(4.13)

and is solved in MATLAB using the command \texttt{fmincon}, which uses a sequential quadratic programming (SQP) algorithm to solve the optimization program. A logarithmic scaling is used for the design variables and the objective function, as is presented in [11]. This approach is used for all optimization problems in this thesis. With this scaling the derivatives reflect the relative change of the objective function due a relative change in a design variable:

\[
\hat{p}_i = \ln(p_i), \\
\hat{J} = \ln(J), \\
\frac{\delta J}{\delta p_i} = \frac{1}{\hat{p}_i} \frac{\delta J}{\delta \hat{p}_i}
\]

(4.14)

Six different starting points for the values of the design variables are used. This is done to decrease the chance of finding a solution at a local minimum of the objective function, while keeping the computational time within limits.

The results are shown in Figure 4.5, where the values of the objective function are plotted for the different values of the available working space \( S_{max} \). The identified quarter-truck model is also indicated in this figure, showing that a possible comfort improvement of 1.4 % can be achieved at \( \pm 0.04 \text{ [m]} \) available working space. The damping coefficient of the linear suspension system is plotted as a function of \( S_{max} \) in Figure 4.6. This shows that the optimal value for \( d_c \) at \( S_{max} = 0.04 \text{ [m]} \) is \( 1.329 \cdot 10^4 \text{ [Ns/m]} \), which is almost the same as the value for the identified quarter-truck model. For all values of \( S_{max} \) the optimal suspension stiffness \( k_c \) lies on the lower limit, \( 4 \cdot 10^4 \text{ [N/m]} \). From this results can be concluded that the identified passive cabin suspension using linear springs and dampers can be considered close to optimal. The linear quarter-truck model with \( k_c = 4 \cdot 10^4 \text{ [N/m]} \) and \( d_c = 1.329 \cdot 10^4 \text{ [Ns/m]} \) will be referred to as the benchmark model from now on.
4.6 FSD cabin suspension system

The principle and modeling of the FSD suspension concept is discussed in Section 2.3.1. Herein, the damping force delivered by an FSD damper is modeled as

\[ F_{\text{FSD}} = D_{\text{FSD}} \dot{x}_p : [0.9(1 - e^{-\Gamma \tau}) + 0.1]. \]  

(4.15)

A block scheme of the MATLAB/SIMULINK model used to describe the FSD damper force is shown in Figure 4.7 [28]. Here the sample and hold block is used to count the time the damper piston is moving in one direction. This time is an input to the function block describing the FSD correction factor given by \(0.9(1 - e^{-\Gamma \tau}) + 0.1\). This factor is then multiplied with the piston velocity times the FSD damper constant \(D_{\text{FSD}} \dot{x}_p\) to get the FSD damper force.

In figure 4.8 it can be seen how a change in \(\Gamma\) affects the frequency response of the quarter-truck model. Note that the quarter-truck model incorporating an FSD damper is a nonlinear system. Therefore a frequency response function (FRF) for this system can not be calculated and the variance gain is used as an approximation of the magnitude of the systems FRF, as is described in [74]. The approximate FRFs from road input \(w_r\) to vertical cab acceleration \(a_c = \ddot{z}_c\) and vertical cab suspension deflection \(\delta z_c = z_c - z_f\) respectively, are computed using the following steps:

1. The closed-loop system is fed with a finite set of \(N\) pure tones in the frequency range of interest: \(w_{r,i}(t) = A \sin(\omega_i t); \quad i = 1, 2, \ldots N, \quad t \in [0, T];\)
2. The corresponding output signals have been recorded: \(\ddot{z}_{c,i}(t)\) and \(\delta z_{c,i}(t); \quad i = 1, 2, \ldots N, \quad t \in [0, T];\)

- \(\ddot{z}_{c,i}(t)\) and \(\delta z_{c,i}(t)\) are the vertical accelerations and deflections of the cab, respectively.
- \(D_{\text{FSD}}\) is the linear damping coefficient.
- \(f(u)\) is the FSD correction factor.
- \(F_d\) is the damper force.

![Figure 4.7: SIMULINK block scheme the FSD damper model.](image)
3. From each measured output signal the component at \( \omega_i \) is extracted: 
\[
\tilde{z}_{c,i}(t) \approx \Gamma_i \sin(\omega_i t + \Theta_i),
\]
and 
\[
\delta z_{c,i}(t) \approx \Pi_i \sin(\omega_i t + \Theta_i); \quad i = 1, 2, \ldots N, \quad t \in [0, T];
\]

4. The approximate describing function is computed in \( N \) points; it is defined as:
\[
\hat{F}_{acc}(j\omega_i) = (\Gamma_i/A) e^{j\Theta_i}, \quad \hat{F}_{str}(j\omega_i) = (\Pi_i/A) e^{j\Theta_i}; \quad i = 1, 2, \ldots N; \quad (4.16)
\]

5. The approximate variance gain is computed in \( N \) points; it is defined as:
\[
\hat{F}_{acc}(j\omega_i) = \left[ \frac{1}{T} \int_{0}^{T} (\tilde{z}_{c,i}(t))^{2} dt \right]^{1/2}, \quad \hat{F}_{str}(j\omega_i) = \left[ \frac{1}{T} \int_{0}^{T} (\delta z_{c,i}(t))^{2} dt \right]^{1/2}; \quad i = 1, 2, \ldots N
\]

\[
(4.17)
\]

The describing function describes the gain amplification and phase-shift experienced by a pure harmonic signal when passing through the non-linear system. The variance gain provides a complimentary piece of information with respect to the describing function. It describes the square-root of the average power amplification experienced by a pure-tone signal when passing through the non-linear plant. It provides a simple and immediate way of measuring the degree of non-linearity of the I/O behavior of the plant which is given by the distance between \( \hat{F}_{acc}(j\omega_i) \) and \( \hat{F}_{acc}(j\omega) \) at each frequency.

In Figure 4.8a the variance gain from the vertical road velocity input \( w_r \) to the vertical cabin acceleration \( a_c \) is shown. Figure 4.8b shows the variance gain from the vertical road velocity to the cabin suspension stroke \( z_c - z_f \). It can be seen that the transfer from the road input to \( a_c \) decreases strongly at high frequencies when decreasing \( \Gamma \). Meanwhile the transfer from \( w_r \) to the vertical suspension deflection increases over the whole frequency range, with a minimum increase at the antiresonance/resonance around 10 [Hz]. Furthermore should be noted the for \( \Gamma = 0.5 \) the damping becomes so low that the transfer from \( w_r \) to \( a_c \) increases around the resonance frequency of \( \Gamma \) [Hz]. In this case two peaks can be distinguished, which correspond with the eigenfrequency of the rigid-body mode of the model and the eigenfrequency of the cabin resonance mode.

Figure 4.9 shows the effects of a change in \( D_{\text{lin}} \) for a constant \( \Gamma \) in the same manner as Figure 4.8 does for a change in \( \Gamma \). Here the same effects as in Figure 4.8 can be seen.

![Figure 4.8: Quarter-truck variance gain from for varying \( \Gamma \).](image)

(a) From road input \( w_r \) to vertical acceleration \( a_c \).

(b) From road input \( w_r \) to cabin suspension displacement \( z_c - z_f \).
The optimization of the FSD cabin suspension system for the quarter-truck model is a problem with three design variables: the cabin suspension stiffness $k_1$, the FSD damping constant $D_{FSD}$ and the shape function $\Gamma$. When solving the optimization problem using the approach described in Section 4.5, the value of $\Gamma$ does not change, while the values for $k_1$ and $D_{FSD}$ do. This might indicate a problem with the scaling of the design variables. However, different scaling approaches have been investigated, none of which lead to a change in $\Gamma$ during the optimization.

Since the emphasis of this thesis is not on the design of the optimal FSD damper, the optimization problem is instead solved for three fixed values for $\Gamma$. These values are $\Gamma = 0.5$, $\Gamma = 5$ and $\Gamma = 50$. Through this parameter study some insight can be gained on the effect of a change in $\Gamma$ on different solutions of the remaining optimization problem. The stiffness is again bounded by $4 \cdot 10^4 \leq k_1 \leq 12 \cdot 10^4$ [N/m] and $D_{FSD}$ is bounded by $1 \cdot 10^3 \leq D_{FSD} \leq 1 \cdot 10^6$ [Ns/m]. Again different sets of starting values for the design variables are used to avoid local minima in the optimization problem.

The results are shown in Figure 4.10, where the values of the objective functions are plotted for the different values of the available working space $S_{max}$, together with the values for the benchmark system. It can be seen that the FSD system with $\Gamma = 50$ shows the best results for $S_{max} = 0.04$, $S_{max} = 0.05$ and $S_{max} = 0.06$ [m]. For $S_{max} = 0.02$ and $S_{max} = 0.03$ the objective function value for the system with $\Gamma = 0.5$ is slightly lower. Figure 4.11 shows the variance gain from $w_r$ to $a_c$ for the optimized systems at $S_{max} = 0.04$ [m] and the benchmark system. On first sight it seems that the
variance gain of the system with $\Gamma = 5$ is most favorable, regarding the smaller variance gain at high frequencies. The fact that the system with $\Gamma = 50$ has the smallest objective function value in Figure 4.10 can be explained by zooming in on the peak in the variance gain around 1.7 [Hz] (Figure 4.12). Here we can see that the peak for the FSD suspension system is lowest when $\Gamma = 50$. From this can be concluded that the high frequency behavior of the suspension system is of less importance regarding the ride index than the behavior around the resonance frequency at 1.7 [Hz] for $S_{\max} = 0.04$ [m].

Figure 4.10 also shows that for $S_{\max} = 0.04$, $S_{\max} = 0.05$ and $S_{\max} = 0.06$ the FSD suspension does not perform better than the benchmark system. In order to fulfill the working space constraints, the optimized damping factor $D_{FSD}$ has to be higher than the optimal damping coefficient $d_c$ of the linear suspension to compensate for the reduction of the damper force at higher frequencies. This means however that the damping at lower frequencies is larger for the FSD system than for the linear system. This results in a higher resonance peak around 1.7 [Hz]. When less working space is available – $S_{\max} = 0.02$, $S_{\max} = 0.03$ [m] – the decrease of variance gain at high frequencies due to the FSD system seems to have more influence on the ride index. This causes the FSD suspensions with $\Gamma = 0.5$ and $\Gamma = 50$ to perform slightly better than the benchmark system.

Note that the optimal FSD cabin suspension system for the quarter-truck model is not discussed here, since $\Gamma$ was no design variable in the FSD optimization problems. However the goal of this investigation is not to design the optimal FSD suspension system, but to examine the potential of using the FSD concept in a cabin suspension, to improve the driver comfort. This potential appears to be low, based on the resulting configurations for different values of $\Gamma$. Also, from a physical point of view the limited potential of an FSD damper in this system is understandable. The FSD concept reduces the transfer of vibrations for higher frequencies, while in the quarter-truck system the low frequency modes are more important when regarding the driver comfort than in the quarter-car system (also recall Figure 4.2).

### 4.7 Stroke dependent damping

Another passive suspension concept with the potential to improve the driver comfort is stroke dependent damping. In [87] this concept is used to reduce the suspension deflection at the rear axle of a truck, while maintaining driver comfort. In this research the concept is used to increase the driver comfort, while maintaining a certain maximum suspension working space. Also in this case the goal is not to design the optimal stroke dependent damping suspension, but to assess the potential of such a system to increase driver comfort. Therefore the investigated characteristics and the modeling of the stroke dependent damper are kept relatively simple in this section.
The force of the stroke dependent damper $F_{\text{SDD}}$ is given by

$$F_{\text{SDD}} = D_{\text{SDD}} \dot{x}_p \cdot \text{sat}_{[0,\delta]}(\beta|x_p|^\gamma + 1),$$

(4.18)

where $\text{sat}_{[0,\delta]}(\cdot)$ is a saturation operator and $\delta$ is the value of the function $\beta|x_p|^\gamma + 1$ at $S_{\text{max}}$. $D_{\text{SDD}}$ is a damping constant which, when multiplied by the piston velocity $\dot{x}_p$, determines the damping force around $x_p = 0$. The function $\beta|x_p|^\gamma + 1$ describes the factor the force $D_{\text{SDD}} \dot{x}_p$ is multiplied with, as a function of the piston position $x_p$. In this way the damper force is increased as the suspension moves towards the end of its working space. The parameters $\beta$ and $\gamma$ are chosen in such a way that three different shapes of the function $\beta|x_p|^\gamma + 1$ are investigated, shown in Figure 4.13. For these functions $\delta = 3$ is chosen, which means that the force at $x_p = -S_{\text{max}}$ and $x_p = S_{\text{max}}$ is three times the force at $x_p = 0$, for the same piston velocity $\dot{x}_p$. In Table 4.3 the values of $\beta$ for different values of $\gamma$ and $S_{\text{max}}$ are given.

<table>
<thead>
<tr>
<th>$S_{\text{max}}$ [m]</th>
<th>$\gamma$</th>
<th>0.02</th>
<th>0.03</th>
<th>0.04</th>
<th>0.05</th>
<th>0.06</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td></td>
<td>0.5000</td>
<td>0.2222</td>
<td>0.1250</td>
<td>0.0800</td>
<td>0.0556</td>
</tr>
<tr>
<td>3</td>
<td></td>
<td>25.000</td>
<td>7.4074</td>
<td>3.1250</td>
<td>1.6000</td>
<td>0.9259</td>
</tr>
<tr>
<td>4</td>
<td></td>
<td>1250.0</td>
<td>246.91</td>
<td>78.125</td>
<td>32.000</td>
<td>15.432</td>
</tr>
</tbody>
</table>

The optimization of both the linear and FSD quarter-truck cabin suspension systems have shown that the optimal value for the stiffness $k_c$ always lies on the lower limit $k_c = 4 \cdot 10^4$ [N/m]. Therefore this is chosen as a fixed value for $k_c$ when solving optimization problem (4.12) for the stroke dependent damping suspension. This makes the optimization procedure simpler and faster. Since the optimization is done for three fixed shapes of the function $\beta|x_p|^\gamma + 1$, the only design variable in this case is $D_{\text{SDD}}$. This variable is limited by $1 \cdot 10^2 \leq D_{\text{SDD}} \leq 1 \cdot 10^6$ [Ns/m]. Five different starting values for $D_{\text{SDD}}$ are chosen to avoid local minima in the solution of the optimization problem. Figures 4.14 and 4.15 show the results of the optimization. In Figure 4.14 the minimum objective function value at each value of $S_{\text{max}}$ is shown for the different shapes of $\beta|x_p|^\gamma + 1$, along with the benchmark system objective function values. In Figure 4.15 the optimal value for $D_{\text{SDD}}$ at each value of $S_{\text{max}}$ is shown for the different shapes of $\beta|x_p|^\gamma + 1$.

It can be seen that the difference in performance for the various shapes of $\beta|x_p|^\gamma + 1$ is very small. For the smaller values of $S_{\text{max}}$ can be seen that the system with $\gamma = 2$ performs best and the performance of the system with $\gamma = 4$ is the worst.
In the above optimization procedure the values for $\beta$ and $\gamma$ are chosen such that $\delta = 3 \, [-]$, as can be seen in Figure 4.13. Figure 4.16 shows the minimum objective function values for $\gamma = 2$ and $\beta$ is chosen such that the multiplication factor $\delta$ varies between 2 and 10 for $S_{max} = 0.04 \, [m]$. It can be seen that this relation has a minimum of 0.935 around $\delta = 3.75$, which is therefore the optimal multiplication factor for a stroke dependent damping system with $\gamma = 2$ and $S_{max} = 0.04 \, [m]$. In this case an improvement in driver comfort of 5.1% with respect to the benchmark system can be achieved.

As mentioned before, the results plotted in Figure 4.14 show that there is little difference in performance for the systems with a different value for $\gamma$. Therefore, it can be assumed that the achievable driver comfort improvement for the stroke dependent damping system investigated here lies around 5%

4.8 Cabin suspension with inerter

The inerter concept is discussed in Section 2.3.2. Herein, it is mentioned that the suspension configuration as shown in Figure 2.5 is preferred in case of minimization of the sprung mass acceleration, when using the inerter in a vehicle primary suspension. Since information of a comparable research for secondary suspension system is not available, the same configuration is used in the current section
to investigate the potential of using an inerter in a truck cabin suspension. The optimization problem (4.12) is therefore solved for three design variables: damping constant $d$, inertance $b$ and centering spring stiffness $k_1$. As is the case in the optimization of the stroke dependent damping suspension, for the suspension stiffness a fixed value of $k = 4 \cdot 10^4$ [N/m] is chosen. The design variables are limited by $1 \cdot 10^2 \leq k_1 \leq 1 \cdot 10^6$ [N/m], $1 \cdot 10^2 \leq d \leq 1 \cdot 10^6$ [Ns/m] and $1 \cdot 10^1 \leq b \leq 1 \cdot 10^6$ [kg].

To get an indication of a suitable set of starting values, the design variables are first chosen in a way that the frequency response of the quarter-truck model including the suspension with inerter approximates the frequency response of the identified quarter-truck model. In Figures 4.17 and 4.18 the variance gain of the identified quarter-truck model and the model with the inerter is shown, from vertical road velocity to vertical cabin acceleration and secondary suspension deflection respectively. Here the design variables have the values $k_1 = 1 \cdot 10^4$ [N/m], $d = 2 \cdot 10^4$ [Ns/m] and $b = 1 \cdot 10^4$ [kg]. This set is used as a set of starting values for solving (4.12), along with nine variations on this set, to avoid local minima in the solution.

In Figure 4.19 the results of the optimization are shown. This figure shows the minimum values of the objective function as a function of the maximum allowable absolute suspension deflection $S_{\text{max}}$. It can be seen that at $S_{\text{max}} = 0.04$ [m], the cabin suspension with inerter has an improvement in comfort of 2.1% with respect to the identified quarter-truck model. This is only 0.7% with respect to the benchmark. The largest improvement can be found at $S_{\text{max}} = 0.02$ [m], where the improvement
in comfort is 0.9% with respect to the benchmark. At $S_{\text{max}} = 0.06$ [m] the cab suspension with inerter performance is worse than the benchmark with a 0.5% decrease in comfort. It can be concluded that applying the inerter concept in a truck cabin suspension, in the configuration used here, shows low potential for improving the driver comfort.

4.9 Summary

For the investigation of the vertical dynamics of a truck cabin, the 4 DOF quarter-truck model is presented in this chapter. In addition to the well-known 2 DOF quarter-car model, this model includes a secondary suspension and a mass representing the cabin, as well as a separate mass representing the truck engine. The input of this model is the disturbance signal on the velocity of the vertical tire deflection induced by the road. Since nonlinear suspension concepts are investigated, the model has to calculate the vehicle heave dynamics in the time-domain. A time-domain description for the input signal is therefore also derived in this chapter.

When optimizing the cabin suspension, the goal is to maximize driver comfort while keeping the suspension deflection within limits. In this chapter an objective function and constraints are selected which are representative for this optimization problem. The objective function to be minimized is the normalized ISO weighted vertical cab acceleration for two different road input signals. The constraint function limits the absolute suspension deflection according to the chosen available working-space.

The optimization problem is used to optimize the stiffness and damping constant of the cab suspension in the linear quarter-truck model. The performance of this linear passive cab suspension is used as a benchmark for the other suspension systems to be evaluated. In this chapter the passive suspension systems including respectively Frequency Selective Damping (FSD), Stroke Dependent Damping (SDD) and the inerter are evaluated.

It should be mentioned that not all parameters of the FSD and SDD concepts are used as design variables in the optimization, and only one configuration of a suspension system including an inerter is considered. Nevertheless the results of the optimization give an indication of the potential of these systems to increase the driver comfort. This potential appears to be relatively low, while the largest improvement, achieved with the SDD concept is approximately 5% with respect to the benchmark system, which is just noticeable by the driver.
Chapter 5

Semi-active suspension systems

5.1 Introduction

To fully use the potential of a semi-active suspension system, a control strategy for this suspension has to be designed. This control strategy should ensure that the driver comfort is optimal, while regarding the constraints on the available working space. Controllers which are able to accomplish this objective by actuating the variable rate damper in a semi-active cabin suspension are investigated in this thesis.

In Chapter 3 several control strategies for semi-active vehicle primary suspensions are discussed. It is worthwhile to investigate to what degree these strategies can increase driver comfort when used in a semi-active cabin suspension, since little information on the control of secondary semi-active suspension systems is available. Also, a controller designed for the particular semi-active cabin suspension system in a quarter-truck model should be investigated. To be able to compare these control strategies, their control parameters should be optimized to the same objective function and constraints. Therefore, they all should be applied to the same vehicle model. In this thesis, where the focus lies on the driver comfort improvement in the heave direction, this vehicle model will be the quarter-truck model, including a variable rate damper in the cabin suspension.

In this chapter, first the variable rate damper model used in this thesis is introduced. Section 2.4 shows that a variety of more and less complex models for variable rate dampers are available in the literature. The Bouc-Wen model is the state-of-the-art, however it is too complex to use in this research. The selected damper model is kept relatively simple. This keeps the simulation time within limits, while the model describes the variable rate damper accurately enough for controller design. Complex damper characteristics like hysteresis are not included, since information on these subjects is not available for variable rate dampers used in cab suspensions. Furthermore, the influence of these characteristics on the performance of the semi-active suspension system remain unclear.

Next, the 2-state skyhook and Acceleration Driven Damping (ADD) control strategies from Chapter 3 are compared to a newly developed approach. This control strategy for the semi-active cabin suspension is named the LQ/LPV-approach and is based on the $H_{\infty}$/LPV-approach discussed in Section 3.3. A controller is designed for the cabin suspension of the quarter-truck model using LQ-optimal control theory. Since the actuator is a semi-active damper in this system, some constraints exist on the control force. These constraints are taken into account during the controller design process by means of LPV theory. It is mentioned in Chapter 3 that this is preferred to “clipping” a control strategy for a fully active system, which does not guarantee optimal performance for a semi-active system. As the LQ/LPV strategy requires full state feedback, a state estimator for the quarter-truck model is also derived.

After the comparison of the three control strategies, the possibility of using road adaptive parameter scheduling in each of these strategies is briefly discussed. Also the influence of the dynamics of the variable rate damper is investigated.

It is shown that the LQ/LPV strategy performs significantly better compared to the 2-state skyhook and ADD strategies. These strategies show hardly any improvement in driver comfort, only $1 - 4\%$ compared to the benchmark system. This conclusion is drawn after solving the optimization problem.
from Section 4.4 for a maximum allowable suspension deflection of 0.04 [m] and comparing the results in terms of driver comfort. It is chosen to solve the problem only for this maximum allowable suspension deflection in this chapter, since the effect of changing the available working space on the performance of a suspension system is already shown in Chapter 4.

This chapter is organized as follows. The variable rate damper model used for the design and evaluation of the different control strategies is presented in Section 5.2. In Section 5.3 the 2-state skyhook control strategy is discussed and optimized to the chosen criteria and constraints. The same is done for the ADD strategy in Section 5.4. In Section 5.5 the LQ/LPV controller is derived, optimized and compared to the other two strategies. After that road adaptive control is discussed in Section 5.6. Finally, in Section 5.7 a state estimator for the quarter-truck model with LQ/LPV control is designed.

5.2 Semi-active damper model

A semi-active damper model is needed for the design and evaluation of different semi-active suspension control strategies. Different possibilities on modeling semi-active dampers (both variable orifice and ER/MR variants) are discussed in Section 2.4. Herein is concluded that both can be approximated by basic linear models which describe the actuator limitations like force limitation, the passivity constraint and limited bandwidth. For the description of the nonlinear behavior of the semi-active damper, i.e. nonlinear force-velocity-voltage mappings and (frequency-dependent) hysteresis, more complex models are necessary. However, representative parameter sets for semi-active dampers in a truck cabin suspension are not available for these models. Therefore, in this research is chosen to use a relatively simple model.

The force $F_{\text{SA}}$ delivered by the semi-active damper is assumed to be a linear combination of the minimum and maximum damper force as function of the piston velocity, dependent on the normalized input voltage of the damper $V$. This means that $F_{\text{SA}}$ can be written as

$$F_{\text{SA}} = V \cdot F_{\text{max}} + (1 - V) \cdot F_{\text{min}},$$

with $F_{\text{min}} = F_{\text{min}}(\dot{x}_p)$ and $F_{\text{max}} = F_{\text{max}}(\dot{x}_p)$ being respectively the minimum and maximum damper force as a function of the piston velocity $\dot{x}_p$. The damper dynamics are modeled as a first order filter, as is done in [76] and [50], using

$$\dot{V} = -\eta (V - V^{\text{ref}}),$$

where $V^{\text{ref}}$ is the normalized reference voltage applied to the semi-active damper. Following [50] $\eta = 190$ [s$^{-1}$] is chosen. In Section 2.4 it is mentioned that the values of the parameters used for the description of the variable rate damper dynamics vary among different references. However, in Appendix C it is shown that varying these parameters has little effect on the performance of the suspension system.

A typical shape of the velocity-force characteristics of both ER/MR dampers, [50], [44], [13] and variable orifice dampers [40] is shown in Figure 5.1. The characteristics shown in this figure are described by

$$\begin{cases} F_{\text{min}} = d_1 \dot{x}_p & \text{if } -\alpha < \dot{x}_p < \alpha \\ F_{\text{max}} = d_2 \dot{x}_p + D & \text{if } \dot{x}_p \geq \alpha \\ F_{\text{max}} = d_2 \dot{x}_p - D & \text{if } \dot{x}_p \leq -\alpha \end{cases},$$

where $d_1 = 1 \cdot 10^5$ [Ns/m], $d_2 = 4 \cdot 10^5$ [Ns/m], $D = 6000$ [N] and $\alpha = 0.0625$ [m/s]. Using these parameters the $F_{\text{max}}$ characteristic of the semi-active damper lies well above the identified passive linear damper characteristic for $-0.5 \leq \dot{x}_p \leq 0.5$ [m/s]. The identified passive linear damper characteristic is also shown in Figure 5.1. Besides the characteristics for the minimum and maximum damper force this figure shows how the damper characteristic is changing for different input voltages $V$. This is indicated by the grey lines.
The inputs of this semi-active damper model are the piston velocity $\dot{x}_p$ and the reference voltage $V^{ref}$. However, controllers for semi-active suspension systems are often designed to calculate a reference force, which is to be reproduced by the semi-active damper, instead of a reference voltage which can be directly applied to the damper. In this case the semi-active damper needs to be equipped with an internal controller, which is able to calculate the voltage that is required to deliver the reference force, given the piston velocity at that instant. In [55] different control algorithms to perform this task are compared. Here, the most effective algorithm is found to be

$$
\begin{cases}
V^{ref} = V_{\text{min}} & \text{if } G(F_{\text{ref}}^{SA} - B \cdot F_{\text{SA}}) \text{ sgn}(F_{\text{SA}}) < V_{\text{min}} \\
V^{ref} = V_{\text{max}} & \text{if } G(F_{\text{ref}}^{SA} - B \cdot F_{\text{SA}}) \text{ sgn}(F_{\text{SA}}) > V_{\text{max}} \\
V^{ref} = G(F_{\text{ref}}^{SA} - B \cdot F_{\text{SA}}) \text{ sgn}(F_{\text{SA}}) & \text{if } V_{\text{min}} \leq G(F_{\text{ref}}^{SA} - B \cdot F_{\text{SA}}) \text{ sgn}(F_{\text{SA}}) \leq V_{\text{max}}
\end{cases},
$$

where sgn(·) is the sign-operator, and $V_{\text{min}}$ and $V_{\text{max}}$ are the minimum and maximum voltage respectively. In [55] is chosen for $G = 0.021$ and $B = 1$.

For the damper model described by (5.1) – (5.3) these values also work properly. This can be seen in Figures 5.2 and 5.3. Here the results are shown of a simulation with the quarter-truck model, using the semi-active damper model controlled by the 2-state skyhook algorithm (3.10) mentioned in section 3.2. In this case the values for the minimum and maximum damping coefficients are $c_{\text{min}} = 7 \cdot 10^3$ [Ns/m] and $c_{\text{max}} = 1.7 \cdot 10^4$ [Ns/m]. Figures 5.2 and 5.3 show both the reference force and the actual damping force as a function of time and piston velocity respectively. The reference force is calculated by the 2-state skyhook algorithm in a simulation where the quarter-truck model drives over a Type II road. The actual force is the output of the damper model, using the internal controller of (5.4) It can be seen that the actual damper force delivered by the semi-active damper model is able to follow the
calculated reference force closely. Only at time instant \( t = 3.1 \) [s] the actual force lies slightly above the reference force as can be seen in Figure 5.2. This corresponds with the point at 0.054 [m/s] piston velocity in Figure 5.3 where the cross representing the actual force and the circle representing the reference force do not coincide.

Since the internal controller can track the reference force very well, it is chosen to simplify the combination of the model described by equations (5.1) – (5.3) and the internal control algorithm (5.4), in case the semi-active controller prescribes a reference force instead of a reference voltage. The semi-active damper model is then written as

\[
\dot{F}_{SA} = -\eta (F_{SA} - F_{SA}^{ref}),
\]

with the reference force \( F_{SA}^{ref} \) as input and the actual damper force \( F_{SA} \) as output. Note that this is only allowed as long as the reference force prescribed by the semi-active suspension controllers satisfies the passivity constraint and other force limitations of the variable rate damper. The references forces prescribed by the control algorithms investigated in this chapter are limited to these constraints within the algorithms themselves.

5.3 2-state skyhook control

The 2-state skyhook control algorithm is the first of the three semi-active control algorithms evaluated for a semi-active cab suspension system in this chapter. It is a so-called clipped control strategy. This means it is a semi-active approximation of the skyhook control strategy, which is the LQ-optimal control strategy for a 1 DOF, fully active suspension system (see Section 3.2). The skyhook control strategy and its semi-active approximations are discussed extensively in section 3.2.

For the quarter-truck model, the 2-state skyhook control algorithm is given by

\[
F_{SA}^{ref} = \begin{cases} 
F_{SA}^{max} & \text{if } \dot{z}_c(\dot{z}_c - \dot{z}_f) > 0 \\
F_{SA}^{min} & \text{if } \dot{z}_c(\dot{z}_c - \dot{z}_f) \leq 0
\end{cases}
\]

where \( F_{SA}^{ref} \) is the reference force, which is to be delivered by the semi-active damper and \( \dot{z}_c \) and \( \dot{z}_f \) are the vertical velocity of the cabin and the frame of the quarter-truck model respectively. \( F_{SA}^{max} \) and \( F_{SA}^{min} \) are a maximum and a minimum damper force respectively.

Note that in [51], where the semi-active skyhook control algorithm is first described, \( F_{SA}^{min} = 0 \). However, the absence of the damping force at some instances means in practice that the working space requirements that hold for the passive system (in this thesis \( |S_{max}| = 0.04 \) [m] for \( k_c = 4 \cdot 10^4 \) [N/m]...
and \(d_c = 1.329 \cdot 10^4 [\text{Ns/m}]\) can not be met by the semi-active system without increasing the cabin stiffness \(k_c\). In Chapter 4 it is shown that the stiffness should be kept as low as possible for optimal driver comfort. Therefore, in this thesis \(F_{\text{SA}}^{\text{max}}\) and \(F_{\text{SA}}^{\text{min}}\) will be optimized such that the working space requirements are met while the stiffness \(k_c = 4 \cdot 10^4 [\text{N/m}]\) remains unchanged. This means that \(F_{\text{SA}}^{\text{min}} > 0\) and the semi-active damper controlled by (5.6) can be seen as a passive damper with velocity-force characteristic \(F_{\text{SA}}^{\text{min}}\) with a semi-active skyhook damper according to [51] in parallel.

When regarding the damper model presented in Section 5.2, different possibilities exist for finding the optimal values for \(F_{\text{SA}}^{\text{max}}\) and \(F_{\text{SA}}^{\text{min}}\). One possibility is to search for the optimal values of constant input voltages \(V_{\text{SA}}^{\text{min}}\) and \(V_{\text{SA}}^{\text{max}}\), which describe \(F_{\text{SA}}^{\text{min}}\) and \(F_{\text{SA}}^{\text{max}}\) respectively. This results in \(F_{\text{SA}}^{\text{min}} = F_{\text{SA}}(\dot{z}_c - \dot{z}_f, V_{\text{SA}}^{\text{min}})\) and \(F_{\text{SA}}^{\text{max}} = F_{\text{SA}}(\dot{z}_c - \dot{z}_f, V_{\text{SA}}^{\text{max}})\). This means that the nonlinear behavior of the semi-active damper with respect to the piston velocity is preserved. This can be seen in Figure 5.4, where a typical velocity-force diagram for the semi-active damper model is shown in case it is controlled with the strategy using \(V_{\text{SA}}^{\text{min}}\) and \(V_{\text{SA}}^{\text{max}}\). This strategy will be referred to as the optimized-V strategy.

However, it is uncertain whether this particular nonlinear characteristic is desirable. It is known that the dampers in a passive cab suspension of a truck can be well approximated by a linear damper model with a constant damping coefficient. Therefore another possibility for the optimization of the 2-state skyhook control strategy is also considered. In this case is is searched for the optimal values of \(d_{\text{SA}}^{\text{min}}\) and \(d_{\text{SA}}^{\text{max}}\), which yield \(F_{\text{SA}}^{\text{min}} = d_{\text{SA}}^{\text{min}} \cdot (\dot{z}_c - \dot{z}_f)\) and \(F_{\text{SA}}^{\text{max}} = d_{\text{SA}}^{\text{max}} \cdot (\dot{z}_c - \dot{z}_f)\) respectively. The 2-state skyhook algorithm now selects a reference force which is to be tracked by an internal controller which applies the necessary input voltage to the semi-active damper. A typical velocity-force diagram for the semi-active damper model in case it is controlled with the strategy using \(d_{\text{SA}}^{\text{min}}\) and \(d_{\text{SA}}^{\text{max}}\) is shown in Figure 5.5. This strategy will be referred to as the optimized-\(d\) strategy.

As mentioned before, the parameters for both strategies are found by solving problem (4.12) for \(|S_{\text{max}}| = 0.04 [\text{m}]\). For the optimized-V strategy this results in a minimal value of the objective function of \(J = 1.3399\) for \(V_{\text{SA}}^{\text{min}} = 0.4175 [\text{V}]\) and \(V_{\text{SA}}^{\text{max}} = 0.7625 [\text{V}]\). Six different sets of starting values for the design variables are used.

The optimization of the optimized-\(d\) strategy results in a minimal objective function value of \(J = 0.9638\) for \(d_{\text{SA}}^{\text{min}} = 0.975 \cdot 10^4 [\text{Ns/m}]\) and \(d_{\text{SA}}^{\text{max}} = 1.640 \cdot 10^4 [\text{Ns/m}]\). In this case, (5.6) is rewritten into

\[
F_{\text{SA}}^{\text{ref}} = \begin{cases} 
  d_{\text{SA}}^{\text{max}} \cdot (\dot{z}_c - \dot{z}_f) & \text{if } \dot{z}_c(\dot{z}_c - \dot{z}_f) > 0 \\
  d_{\text{SA}}^{\text{min}} \cdot (\dot{z}_c - \dot{z}_f) & \text{if } \dot{z}_c(\dot{z}_c - \dot{z}_f) \leq 0 
\end{cases} 
\]

(5.7)

and (5.5) is used for the damper model. Six different sets of starting values for \(d_{\text{SA}}^{\text{min}}\) and \(d_{\text{SA}}^{\text{max}}\) are used to solve the optimization problem. Five of these starting values result in values for the design variables which lie within 10% of each other.

![Figure 5.4: Semi-active damper velocity-force diagram for controller using \(V_{\text{SA}}^{\text{min}}\) and \(V_{\text{SA}}^{\text{max}}\).](image1)

![Figure 5.5: Semi-active damper velocity-force diagram for controller using \(d_{\text{SA}}^{\text{min}}\) and \(d_{\text{SA}}^{\text{max}}\).](image2)
In Figure 5.6 the suspension deflection for both 2-state skyhook strategies as a function of time, is shown for the Type II road conditions and discrete obstacle input signals respectively. It can be seen that the available working space is fully used by the optimized-\(d\) strategy, unlike the optimized-\(V\) strategy. For this latter strategy the constraint \(\max |z_c - z_f|_{\text{bump}} \leq |S_{\text{max}}|\) is active, as can be seen in Figure 5.6b. The discrete obstacle input requires a high damper force at a high piston velocity, for the optimized-\(V\) strategy this automatically results in high damper forces at lower piston velocities due to the nonlinear characteristics. These higher damper forces result in a decrease the comfort with respect to the optimized-\(d\) strategy, which is shown in Figure 5.7 were the vertical cabin acceleration is shown as a function of time for Type II road input conditions. When the discrete obstacle constraint is omitted and the optimization problem is solved using only the constraint at Type II road disturbance input conditions, the objective function value of the optimized-\(V\) strategy is still larger than the value for the optimized-\(d\) strategy. It is therefore decided to use the optimized-\(d\) strategy as the 2-state skyhook control strategy in this research.

With a minimum objective function value of \(J = 0.9638\) a semi-active cabin suspension system using this 2-state skyhook control strategy results in a 3.6% improvement in driver comfort. This is very low for a semi-active system, since the passive stroke dependent damping system of Section 4.7 can achieve an improvement of approximately 5% and does not require any power input.

### 5.4 ADD control

The ADD, or Acceleration Driven Damping, control algorithm is extensively discussed in Section 3.2. It is an LQ-optimal control strategy derived for a semi-active 2 DOF quarter-car system. For comparison, the 2-state skyhook strategy is an approximation of an LQ-optimal control strategy, derived for a fully active, 1 DOF system. Due to the fact that the ADD algorithm is explicitly derived for a semi-
active suspension system, it is worthwhile to investigate its performance in a semi-active truck cabin suspension.

The original ADD algorithm is given by (3.16). However, during the derivation of the ADD control strategy, working space constraints were not taken into account. Still, it is desired to keep the maximum suspension displacement within the desired range of $|S_{max}| = 0.04$ [m]. Therefore the algorithm is altered such that a passive damper force acts parallel to the original control algorithm, analogous to what is done in the 2-state skyhook algorithm in the previous section. The altered ADD algorithm can now be written as

$$ F_{SA} = \begin{cases} F_{max}^SA & \text{if } \ddot{z}_c(\dot{z}_c - \dot{z}_f) \geq 0 \\ F_{min}^SA & \text{if } \ddot{z}_c(\dot{z}_c - \dot{z}_f) < 0 \end{cases} $$(5.8)

where $F_{SA}$ is the force delivered by the semi-active damper and $\ddot{z}_c, \dot{z}_c$ and $\dot{z}_f$ are the vertical velocity of the cabin and the frame of the quarter-truck model respectively. $F_{max}^SA$ and $F_{min}^SA$ are a maximum and a minimum damper force respectively.

For the ADD strategy, simulations also show that a linear velocity-force damper characteristic is preferable to the nonlinear characteristic of the semi-active damper model. Therefore the design variables are $d_{min}$ and $d_{max}$, when optimizing the ADD controlled semi-active suspension. These variables describe the minimum and maximum damper forces as $F_{min} = d_{min} \cdot (\dot{z}_c - \dot{z}_f)$ and $F_{max} = d_{max} \cdot (\dot{z}_c - \dot{z}_f)$ respectively.

When optimizing the ADD controlled semi-active cab suspension by solving (4.12) for $|S_{max}| = 0.04$ [m], a minimum objective function value of $J = 0.9894$ [-]. The corresponding optimal values for the design variables are $d_{min} = 1.298 \cdot 10^4$ [Ns/m] and $d_{max} = 2.6919 \cdot 10^7$ [Ns/m]. For the optimization six different sets of starting values for the design variables are used and the design variables are limited by $1 \cdot 10^2 \leq d_{min}, d_{max} \leq 1 \cdot 10^6$ [Ns/m]. The resulting design variable values of four of the six starting value sets lie within 10% of each other.

The increase in comfort that can be achieved here is only 1.1% compared to the benchmark model. This can be explained by the fact that the minimum damping constant $d_{min}$ is almost equal to the optimal linear passive damping constant calculated in Section 4.5, which is $d_c = 1.329 \cdot 10^4$ [Ns/m]. The maximum damping coefficient on the other hand is approximately 20 times higher.

In [75] the 2-state skyhook strategy is compared to the ADD strategy for a 2 DOF quarter-car model. Here is shown that the transfer from the road profile to the sprung mass is smaller for the skyhook strategy than for the ADD strategy, at low frequencies. At higher frequencies this transfer is smaller for the ADD strategy. Based on this observation, a controller is proposed which uses the skyhook strategy in the low-frequency range and the ADD strategy in the higher-frequency range.

Figure 5.8 shows the variance gain from vertical velocity of the road disturbance input $v_r$ to the vertical cabin acceleration for the quarter-truck model. The grey line indicates the semi-active system

![Figure 5.8: Variance gain from vertical velocity of the road disturbance input $v_r$ to the vertical cabin acceleration $a_c$.](image-url)
controlled by the ADD strategy. The black dashed line indicates the system with the 2-state skyhook strategy of the previous section. It can be seen that the variance gain for the skyhook controlled system is lower than or equal to the ADD controlled system for all frequencies. A mixed-skyhook-ADD strategy as proposed in [75] is therefore infeasible. Another conclusion, which can also be drawn from the minimum objective function values, is that the 2-state skyhook strategy is preferable to the ADD strategy for a quarter-truck system with \( |S_{\text{max}}| = 0.04 \text{[m]} \).

### 5.5 LQ/LPV control

The LQ/LPV control strategy is inspired by the \( \mathcal{H}_\infty/LPV \) strategy, which is discussed in detail in Section 3.3. In this strategy, LPV theory is used to describe the nonlinear closed loop cab suspension system as a linear system, through the introduction of a parameter which captures the nonlinearity of the system. The closed loop cabin suspension system includes a semi-active damper and a control strategy. Using the LPV theory, linear control theory can be applied to the system for fixed values of the introduced parameter. In this section, this is used to design a controller for the nonlinear semi-active cab suspension system based on LQ-optimal control theory.

In the LQ/LPV concept the force delivered by the semi-active damper consists of a linear, passive part, dependent on damping coefficient \( d_c \), and a part which is calculated by the controller. This means that the desired semi-active damper force \( F_{s\text{a}}^{\text{ref}} \) can be written as

\[
F_{s\text{a}}^{\text{ref}} = d_c(\dot{z}_c - \dot{z}_f) + F_c
\]

Here \((\dot{z}_c - \dot{z}_f)\) is the relative vertical velocity between the cabin and the frame and \( F_c \) is the additional force calculated by the controller. For \( d_c \) the optimal damping constant found in section 4.5 is an appropriate choice, since this is the minimum constant value for \( d_c \) which guarantees that the constraints on the suspension working space are satisfied at each time instant in the optimization. This force \( F_c \) can be calculated by an LQ-optimal controller for a fixed value of the LPV parameter. The performance variables for this controller are the relative vertical displacement between cabin and frame \( z_f - z_c \) and the ISO weighted vertical cabin acceleration \( \ddot{z}_{c\text{ISO}} \).

In order to calculate the gains of the LQ-optimal controller, the quarter-truck model, including the ISO 2631 weighting on the cabin acceleration approximated by (5.10), is written as

\[
\begin{align*}
\dot{x} &= Ax + Bu + Gw \\
z &= Fx
\end{align*}
\]

where \( \mathbf{x} = [\mathbf{x}_q, \mathbf{x}_{ISO}]^T \) is the state-vector with quarter-truck states

\[
\mathbf{x}_q = [z_c - z_f, \; z_f - z_c, \; z_f - z_a, \; z_a - z_r, \; \dot{z}_c, \; \dot{z}_f, \; \dot{z}_a]^T
\]

and ISO weighting filter states

\[
\mathbf{x}_{ISO} = [x_{12}, \; x_{13}, \; x_{14}]^T
\]

The control input \( u \) is the force \( F_c \). Furthermore the disturbance \( w \) is the vertical velocity of the road surface \( \dot{z}_r \) which acts as a white-noise disturbance signal on the system and \( z = [z_c - z_f, \; \ddot{z}_{c\text{ISO}}]^T \) is the vector of performance outputs. The matrices of the state equation are given by

\[
\begin{align*}
A &= \begin{bmatrix} \mathbf{A}_q & 0 \\ B_{ISO}A_{\text{iso}}(5,:) & \mathbf{A}_{\text{iso}} \end{bmatrix}, \\
B &= \begin{bmatrix} B_q \\ B_{ISO}B_{\text{iso}}(5,:) \end{bmatrix}, \\
G &= \begin{bmatrix} G_q \\ B_{ISO}G_{\text{iso}}(5,:) \end{bmatrix},
\end{align*}
\]

where \( \mathbf{A}_q, B_q \) and \( G_q \) are given in Section 4.2 and

\[
\begin{align*}
\mathbf{A}_{\text{iso}} &= \begin{bmatrix} -80 & -2264 & -7172 & -21196 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}, \\
B_{ISO} &= \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix}^T.
\end{align*}
\]

The notation \( \mathbf{A}_q(5,:) \) indicates the 5th row of the matrix \( \mathbf{A}_q \). Note that the passive part of the force delivered by the semi-active damper according to (5.9) is captured by the parameter \( d_c \) in \( \mathbf{A}_q \), while
the part calculated by the controller is represented by \( u \), as is mentioned before. The ISO weighted vertical cabin acceleration \( z_{\text{ISO}} \) is given by \( z_{\text{ISO}} = C_{\text{ISO}} x_{\text{ISO}} \), so
\[
E = \begin{bmatrix} 1 & 0_{[1 \times 1]} \\ 0_{[1 \times 8]} & C_{\text{ISO}} \end{bmatrix},
\]
where \( 0_{[i \times j]} \) is a zero matrix with size \([i \times j]\) and
\[
C_{\text{ISO}} = \begin{bmatrix} 81.89 & 796.6 & 1937 & 0.1446 \end{bmatrix}.
\]

An LQ-optimal controller for this system can be found by solving the algebraic Riccati equation
\[
A^T X + X A - X B R^{-1} B^T X + Q = 0,
\]
with
\[
Q = E^T \begin{bmatrix} q_1 & 0 \\ 0 & q_2 \end{bmatrix} E,
\]
where \( q_1 \) is the weight on the suspension working space, scaled by the maximum working space \( 0.04 \) [m] and \( q_2 \) is the weight on the frequency weighted cab acceleration, scaled by \( 1 \) [m/s²]. Furthermore, \( R \) is the weight on the control effort. The controller gain matrix is now given by
\[
K = R^{-1} B^T X.
\]
and the additional control force \( F_c \) is calculated by
\[
F_c = u = -K x.
\]

As mentioned before, an LPV parameter is introduced in the closed loop system to capture the nonlinear characteristics of this system. In this case this nonlinearity consists of the limited maximum and minimum force and the passivity constraint of the semi-active damper. This means that not every force \( F_{\text{sa}}^{\text{ref}} = d_c(\dot{z}_c - \dot{z}_f) + F_c \) can be delivered by this damper. Therefore the parameter \( \rho \) is introduced in the weight on the control effort \( R \) through
\[
R(\rho) = 1 \cdot 10^{-3} + \rho \cdot 10^3.
\]

This means that for \( \rho = 0 \), the weight on the control effort is very small, while for \( \rho > 0 \) this weight increases proportionally with \( \rho \). For the control force it holds that \( F_c \to 0 \) for large values of \( R \), causing the semi-active suspension system to act as a passive system with \( F_{\text{sa}}^{\text{ref}} = d_c(\dot{z}_c - \dot{z}_f) \). Using (5.10), the closed-loop suspension system including the road disturbance \( w \) can be written as
\[
\dot{x} = [A - BK(\rho) \ G] \begin{bmatrix} x \\ w \end{bmatrix}.
\]

For \( \rho \in [\rho_{\text{min}}, \rho_{\text{max}}] \), this results in an infinite number of linear systems, with an infinite number of LQ-optimal controllers \( K(\rho) \). According to [68] this problem can be reduced by using the so-called polytopic approach, leading to
\[
K(\rho) = \frac{|\rho - \rho_{\text{max}}|}{\rho_{\text{max}} - \rho_{\text{min}}} K(\rho_{\text{min}}) + \frac{|\rho - \rho_{\text{min}}|}{\rho_{\text{max}} - \rho_{\text{min}}} K(\rho_{\text{max}}),
\]
where \( K(\rho_{\text{min}}) \) is the LQ-optimal controller when \( \rho = \rho_{\text{min}} \) and \( K(\rho_{\text{max}}) \) is the LQ-optimal controller when \( \rho = \rho_{\text{max}} \).

The LQ/LPV controller verifies whether the force \( F_{\text{sa}}^{\text{ref}} \) calculated for \( K(\rho_{\text{min}}) \) lies within the static boundaries of the semi-active damper. The parameter \( \epsilon \) is the difference between the requested force \( F_{\text{sa}}^{\text{ref}} \) and the closest achievable boundary force at the current piston velocity of the damper, when \( F_{\text{sa}}^{\text{ref}} \) lies outside the static boundaries. When \( F_{\text{sa}}^{\text{ref}} \) lies inside these boundaries \( \epsilon = 0 \). Figure 5.9 shows
the static force boundaries for the damper model used in this research and indicates the value that is assigned to \( \epsilon \) for every region.

Now for the parameter \( \rho \) is chosen

\[
\rho(\epsilon) = 10 \frac{\mu |\epsilon|^4}{|\epsilon|^4 + 1/\mu}.
\]

In [68], a value of \( \mu = 10^8 \) is regarded to be sufficiently large. Using this definition for \( \rho \), a sort of smooth switching behavior from an active to a passive linear suspension system is created at the limits of the semi-active damper force characteristics. Other possible formulations for \( \rho \) can be found in [67] and [5]. Figure 5.10 shows \( \rho \) as a function of \( \epsilon \). In this case \( \rho_{\text{min}} = 0 \) and \( \rho_{\text{max}} = 10 \). Here can be seen that \( \rho \) increases rapidly when \( F_{\text{ref}}^{SA} \) lies outside the boundaries of the damper model. This means that the controller \( K(\rho) \rightarrow K(\rho_{\text{max}}) \) when the force \( F_{\text{ref}}^{SA} \) calculated for \( K(\rho_{\text{min}}) \) lies outside the boundaries of the damper model, and the semi-active damper acts as a passive linear damper with damping coefficient \( d_c \). In case the force \( F_{\text{ref}}^{SA} \) calculated for \( K(\rho_{\text{min}}) \) lies inside the boundaries of the damper model, this force is used as reference force for the internal controller of the semi-active damper.

Analogous to the 2-state skyhook controller and the ADD controller, the LQ/LPV controller is also optimized according to (4.12) for \( |S_{\text{max}}| = 0.04 \) [m], using the design variable \( q_1 > 0 \) and \( q_2 > 0 \). Also, one additional constraint to limit the reference force calculated by the LQ/LPV controller is used:

\[
F_{\text{ref}}^{SA} \leq 8000.
\]

The choice for the weight on the control effort \( R \) is determined by (5.15). This leaves \( q_1 \) and \( q_2 \) from (5.12) as design variables. Without the additional constraint, these two weighting factors can become very large with respect to \( R \). This means that \( F_{\text{ref}}^{SA} \) can become so large that the reference force lies outside the static boundaries of the semi-active damper very often, and therefore \( \rho_{\text{max}} \rightarrow 10 \) which makes the semi-active damper act as a passive damper. This limits the possibility to make use of the variable rate characteristics of the semi-active damper.

Various other choices for the constraint function on \( F_{\text{ref}}^{SA} \) can be considered, e.g. (5.19) with limit force higher or lower than 8000 [N], or a constraint function dependent on the damper piston velocity. These different options are not investigated in this thesis however and further research on how the choice of this constraint function influences the maximum achievable driver comfort is required.

Six different sets of starting values for \( q_1 \) and \( q_2 \) are used to solve the optimization problem, using (5.5) for the damper model. Analogous to the optimization of the passive suspension systems, this is done to reduce the possibility of finding a local minimum in the objective function, while keeping
the computational time within limits. Different local minima are found, therefore the optimization is repeated with six new sets of starting values. These values are located around values of the minimum objective function value found in the first optimization. This results in a minimum objective function value of $J = 0.8757$ for $q_1 = 1.48 \cdot 10^4$ and $q_2 = 0.164 \cdot 10^4$. Figure 5.11 shows the suspension deflection stays within the prescribed limits at all times. Figure 5.12 shows the value of $\rho$ while driving under Type II road conditions. It can be seen that the parameter practically switches between the values 0 and 1, this means that the semi-active controller $K(\rho)$ switches between the two LQ-optimal controllers $K(\rho_{\text{min}})$ and $K(\rho_{\text{max}})$.

For the sake of comparison, the optimization problem is also solved for a fully active cab suspension system, controlled by an LQ-optimal state-feedback controller. The weight on the control effort is kept very low; $R = 1 \cdot 10^{-10}$. This means actuator limitations do not play a role here and the performance of the active system is an indication of the maximum achievable driver comfort (using LQ-optimal control), regarding the chosen working space constraint $|S_{\text{max}}| = 0.04$ [m]. Since $R$ is chosen very small, only the ratio between the working space weight $q_1$ and acceleration weight $q_2$ is relevant, if both are chosen large with respect to $R$. Therefore $q_1 = 1$ is chosen fixed and the only design variable in the optimization problem is $q_2$. For $q_2 = 0.0815$, the maximum absolute suspension deflection $|S_{\text{max}}| = 0.04$ [m] and $J = 0.7782$. This means that an LQ-controlled active suspension system can cause an improvement in driver comfort of approximately 22.2%. The improvement using a semi-active suspension system is 12.4% when using the LQ/LPV controller and a 3.6% improvement for the 2-state skyhook controlled system.

It can be concluded that the LQ/LPV control strategy is a good choice for controlling a semi-active truck cab suspension, when aiming for driver comfort improvement in heave direction without in-
creasing the suspension working space. The LQ/LPV controller performs significantly better than the well-known semi-active skyhook algorithm. Nevertheless, the use of an LQ-optimal controlled active suspension system can further increase driver comfort almost another 10%. However, when using an active system instead of a semi-active system will typically also result in a significantly higher energy consumption.

5.6 Road adaptive control

In the previous sections is shown that semi-active suspension systems can considerably improve driver comfort with respect to passive suspension systems. However, the semi-active control strategies discussed so far all have fixed parameters. These parameters are optimized to provide maximum driver comfort under both good and poor road input conditions. Meanwhile they assure that the suspension deflection is kept within the limits of the available working space, even under worst-case conditions. As a result, the semi-active suspension system does not fully use the available working space while driving under less demanding road input conditions. This can be seen in Figure 5.13 for a semi-active suspension system with an LQ/LPV controller with weights \( q_1 = 1.48 \cdot 10^4 \) and \( q_2 = 0.164 \cdot 10^4 \). Here the working space deflection is shown as a function of time for the road disturbance input signal classified as Type I in Section 4.4.

In chapter 4 it is already shown that the available (or in this case utilized) suspension working space has a large influence on the performance of the suspension system in terms of driver comfort. Also, Hrovat acknowledged already in 1984 that “the full advantage of active suspensions stems from possible adaptive tuning (or gain scheduling) of controller parameters, depending on the driving condition”[45]. A statement which is also plausible for semi-active controllers. Therefore, the effect of using adaptive parameters in the 2-state skyhook and LQ/LPV semi-active control strategies, discussed in Sections 5.3 and 5.5 respectively, is investigated here. The ADD control strategy is not further investigated since it is already shown in Section 5.4 that its potential is low compared to the other two semi-active strategies.

The controller parameters are assumed to be variable as a function of the road input conditions. How these conditions are measured or recognized is beyond the scope of this research. To find the optimal values of the controller parameters for different road input conditions, the optimization problem

\[
\min J = a_w(w_{r,i}(t), x) \\
\text{s.t. } \max |z_c - z_f|i \leq 0.04
\]

is solved for the different control strategies. The control strategies are optimized for the ride index \( a_w \) for five different road disturbance input signals \( w_{r,i}(t) \), with \( i = 1, 2, 3, 4, 5 \). These signals all have different degrees of road roughness \( S_{zr,i}(n_0) \). The values of \( S_{zr,i}(n_0) \) are linearly distributed between \( S_{zr1}(n_0) = 16 \cdot 10^{-6} [m^3] \) and \( S_{zr5}(n_0) = 112 \cdot 10^{-6} [m^3] \). Furthermore, \( \max |z_c - z_f|i \), is the maximum absolute suspension deflection as a result of \( w_{r,i}(t) \). This means that the controller parameters are optimized to the ride index, while the available working space of 0.04 [m] can be fully utilized at each of the five road disturbance input signals. Note that the suspension deflection when driving over a discrete obstacle is not regarded in the constraints, because this would greatly reduce the achievable performance of the adaptive suspension system under good road input conditions.

![Figure 5.13: Cabin suspension deflection as a function of time for good road input conditions.](image)
The optimization problem (5.20) is solved for the different semi-active controllers in a similar way as is discussed in Sections 5.3 and 5.5. Note that for the LQ/LPV controller, the optimal value for $d_c$ in (5.9) is also dependent on the road input conditions. This value is chosen by solving optimization problem (5.20) for a linear passive quarter-truck model, with design variable $d_c$, the cabin suspensions damping coefficient.

The final results are shown in Figure 5.14, where the ride index for the optimized controllers is plotted against the values of $S_{zr}(n_0)$ used in the optimization problem. Table 5.1 shows the controller parameters for the three controllers optimized at the different values of $S_{zr}(n_0)$. The results of the optimization of the passive linear cabin suspension, mentioned above to find the optimal $d_c$ for the LQ/LPV controller, are also shown in Figure 5.14 and Table 5.1. They show the potential improvement in driver comfort when the semi-active damper would function as a linear passive damper with an adaptive damping coefficient. For the sake of comparison, Figure 5.15 shows the ride index at the different road input conditions for the control strategies with fixed controller parameters, found in Sections 5.3 and 5.5.

Figures 5.14 and 5.15 clearly show the potential of a road adaptive control strategy. When the semi-active damper would function as linear damper with a road dependent damping coefficient, the ride index will decrease at less demanding road input conditions. At worst-case road input conditions, the performance of such a road adaptive linear damper will not differ much from the performance of the identified quarter-truck model. This can be easily explained when regarding that the worst-case conditions are defined in such a way that the limits of the suspension working space for the identified quarter-truck model are approached under these conditions.

### Table 5.1: Adaptive controller parameter values.

<table>
<thead>
<tr>
<th>Control strategy</th>
<th>Parameters</th>
<th>$10^{-6}$m³</th>
<th>16</th>
<th>40</th>
<th>64</th>
<th>88</th>
<th>112</th>
</tr>
</thead>
<tbody>
<tr>
<td>2-state skyhook</td>
<td>$d_{min}$</td>
<td>[Ns/m]</td>
<td>100.0</td>
<td>3131</td>
<td>5679</td>
<td>8224</td>
<td>9760</td>
</tr>
<tr>
<td></td>
<td>$d_{max}$</td>
<td>[Ns/m]</td>
<td>2932</td>
<td>7976</td>
<td>11392</td>
<td>13406</td>
<td>16399</td>
</tr>
<tr>
<td>LQ/LPV</td>
<td>$w_1$</td>
<td>[-]</td>
<td>998</td>
<td>4025</td>
<td>8510</td>
<td>11352</td>
<td>17185</td>
</tr>
<tr>
<td></td>
<td>$w_2$</td>
<td>[-]</td>
<td>1514</td>
<td>1688</td>
<td>1778</td>
<td>1653</td>
<td>1796</td>
</tr>
<tr>
<td>Adaptive linear passive</td>
<td>$d_c$</td>
<td>[Ns/m]</td>
<td>2469</td>
<td>6370</td>
<td>9193</td>
<td>11401</td>
<td>13292</td>
</tr>
</tbody>
</table>

65
This also explains that the performance of the adaptive 2-state skyhook and adaptive LQ/LPV controllers is approximately equal to the performance of the non-adaptive implementations at the road input conditions with \( S_{zr}(n_0) = 112 \cdot 10^{-6} \text{ [m}^3\text{]} \). The ride index as a function of \( S_{zr}(n_0) \) for non-adaptive 2-state skyhook and LQ/LPV controllers follows the same trend as the identified quarter-truck model. The improvement in driver comfort is approximately 4% and 15% respectively under all road input conditions. However, the ride index for the adaptive 2-state skyhook and LQ/LPV strategies decreases much faster and almost linear when decreasing \( S_{zr}(n_0) \). For low values of \( S_{zr}(n_0) \) the semi-active suspension systems can operate using a much “softer”, and therefore more comfortable, damper setting while still respecting the working space limitations. This results in a maximum improvement of driver comfort of 38% for the adaptive 2-state skyhook strategy and 45% for the adaptive LQ/LPV strategy respectively at \( S_{zr}(n_0) = 16 \cdot 10^{-6} \text{ [m}^3\text{]} \), which is very large.

5.7 A state-estimator for LQ/LPV control

In Section 5.5 it is shown that the (fixed gain) LQ/LPV-controlled semi-active cab suspension can increase driver comfort with 12.4%, when full state knowledge is assumed. In practice only a limited number of the states of the truck is available through measurements. A state-estimator is therefore needed to obtain knowledge of the unmeasured states. The design of this state-estimator is discussed in this section.

The quarter-truck system with ISO weighting filter is described by (5.10) in Section 5.5. This system can be extended with an output equation describing the outputs measured by various sensors and the addition of white noise-terms describing system and measurement disturbances. This results in

\[
\dot{x} = Ax + Bu + Gw \\
z = Ex \\
y = Cx + Du + Hw + v
\]

(5.21)

where \( x, z, A, B \) and \( u \) are equal to the matrices, vectors and signals given in Section 5.5. Furthermore \( w \) is a vector of white-noise system disturbances, in this case the vertical velocity of the road surface and, additionally, the control uncertainty. The vector \( v \) represents the measurement noise. The matrix \( G \) is in this case given by

\[
G = \begin{bmatrix} G_\star \\ B_{ISO}C_\star (5,:) \end{bmatrix},
\]

with

\[
G_\star = \begin{bmatrix} 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{m_c} & 0 & 0 \\ 0 & 0 & 0 & 0 & -\frac{1}{m_f} & 0 & 0 \end{bmatrix}^T.
\]

The matrices of the output equation depend on the chosen sensor configuration. In practice it is difficult to measure the absolute velocity of a mass, since accurate velocity sensors are very expensive and integrating the signal of an accelerometer can result in a constant offset. Therefore only sensors measuring relative displacement and absolute acceleration are regarded here. Two sensor configurations are considered in this research. The first configuration uses two sensors and measures the absolute vertical cabin acceleration \( \ddot{z}_c \) and the relative displacement between cabin and frame \( z_c - z_f \). The second configuration uses four sensors and measures the absolute vertical acceleration of the frame \( \ddot{z}_f \) and the relative displacement between the frame and the front axle \( z_f - z_a \) in addition to the signals measured by the 2-sensor configuration. The matrices \( C_{2s}, D_{2s} \) and \( H_{2s} \) describe the output equation for the 2-sensor configuration, for the 4-sensor configuration the output equation is
described by $C_{4s}$, $D_{4s}$, and $H_{4s}$. These matrices are given by

$$
C_{4s} = \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & A_4(5,:) \\
0 & 0 & 0 & A_4(8,:)
\end{bmatrix},
\quad
D_{4s} = \begin{bmatrix}
0 \\
B_4(5,:) \\
B_4(8,:)
\end{bmatrix},
\quad
H_{4s} = \begin{bmatrix}
G(1,:) \\
G(3,:) \\
G(5,:) \\
G(8,:)
\end{bmatrix}.
$$

The optimal estimator for a linear time invariant system is the Kalman filter. Although the quarter-truck including the LQ/LPV-controlled semi-active cabin suspension is a nonlinear system, the use of a Kalman filter to estimate the quarter-truck states is allowed because of the well-known separation principle. This principle states that the controller and the observer for a linear system can be designed independently of each other. When regarding the cabin suspension as a spring in parallel with an actuator delivering the control force $u$, the quarter-truck model is a linear system.

The states of the system $\{A, B, C, D\}$ can be estimated asymptotically when the pair $\{A, C\}$ is detectable [18]. Detectability can be proven using the Hautus-test. This test states that the pair $\{A, C\}$ is detectable if and only if

$$
\text{rank} \begin{bmatrix} A - sI & C \end{bmatrix} = n
$$

for every complex number $s$ in the right half plane. Here $I$ is the identity matrix and $n$ is the number of states of the system. It can be easily shown that the system is detectable with both $C = C_{2s}$ and $C = C_{4s}$.

Using a Kalman filter, the dynamics of the state estimate $\hat{x}$ are given by

$$
\dot{\hat{x}} = A\hat{x} + Bu + L(y - C\hat{x} - Du),
$$

where $C = C_{2s}$, $C = C_{4s}$, and $D = D_{2s}$, or $D = D_{4s}$, depending on the sensor configuration. The optimal choice for $L$ is given by

$$
L = \mathcal{Y}C^TV^{-1},
$$

where $\mathcal{Y}$ is the solution of the algebraic Riccati equation

$$
A^T\mathcal{Y} + \mathcal{Y}A - \mathcal{Y}CV^{-1}C\mathcal{Y} + W = 0.
$$

Here

$$
\mathcal{Y} = \mathcal{Y}_N + HW_NH^T
$$

and

$$
W = GW_NG^T.
$$

The matrix $W_N = E(ww^T)$ is a diagonal matrix with the mean square values of the system disturbances on its diagonal and $\mathcal{Y}_N = E(vv^T)$ is a diagonal matrix with the mean square values of the sensor disturbances on its diagonal.

In this case $w = [w_1, w_2]^T$, where $w_1$ is the vertical velocity of the road surface with $E(w_1^2) = 2.1703$ [m/s], based on the RMS value of the vertical velocity under Type II road conditions used in the suspension optimization problem. Furthermore $w_2$ is the control uncertainty, with $E(w_2^2) = 1 \times 10^{-6}$ [N]. The mean square value for the measurement noise is chosen to be $0.01$ [m/s$^2$] for the acceleration sensors and $1 \times 10^{-6}$ [m] for the relative displacement sensors, in accordance with [26].

Using the values presented above, the filter gains $L = L_{2s}$ and $L = L_{4s}$ are calculated for the 2-sensor and 4-sensor configuration respectively. The performance of the two sensor configurations is evaluated using two different scenarios for the input of the system.

In the first scenario the actuator in the cabin suspension system is replaced with a linear passive damper. The vertical road surface velocity input is $0$ [m/s] for one second, then instantaneously increases to a constant $0.2$ [m/s] for four seconds, after which it instantaneously decreases back to $0$ [m/s]. This is a simple manoeuvre to check the response of the estimator to a step input.

For this manoeuvre, some states can be estimated well by both the 2-sensor configuration and the 4-sensor configuration. This can be seen in Figure 5.16a, where the absolute vertical cabin velocity
\( v_c = \dot{z}_c \) is shown as a function of time. However, for some other states the error between the actual state value and the value estimated with the 2-sensor configuration is a lot larger. This can be seen in Figure 5.16b, where the relative displacement between the front axle and the road surface is shown as a function of time.

For the second scenario, the Kalman filters with 2-sensor and 4-sensor configuration are used in a quarter-truck system with the LQ/LPV-controlled semi-active suspension system. The road input is the same signal as is used to describe the Type II road conditions in the suspension optimization problem. The state equation of the filter given in (5.23) transforms in

\[
\dot{x} = [A - BK - L(C + DK)] \dot{x} + Ly,
\]

where \( C = C_{2x} \), \( D = D_{2x} \), and \( L = L_{2x} \), or \( C = C_{4x} \), \( D = D_{4x} \), and \( L = L_{4x} \). Furthermore \( K = K_{\rho_{\text{min}}} \) or \( K = K_{\rho_{\text{max}}} \), where \( K_{\rho_{\text{min}}} \) is the matrix with control gains in case the desired control force can be delivered by the semi-active damper and \( K_{\rho_{\text{max}}} \) is the matrix with control gains in case the desired control force can not be delivered by the semi-active damper. Note that the filter becomes unstable for the combination of \( C = C_{2x} \), \( D = D_{2x} \), \( L = L_{2x} \), and \( K = K_{\rho_{\text{min}}} \). This is shown by the fact that the matrix \( A - BK - L(C + DK) \) has two eigenvalues with positive real parts. However, it is assumed that the whole quarter-truck system with LQ/LPV controlled semi-active cab suspension can not become unstable since the semi-active suspension system can only dissipate energy.

Figure 5.17a shows the actual and estimated vertical velocity of the front axle as a function of time for the 2-sensor and the 4-sensor configuration. Figure 5.17b shows the errors between the actual state and the estimated states for both configurations. As can be seen the states of the system can be estimated very well for the second scenario when the 4-sensor configuration is used. The error for the 2-sensor configuration is much larger in this case.

This difference in performance is also shown when looking at the ride-index and maximum cabin suspension stroke of the two systems. For the 4-sensor configuration the ride-index is \( a_{\rho_{\text{max}}}^{4s} = 2.0697 \) [m/s²] with a maximum cab suspension stroke of \( S_{\rho_{\text{max}}}^{4s} = 0.0401 \) [m], compared to the full-state feedback values \( a_{\rho_{\text{max}}}^{fs} = 2.0693 \) [m/s²] and \( S_{\rho_{\text{max}}}^{fs} = 0.04 \) [m]. The values for the 2-sensor configuration are \( a_{\rho_{\text{max}}}^{2s} = 2.1701 \) [m/s²] with \( S_{\rho_{\text{max}}}^{2s} = 0.0484 \) [m]. For the 2-sensor configuration the ride-index increases with 4.87% while the maximum suspension stroke is 8.4 [mm] larger than in the full state case. The ride-index of the 4-sensor configuration increases with only 0.02%, while the maximum suspension stroke is only 0.1 [mm] larger than the \( S_{\rho_{\text{max}}}^{fs} \). For this reason the 4-sensor configuration seems preferable to the 2-sensor configuration, despite the extra costs of two additional sensors.

The performance of the Kalman filter using 4 sensors is also evaluated for a third scenario. In this scenario, environmental disturbance forces are exerted on the cabin mass and frame mass of the
quarter-truck model. In this case these forces represent the inertial forces occurring during braking or steering. Figure 5.18 show the applied forces on the cabin $F_{cab}$ and frame $F_{frame}$ as a function of time. For the application of these forces on the quarter-truck model, the matrix $G_q$ is extended to

$$G_q = \begin{bmatrix}
0 & 0 & 0 & -1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{1}{m_c} & 0 & 0 \\
0 & 0 & 0 & -\frac{1}{m_c} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -\frac{1}{m_f} & 0 & 0
\end{bmatrix}^T.$$

The system disturbance vector is in this case $w = [w_1, w_2, F_{cab}, F_{frame}]$.

Although in practice (and in this scenario) the inertial forces will not be stochastic processes, here the matrix $W_N$ is extended with $E(F_{cab}^2) = E(F_{inertial}^2)$ and $E(F_{frame}^2) = E(F_{inertial}^2)$ on the diagonal. The matrix $L_{4s}$ is calculated for $E(F_{inertial}^2) = 1$ [N], $E(F_{inertial}^2) = 1 \cdot 10^3$ [N] and $E(F_{inertial}^2) = 1 \cdot 10^6$ [N]. The performance of the Kalman filters for the different values of $E(F_{inertial}^2)$ is compared.

In Figure 5.19 the actual and estimated values of six different states of the quarter-truck model are shown as a function of time, in case the model is subjected to the forces shown in Figure 5.18. Figure 5.19a shows the relative displacement between the cabin and the frame, $z_c - z_f$. The performance of the Kalman filters using different values of $E(F_{inertial}^2)$ is quite similar for this state, which is directly

$$\begin{align*}
0 & \quad 2 & \quad 4 & \quad 6 & \quad 8 & \quad 10 & \quad 0 & \quad 500 & \quad 2000 & \quad 4000 & \quad 6000 & \quad 7500 \\
\text{time [s]} & \quad \text{Force [N]} \\
F_{cab} & \quad F_{frame}
\end{align*}$$

Figure 5.18: Inertial forces as a function of time.
measured. Only for the value $E(F^2_{\text{inertial}}) = 1 \cdot 10^3$ [N] the estimated deflection is somewhat larger compared to the actual state. Furthermore it can be seen that a larger value for $E(F^2_{\text{inertial}})$ implies a “noisier” estimation. This is logical since a larger $E(F^2_{\text{inertial}})$ results in higher gains in the matrix $L_{4s}$, which means that the measurement noise amplified more. This effect can also be seen in the other plots in Figure 5.19.

The benefit of higher gains in $L_{4s}$ is that the estimated states converge faster to the actual state. This can be seen in Figures 5.19b, 5.19c and 5.19f, where respectively the relative displacement between the front axle and the road surface $z_a - z_r$, the vertical velocity of the frame $\dot{z}_f = v_f$ and the third state of the ISO acceleration filter $x_{i3}$ are plotted. The estimation of these states is poor for
\( \mathbb{E}(F_{\text{inertial}}^2) = 1 \text{ [N]}, \) but improves when this value is increased. On the other hand, Figures 5.19d and 5.19e show that increasing \( \mathbb{E}(F_{\text{inertial}}^2) \) can result in a very poor signal to noise ratio and therefore a very poor estimation. These figures show the vertical velocity of the engine \( \dot{z}_e = v_e \) and the first state of the ISO acceleration filter \( x_{i1} \) respectively.

From the results above it can be concluded that it is difficult to design a Kalman filter which is able to estimate all states of the quarter-truck model properly, when regarding environmental disturbance forces. The increase of the value \( \mathbb{E}(F_{\text{inertial}}^2) \) can improve the estimation of some of the states, but introduces too much noise in the estimation of others. When the environmental forces are driver induced, as was the case in this scenario, developing a disturbance compensator can be considered, as is proposed in [25]. However, more research on this subject is still required.

### 5.8 Summary

In this chapter various control strategies for a semi-active truck cabin suspension system are compared. Two strategies, the so-called 2-state skyhook and ADD algorithms, are selected from literature. These two strategies are developed for respectively 1 degree-of-freedom (DOF) and 2 DOF quarter vehicle models and show great potential in the reduction of the sprung mass acceleration for these systems. The 2-state skyhook and ADD algorithms are applied to a semi-active truck cabin suspension and their control parameters are optimized for the reduction of the ISO weighted vertical cab acceleration, while satisfying a constraint on the maximum absolute suspension deflection. For this case a disappointing 3.6\% and 1.1 \% improvement in driver comfort are found for the 2-state skyhook and the ADD algorithm respectively.

A new semi-active control strategy based on LQ-optimal control in combination with LPV theory is designed. This strategy is developed using the 4 DOF quarter-truck model. Optimization of this strategy using the objective and constraint mentioned above leads to an improvement in driver comfort of 12.4\%.

Since the LQ/LPV strategy requires full-state feedback, a state-estimator is designed for the quarter-truck model. It is shown that the states of the system can be well estimated, with a very minimal increase of the maximum absolute suspension deflection and the weighted cab acceleration.

The potential of road adaptive suspension control is also discussed in this chapter. The results show that improvements up to 45\% in driver comfort can be obtained when using the semi-active LQ/LPV controller with road adaptive parameters.
Chapter 6

Conclusions and recommendations

In this research, the potential of a semi-active cabin suspension system to improve driver comfort is investigated. To this end the following question is posed:

_How much can be gained in terms of driver comfort by using a semi-active suspension for truck cabins and how should it be controlled?_

To answer this question a cab suspension system with a variable rate damper and various control strategies is compared to several passive systems. In order to make a fair comparison, these suspension concepts are all applied in the same vehicle model, representing a truck with cabin suspension. Furthermore the various suspension concepts are optimized to the same performance criteria and constraints.

Regarding the problem stated above, several research objectives are defined:

1. Get an overview of the research and development on cabin suspension systems, both passive and (semi-)active;
2. Select a representative criterion to serve as an objective measure for the performance of a cabin suspension;
3. Design a controller for the semi-active suspension system for optimal performance with respect to the selected criterion;
4. Compare the performance of several passive and semi-active suspension systems, and evaluate the performance potential of these systems.

In this chapter the conclusions and recommendations resulting from this problem statement and these research objectives are discussed.

6.1 Conclusions

The potential of improving driver comfort is significantly larger for a truck cabin suspension system including variable rate dampers, than for the investigated passive concepts.

This is the overall conclusion that can be drawn from this thesis. Several other conclusion are presented below.

The algorithm controlling the cabin suspension system should be designed while regarding the total, semi-active system.

This means that the dynamic behavior of both the primary and secondary suspension, as well as the engine suspension, should be taken into account. A controller designed using such an approach leads
to an increase in performance compared to a semi-active controller based on a primary, fully active suspension system, which is often found in literature.

An extensive literature study on vehicle suspension systems for driver comfort improvement is performed.

The result of this literature study is an overview of different vehicle models, passive suspension concepts, semi-active damper models and optimization strategies for vehicle suspensions. Also a large number of controller for (semi-)active suspension systems is found. A large amount of literature is available on active suspensions of passenger cars. The literature on semi-active cabin suspension systems is limited. However, concepts, models and controllers regarding other vehicle suspension systems can often be of interest.

The quarter-truck model is introduced.

A model able to describe the 1 dimensional heave dynamics of a truck cabin with sufficient accuracy is not found in the literature. Therefore the so-called quarter-truck model is introduced. This is a 4 degree of freedom (DOF) model containing masses which represent the truck front axle, frame, engine and cabin respectively, along with the primary, secondary and engine suspension.

The performance of a (semi-)active suspension system is largely dependent on the used control strategy.

Therefore a short overview of the large amount of control strategies for (semi-)active suspension systems found in literature is presented in this thesis. From this overview two semi-active strategies are selected to be evaluated and compared in the semi-active truck cabin suspension. These strategies are the commonly used 2-state skyhook approach and the so-called Acceleration Driven Damping (ADD) approach. The 2-state skyhook approach is an empirical approximation for semi-active system of the well-known skyhook control strategy. This is the LQ-optimal control strategies for a single degree of freedom, fully active system. The ADD approach is derived using LQ-optimal control on a 2 DOF, semi-active quarter-car system.

Performance criteria to compare nonlinear cabin suspension systems is presented.

In order to make a fair comparison the various suspension concepts are optimized to the same performance criteria and constraints. In this thesis a time-domain based objective function is presented, which is to be minimized in the optimization problem. This objective function is the normalized ISO weighted vertical cab acceleration for two different road input signals. The corresponding constraint function limits the absolute suspension deflection according to the chosen available working-space. This is a convenient feature of the time-domain optimization. In literature a frequency based optimization is often used where the root mean square (RMS) value of the suspension deflection is minimized. In this way, no absolute constraints on the suspension deflection can be guaranteed.

The potential of passive cabin suspension systems appears to be low, while the largest improvement in driver comfort achieved with such a system is 5% with respect to the benchmark system.

Using the selected performance criteria, various passive concepts are optimized and compared. The passive suspension concepts regarded in this thesis are Frequency Selective Damping (FSD), stroke dependent damping and a suspension containing an inerter. The quarter-truck system with an optimized linear passive suspension system consisting of a parallel spring and damper is used as a benchmark. It should be mentioned that not all parameters of the FSD and stroke dependent damping concepts are used as design variables in the optimization, and only one configuration of a suspension system including an inerter is considered. Nevertheless the results of the optimization give an indication of
the potential of these systems to increase the driver comfort. The 5% improvement is achieved with the stroke dependent damping concept.

A new semi-active control strategy based on LQ-optimal control in combination with linear parameter varying (LPV) theory is designed.

The 2-state skyhook and ADD algorithms are applied to the semi-active truck cabin suspension and their control parameters are optimized using the selected performance criteria. For these cases a disappointing 3.6% and 1.1% improvement in driver comfort is found for the 2-state skyhook and the ADD algorithm respectively. In order to achieve a better performance on with the semi-active cab suspension the so-called LQ/LPV strategy is designed. This strategy is developed using the 4 DOF quarter-truck model and takes the limitations of a semi-active system into account on beforehand. Optimization of this strategy using the selected performance criteria leads to an improvement in driver comfort of 12.4%.

A state-estimator is designed for the quarter-truck model.

This is necessary for the LQ/LPV strategy since it requires full-state feedback. It is shown that the states of the system can be well estimated under disturbance of the road surface. The application of the state-estimator in the control system leads to a very minimal increase of the maximum absolute suspension deflection and the weighted cab acceleration.

The potential of road adaptive suspension control is very promising. Improvements up to 45% in driver comfort can be obtained.

This large increase in driver comfort can be realized when using the semi-active LQ/LPV controller with road adaptive parameters. This is due to the fact that, under favorable combinations of road roughness and forward vehicle velocity, the semi-active suspension systems can operate using a much ”softer”, and therefore more comfortable, damper setting. Meanwhile the limitations on the suspension deflection remain maintained.

### 6.2 Recommendations

Many steps are still to be made towards the optimal control strategy for semi-active cabin suspension systems. One very important aspect which is not regarded in this thesis is the power consumption of such a system. Besides the performance in terms of driver comfort improvement, this aspect determines for an important part the feasibility of a semi-active suspension system with respect to passive systems on one end and fully active systems on the other. It is therefore strongly recommended to obtain a semi-active damper model which is able to describe the power consumption in future research. When such a model is available the power consumption can be taken into account in the performance criteria of the optimization problem.

A more detailed damper model will be valuable in general, as semi-active dampers contain nonlinear effects, e.g. hysteresis, which are not described by the model used in this thesis. However, measurements on a semi-active damper suitable for a truck cabin suspension should be performed since parameterized models for such a damper are not available in literature.

The feasibility of a semi-active cab suspension is also determined by the performance that can be achieved by passive suspension systems. Although this thesis has shown that the potential of several passive concepts is low compared to the presented semi-active LQ/LPV concept, further research can be done on this subject. The FSD, stroke dependent damping and inerter concepts are not fully optimized here and perhaps other concepts with better performance are available.

The same holds for different control strategies for semi-active systems. As mentioned before a large amount of literature is available on this subject. The control strategies selected in this thesis show significantly less performance compared to the LQ/LPV approach, but many other strategies exist. This means that total comparison between passive and semi-active systems is only partly made.
in this thesis. However a general picture of the situation is given. To give a complete answer, the passive and semi-active systems not discussed in this thesis should be compared using the method described in this thesis.

Also in the development of the LQ/LPV approach itself is a lot of work left to be done. First of all, the only disturbance on the system that is regarded in this thesis is the disturbance induced by the road. In reality various other disturbances which act on the truck should be taken into account, for example aerodynamic forces, inertial forces during braking and cornering and load changes.

A particular case of a load change is the variation of the cabin mass. This can occur, for example, due to a varying amount of occupants in the cabin. Modern day cabin suspension systems are equipped with a load-leveling system to compensate for this variation in cabin mass. However, this load-leveling system is not taken into account in this thesis. Nevertheless, it may have influence on the characteristics of both the passive and semi-active cabin suspension systems investigated here.

The effects on state-estimator induced by inertial forces acting on the cabin and frame is investigated. The results show that these disturbances largely decrease the performance of the estimator used in this thesis. It is therefore recommended to improve the estimator, so that it is able to cope with such disturbances.

For the LQ/LPV approach a constraint is chosen on the maximum absolute reference force calculated by the control algorithm. However, different choices in this case can be made. It should be investigated in which way the maximum achievable performance of the LQ/LPV approach is influenced by the choice for a certain constraint.

Looking at semi-active suspension systems in general, it is shown in this thesis that road-adaptive control of such a system has a very large potential for the increase of driver comfort. Further investigation of the possibilities of road-adaptive suspension systems is therefore strongly recommended.

Last but not least, the application of the LQ/LPV approach in other degrees of freedom such as roll and pitch should be investigated. The performance of the resulting semi-active cabin suspension system should be validated. This can be done by using a detailed truck model, or, even better, by means of experiments with a test vehicle equipped with the designed suspension system.
Appendix A

ER/MR damper model comparison

In this appendix a comparison is made between the Bouc-Wen ER/MR damper model, the hysteretic Bingham plastic model and the black-box model by Song, all discussed in section 2.4.3. To do this, the force as a function of time, calculated by the Bouc-Wen model for a sinusoidal piston motion at a constant frequency and for a constant input voltage, is used as a reference. The parameters of the hysteretic Bingham plastic model and the black-box model are optimized such that the damper force calculated by these models fits the reference force as good as possible for the same input conditions.

As mentioned in section 2.4.3, the damper force of the hysteretic Bingham plastic model is given by

\[ f(t) = C_{po} f_0(t) + F_y |F_0(t)| + K_x x(t) - \lambda_1 \frac{d}{dt} \left( F_y |F_0(t)| + K_x x(t) \right) + \lambda_2 \frac{d^2}{dt^2} \left( F_y |F_0(t)| + K_x x(t) \right) \]  

(A.1)

Here \( f(t) \) is the reference force calculated by the Bouc-Wen model at a certain constant input voltage. The calculated force according to (2.17) is \( \hat{f}(t) \) and \( t_k \) is the time at which the \( k \)-th sample is taken. The total amount of samples is \( N \). A (linear) relation between the parameters and the input voltage can be found by repeating this procedure for different constant values of the input voltage.

The damper force of black-box model is given by (2.18)–(2.22), with parameters \( a_0 \) up to \( a_4 \), \( b_0 \), \( b_1 \), \( h_0 \) up to \( h_4 \) and \( F_{bias} \). The cost function used to identify these variables is comparable to the one in (A.1) and reads

\[ J(a_0, a_1, a_2, a_3, a_4, b_0, b_1, h_0, h_1, h_2, h_3, h_4, F_{bias}) = \sum_{k=1}^{N} \left[ f(t_k) - \hat{f}(t_k) \right]^2. \]  

(A.2)

The reference force-time history is the calculated by the Bouc-Wen model for a sine-shaped piston motion around its center-position, at a constant input voltage. This motion has a constant frequency of 2 [Hz] and a maximum absolute velocity of 0.05 [m/s]. The value of 2 [Hz] is chosen in approximation of the position of the largest peak in the frequency response function of the quarter-truck model (see Figure 4.2). This frequency will from now on be referred to as the parametrization frequency. Three different parameter sets are identified for the hysteretic Bingham plastic model, for 0, 1 and 2 [V] input voltage. For each parameter a linear relation as a function of the input voltage is found to describe the force-voltage dependency. The black-box model already includes a relation for the dependency of the damper force on the input current. Therefore the black-box model parameter set is identified using only the reference force-time history at 1 [V].

In Figures A.1 – A.3 the velocity-force diagrams at a 2 [Hz] piston motion are shown for the different models. The results are plotted for 0, 1 and 2 [V] input voltage for the Bouc-Wen and the hysteretic Bingham plastic model and for 0, 1 and 1.25 [A] input current for the black-box model respectively.

It can be seen that under these circumstances the hysteretic Bingham plastic model is able to closely match the damper behavior as described by the Bouc-Wen model. Note that the damper force
of the hysteretic Bingham plastic model lies somewhat higher overall, which can be solved by adding a constant offset-term to (2.17). This can be justified since it is found from simulations that the Bouc-Wen model produces a force even when the piston displacement and velocity are both equal to 0 under steady-state conditions.

It also can be seen that the force calculated by the black-box model is a reasonable match to the force calculated by the Bouc-Wen model. However, the black-box model does not match the Bouc-Wen model as well as the hysteretic Bingham plastic model under the prescribed circumstances. In Figure A.4 only velocity-force diagram of the black-box model at 1 [A] is shown. Here it clearly can be seen that the black-box model shows essentially different behavior compared to the other two models. In case of the Bouc-Wen model the damper force characteristic follows the lower part of the hysteresis loop for a positive value of the piston acceleration. In case of a positive value of the piston acceleration and a piston velocity larger than approximately 0.035 [m/s], the damper force becomes larger than the damper force at the same velocities and a negative piston acceleration when using the non-parametric model. In the Bouc-Wen and hysteretic Bingham plastic models the damper force for a positive value of the piston acceleration is always smaller than the damper force for a negative value of the piston acceleration. A similar effect can be seen in the region of negative piston velocity.

The effect of the first order filter as a method to describe the hysteresis loop can be seen in Figures A.5 and A.6 show the velocity-force diagrams of the different model at a 1 and 5 [Hz] piston motion.
respectively. The model parameters for the hysteretic Bingham plastic model and the black-box model, used to create these plots are obtained by identification of the model at a parametrization frequency of 2 [Hz]. These parameters are then used to calculate the damper force when a pure sine with frequency 1 [Hz] and 5 [Hz] respectively is taken as the time history of the piston movement. It can be seen that, for all models, the shape of the hysteresis loop does change for different frequencies of the input signal. However, the for the hysteretic Bingham plastic model, the loop becomes larger when the loop of the other two models becomes smaller, and vice versa. The size of the loop of the black-box model does change in the same direction as the loop of the Bouc-Wen model. However, for this model in both cases the error compared to the Bouc-Wen model increases, especially for higher frequencies. This can be reduced by parameterizing the model at a higher frequency, but this then results in poor performance at lower frequencies. In general the model seems to perform better at lower frequencies, which suggests that it is favorable to do the parametrization at lower frequencies. Also, with a fixed parameter set, the error of the black-box model compared to the Bouc-Wen model is lower when calculating the force at a frequency lower than the parametrization frequency and higher when calculating the force at a frequency higher than the parametrization frequency.

To minimize the error that occurs as a result of the frequency dependent hysteretic behavior of the different models, it is suggested that the model are parameterized at the frequency which is most dominant in the piston velocity input signal to be applied to the model during simulation. Here it is assumed that this input signal is known in advance.
Appendix B

Equivalence of the optimal LQG and $\mathcal{H}_2$ optimization approaches

To show that the optimal LQG optimization approach is a form of structured optimal $\mathcal{H}_2$ control, a linear time-invariant multi-input multi-output system is considered, represented by the block scheme in Figure B.1.

Here $P(s)$ is the transfer matrix of the generalized plant and $F(s)$ is the transfer matrix of the systems controller. The vector $w$ represents all external inputs, such as disturbances (e.g. road irregularities), sensor noise, and reference signals, while the vector $y$ is the criterion or error signal. The vector $v$ is the set of observed variables, in a passive suspension system these are the relative displacement and velocity of the two ends of a spring or damper, respectively. These variables are used by the controller (which can be the suspension system) to compute the control input $u$. The closed loop transfer matrix between $w$ and $y$ is called the lower linear fractional transformation (LFT) of $P$ and $F$ and is denoted $F_l(P, F)$.

\[ \begin{array}{c}
w \\
\downarrow \quad \downarrow \\
P(s) & u \\
\downarrow \\
F(s) & y \\
\downarrow \\
v \\
\end{array} \]

Figure B.1: Linear fractional transformation scheme [19].

The state equations of the system to be controlled can be written as

\[
\begin{align*}
\dot{x}(t) &= Ax(t) + Bu(t) + w(t) \\
z(t) &= Cx(t)
\end{align*}
\]

(B.1)

where $x(t)$ is the state vector as a function of time, $u(t)$ is the control vector as a function of time and $z(t)$ is the output vector as a function of time.

According to [6] the optimal LQ control problem is to find a control vector $u^*(t)$ that minimizes the quadratic cost function

\[
J = \lim_{T \to \infty} \frac{1}{T} E \left\{ \int_0^T (z^T Q z + u^T R u) \, dt \right\},
\]

(B.2)
where $Q \geq 0$ and $R > 0$ are weighting matrices and $x$ is the state vector of the system. In \cite{19} can be found that the equivalent frequency domain problem is to find the state feedback matrix $F^*(s)$, such that the norm $\|F_i(P, F)\|_2$ is minimized, where the transfer matrix of the generalized plant $P(s)$ has the following expression in terms of state space data:

$$P(s) = C(sI - (A))^{-1}B + D \Rightarrow \begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} A & I & B \\ Q^T C & 0 & 0 \\ 0 & 0 & R^T \end{bmatrix}.$$  \hfill (B.3)

Since

$$y(s) = F_i(P, F)w(s),$$  \hfill (B.4)

minimization of the norm $\|F_i(P, F)\|_2$ leads to minimization of $\|y\|_2$. According to \cite{98}, when the system input $w(t)$ is unit white noise characterized by $E[w(t)] = 0$ and $E[w(t)w(t - \tau)] = I\delta(\tau)$ where $\delta$ is the Dirac delta, $\|y\|_2$ is given by

$$\lim_{T \to \infty} \frac{1}{T} E \left\{ \int_0^T (y^T(t)y(t)) dt \right\}^{\frac{1}{2}}.$$  \hfill (B.5)

Knowing from (B.3) that

$$y(t) = \begin{bmatrix} y_1(t) \\ y_2(t) \end{bmatrix} = \begin{bmatrix} Q^T Cx \\ R^T u \end{bmatrix},$$  \hfill (B.6)

and comparing this with (B.2) shows that

$$\|y\|_2^2 = J.$$  \hfill (B.7)
Appendix C

Influence of damper dynamics

In Section 2.4 various ways to model the transient dynamics of a variable rate damper are mentioned. In literature it is found that a first or second order filter with the reference force or voltage as an input and the actual damper force as output is used most often. However, the parameters of these filters vary in different references.

In this thesis a first order filter is used, see (5.5), with \( \eta = 190 \). In this appendix the influence of the damper dynamics on the performance of the semi-active cab suspension system is investigated. Therefore, the first order filter with \( \eta = 190 \) is compared to first order approximations of the fastest and the slowest transient response found in literature, as well as to a second order description of the damper dynamics. Note that, in literature, a distinction is sometimes made between the response when switching the damper from high to low damping and vice versa. This distinction will not be made here.

The fastest transient response is described in [9]. Here it is said that the transient response of the damper settles after 6 [ms], when steady-state is reached. Here this response is approximated by a first order filter with \( \eta = 1000 \). The slowest transient response is 50 [ms] and is mentioned in [39]. This is approximated by a first order filter with \( \eta = 120 \). The second order description of the damper dynamics used for comparison here is found in [9] and is given by

\[
\frac{F_d}{F_s} = \frac{\omega_0^2}{s^2 + 2\zeta\omega_0 s + \omega_0^2},
\]

(\ref{eq:second_order_damper})

with \( \omega_0 = 200\pi \) and \( \zeta = 0.3 \).

The step responses of the four different descriptions are shown in Figure C.1. It can be seen that the first order description with \( \eta = 190 \), used in this thesis, is between the slowest and the fastest

![Figure C.1: Step responses for the various descriptions of the damper transient dynamics.](image-url)
response found in literature. The settling time of the second order description is comparable to the settling time of the first order description with $\eta = 190$. However, the response of the second order filter is faster.

The four different descriptions of the damper dynamics are implemented in the quarter-truck model with the LQ/LPV controlled semi-active cab suspension from Section 5.5. The ride index for the poor road input signal of the optimization problem is calculated for each of the implementations.

The results are shown in Table C. Here the ride index $a_w$ is shown as well as the maximum absolute suspension deflection $|S_{\text{max}}|$ for each of the descriptions. For the first order filter with $\eta = 120$, $|S_{\text{max}}| = 0.04$ [m] remains. The ride index is increased with 2.1% compared to the ride index for the description with $\eta = 190$ used in this thesis. For the first order description with $\eta = 1000$ and for the second order description $|S_{\text{max}}|$ increases slightly, meanwhile the ride index decreases with 3.4% and 3.0% respectively, compared to the first order description with $\eta = 190$.

Table C.1: Ride index and maximum absolute suspension travel for different descriptions of the damper transient dynamics.

| Filter                  | $a_w$ [m/s²] | $|S_{\text{max}}|$ [m] |
|-------------------------|--------------|-------------------------|
| 1st order, $\eta = 190$| 2.1286       | 0.04                    |
| 1st order, $\eta = 120$| 2.1733       | 0.04                    |
| 1st order, $\eta = 1000$| 2.0558      | 0.0406                  |
| 2nd order, $\omega_0 = 200\pi$, $\zeta = 0.3$ | 2.0650 | 0.0405                  |
Bibliography


