Communication-based Cooperative Driving of Road Vehicles by Motion Synchronization

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Master's Thesis

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ABSTRACT

Currently, by means of Cruise Control (CC), Adaptive CC (ACC), Cooperative ACC (CACC) systems, vehicle automation in longitudinal direction is possible and widely used in modern cars. Although their large-scale deployment is still some years ahead, the next step, in the future, is predicted to be combining the longitudinal and lateral behavior, which will result in fully automated vehicles. Therefore, vehicle platoons will be formed not only in longitudinal direction but also in two dimensional plane. Due to this fact, the string stability notion should also be reformulated and generalized to the two dimensional plane.

A single vehicle path tracking controller is designed using “input-output linearization by state feedback”. This controller is capable of trajectory tracking and formed by a PD controller with an acceleration feedforward setting, which is itself not suitable for formation control. Therefore, using techniques from the field of robot synchronization, a formation controller is designed, which is basically formed by the path tracking controller where mutual coupling terms are added for the formation of vehicles. In this control system, the mutual coupling terms determine the trade-off between keeping the formation and tracking the reference trajectory.

String stability of a platoon is systematically analyzed in the frequency domain using transfer functions for ideal vehicle dynamics in the 1D case. The conditions for the mutual coupling terms of the formation controller are derived separately for the two string stability concepts, namely one sided coupling and two sided coupling. Using the formation controller and the conditions derived for ideal vehicle dynamics, analyses are made using the vehicle model in time domain. Apart from the frequency domain analysis for ideal vehicle dynamics, time domain analysis is carried out for the 1D and 2D cases. All simulations in the time domain analyses show that string stability can be achieved with the conditions derived for the mutual coupling terms using ideal vehicle dynamics, which validates the theoretical approach. In summary, a new string stable vehicle following system for 2D vehicular applications and, consequently, a new 2D string stability definition for the assessment of the system are presented in this study.
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CHAPTER 1

INTRODUCTION

This chapter presents the motivation and purpose of this report. In Section 1.1, a general introduction about the current driver assistance systems is given and the notion of cooperative driving and its assessment techniques are explained. The problem definition which reflects the aim of the study is described in Section 1.2. Finally, the outline of this report is presented in Section 1.3.

1.1 Motivation

The world’s mobility problem is getting more and more serious due to the increase in the population and the distances people travel. With the increase in the number of vehicles and, therefore, the increase in traffic, new technologies are sought to overcome this problem. Furthermore, safety is another concern which degrades with the increasing traffic. In the light of these arguments, cooperative driving is seen to have significant potential to solve these problems. With the introduction of so-called Cooperative Driving, automated and communicating vehicles will form adaptive platoons, which, consequently, will increase the so-called road efficiency and the safety by forming smooth flows [VaEs03], [ArDr06], [ZwAr97], [Sh05].

The notion of cooperative driving is a natural extension of Advanced Driver Assistance Systems (ADAS) which primarily focuses on optimizing the individual vehicle behavior based on on-board sensors such as radar and lidar. The first ADAS on this issue is known to be Cruise Control (CC) which currently exists in many of the modern vehicles. It automatically maintains the pre-set desired speed by controlling the throttle. Adaptive Cruise Control (ACC) is an extension of CC, which enables automated vehicle following by keeping a safe distance between vehicles. The primary goal of ACC is to increase the driving comfort and it has traffic safety as a secondary consideration. With the introduction of radar or lidar, ACC measures the inter-vehicle distance and relative velocity with the preceding traffic and controls the throttle and brake to maintain the pre-set desired distance. Since ACC is not primarily a safety system, the driver has to have the control of the vehicle in emergency situations. Therefore, a safe distance, considering the drivers reaction time, should be maintained. However, a small distance has a high importance as it is known that the decrease in following distance results in an increase in the traffic throughput. Furthermore, it also provides decrease in fuel consumption by decreasing the drag force, especially on trucks [Sh05]. In order to optimize the following distance, so-called Cooperative Adaptive Cruise Control (CACC) is developed. The added technology with respect to ACC, is wireless communication. The application of wireless communication extends the vehicle’s horizon, which potentially enables optimization of the traffic in terms of efficiency and safety [NaVu09]. Like the common ACC, CACC takes over the driving task with
respect to throttle and brake in order to keep a desired distance with respect to the preceding vehicle. The addition of wireless communication provides information of the neighboring vehicles that cannot be obtained using on-board sensors. Acceleration and deceleration information of the preceding vehicle can be regarded as the most important ones. Consequently, shorter following distances become possible in view of safety and string stability [NaVu09].

String stability refers to attenuation of disturbances in a platoon (‘string’) of vehicles. ACC is not sufficient to achieve string stable platoons with small inter-vehicle distances, whereas CACC is capable of forming string stable platoons even with small inter-vehicle distances, which is not possible in human controlled vehicles. String unstable behavior is an important source of traffic jams. In string unstable platoons, any disturbance in the leading vehicle may cause the vehicle at the back to stop. Thus, string stability is an indispensible requirement when the traffic throughput, fuel economy and safety are the concerns [Sw97].

1.2 Problem definition

Currently, by means of CC, ACC, CACC systems, vehicle automation in longitudinal direction is possible and widely used in modern cars. Although their large-scale deployment is still some years ahead, the next step, in the future, is predicted to be combining the longitudinal and lateral behavior, which will result in fully automated vehicles, which is also referred to as swarming control in literature [SaMa05]. Therefore, vehicle platoons will be formed not only in longitudinal direction but also in two dimensional plane. Due to this fact, the string stability notion should also be generalized to the two dimensional plane.

For this purpose, firstly, a vehicle model and a controller for a single vehicle, which can track its pre-defined trajectory, should be designed. The studies made in the field of robotics, i.e. wheeled mobile robots, may offer a relevant approach to obtain a fully automated vehicle.

In order to combine the longitudinal and lateral behavior, a new design approach is required in the design of the formation controller. The field of machine synchronization commonly used in robotics provides a useful approach for formational control problem. Mutual synchronization of robots can be implemented for vehicle platoons by reconstructing the control scheme, which results in an entirely new vehicle control strategy for vehicular applications.

Furthermore, a reformulation of the notion of string stability is required for two dimensional behavior since a definition for the 2D applications is not present in the literature. The string stability notion for longitudinal control in the literature can be used as a preliminary.

1.3 Outline of the report

This report is organized as follows. In Chapter 2, an overview of the main literature in the field of wheeled mobile robotics, driver assistance systems in vehicle following and synchronization of robots are presented. In Chapter 3, the design of the vehicle model and its controller for path tracking are explained. Chapter 4 mainly constructs the core of this study. The structure of the system and the notion of string stability to be used in the study are explained in detail. Furthermore,
the formation controller is given without detail and the string stability assessment procedure is presented. At the end, the conditions for the string stability are derived using a model with ideal vehicle dynamics. In Chapter 5, the formation controller is explained in detail and the reference trajectory design for 1D and 2D are separately presented. The assessment of the newly designed formation controller in the view of string stability is carried out using time domain analyses. Two different time domain analyses are performed for 1D and 2D using the designed vehicle model. Finally, in Chapter 6, the main conclusions about the obtained results and also recommendations for future studies are given.
CHAPTER 2

LITERATURE SURVEY

This chapter provides the overview of the literature that is available in the field of wheeled mobile robots, driver assistance systems in vehicle following and synchronization. In Section 2.1, the model derivation approaches for Wheeled Mobile Robots (WMRs) in the literature are presented. Section 2.2 describes the vehicle following systems that are in use or in the development phase. The third literature study subject which is synchronization is given in Section 2.3.

2.1 Wheeled mobile robots-bicycle model

WMRs are used mainly in the automation of industrial processes and in several other areas. They are mainly involved in reference tracking; therefore most of them include the design of an appropriate controller. A WMR is basically a mass with attached wheels which make movement in longitudinal direction possible. By means of steerable wheels, movement to any position in 2D can be achieved. However, there are certain constraints that limit this motion which are position and velocity constraints [CaBa96]. The most important constraints that are used in the derivation of WMR model are the slip and rolling velocity constraints. The slip constraint states that the wheel of a WMR cannot move in lateral direction and the rolling constraint relates the longitudinal velocity to the angular velocity of the wheel ignoring slip.

[CaBa96] defines the concepts of degree of mobility (number of WMR velocities that can be assigned independently) and degree of steerability (number of orientable wheels that can be steered independently) in order to classify and categorize them.

After the categorization four state space models are presented which define behavior of WMR:

- The posture kinematic model
- The configuration kinematic model
- The configuration dynamic model
- The posture dynamic model

The posture kinematic model is the simplest state space representation of a system with posture coordinates (position, orientation angle and steering angle). This model can be regarded as the subsystem of the posture dynamic model, where just the physical control inputs are included. Therefore, this model just deals with the geometric movement of the system in 2D plane using velocity constraints.
The only difference of the configuration kinematic model from the posture kinematic model is the addition of the description of the wheel rotation variables. The added velocity constraint is the rolling without slip constraint which is called to be the nonholonomic velocity constraint [CaBa96], [Vi05].

The configuration dynamic model adds dynamics to the system, which are the mass and rotational inertia of the wheels. Instead of using velocities as the inputs, the real inputs; torques, are used to define the motion. Since one of the wheels of the bicycle model can be steered, there are two torque inputs in the system.

The posture dynamic model again describes the system dynamics; however, it is in a more handy and useful form. This is nothing but the feedback equivalent form of the configuration dynamic model where just the posture coordinates are taken into account. In order to achieve it, a new input vector is defined and a relation between this input and the velocities are found. Since this is the simplest dynamic model, it is commonly used. For instance, it is also derived in [LuOr95], [WaXu03], [CaBa96] and [Vi05].

The instantaneous Center of Rotation (ICR) is an important concept which is used in the model derivation. It is defined as the point in the space where the velocity at that point is zero. This point is located at the intersection of the lines drawn perpendicular to wheel planes. The ICR gives important relations between the angles that define the model. Furthermore, since the ICR mentions that all wheels should be perpendicular to it, a bicycle model is sufficient to denote a 4 wheel vehicle.

The first aim of this report is to design a tracking controller which tracks two of the three posture coordinates which are the positions and the orientation angle. [WaXu03] and [NoCa95] propose input-output linearization which will enable the use of linear control techniques. This technique will be used in the controller design in Chapter 3.

### 2.2 Advanced driver assistance systems

With the development in the information and communication systems technology and their application to the transportation, a new concept, Intelligent Transportation System (ITS) is introduced. Advanced Driver Assistance Systems (ADAS), which mainly help the driver in its driver process, form one of the subgroups of ITS. Some of these driver assistance systems just warn or help the driver without doing an action to the state of the vehicle, whereas others directly take over the driver behavior such as accelerating or braking.

In Table 2.1, some examples are given for ADAS. The so-called Cooperative Adaptive Cruise Control (CACC) system will be the point of interest of this report. Although today’s designed CACC systems only deals with the longitudinal control of vehicles where the driver just controls the steering wheel, it is desired to extend it also to the lateral direction by using communication. Therefore, it is wanted to obtain fully automated vehicle platoons.
2.2.1 Technical issues

The focus of this section are the technical issues/components that should be present in the vehicles in order to obtain the desired system which are: wireless communication, radar and positioning sensors. These three issues basically constitute the practicability and feasibility of the designed system.

The most important and the challenging part is the wireless communication. There are three main types of communications: vehicle to infrastructure (V2I), infrastructure to vehicle (I2V) and vehicle to vehicle (V2V) communication. Reliability, capacity and latency are the main concerns about this technology. Here, latency is defined as the time required from the beginning of packet transmission till the beginning of packet reception and depends on the packet size and the communication structure. Increase in the number of communication links also increases the latency since transmission of a packet should wait until the previous one is delivered. A sudden cut-off or delays in the communication can lead to severe accidents since the system is designed to be fully automated. Capacity is rather important as the increase in the number of vehicles using communication should be predicted beforehand and required measures should be taken.

For the realization of this project, at least V2V or V2I&I2V communication should be present in the system. Each vehicle in the platoon should be equipped with this technology. All vehicles receive the information from the leading vehicle or from the infrastructure, if present. Having a wireless infrastructure which behaves as a command center will be more beneficial in the automated highways since the reference trajectory is, then, defined in a more systematical way; however, it makes the realization more challenging and requires more investment. If smaller inter-vehicle distances are desired, the acceptable latency in the wireless communication should be smaller accordingly. Perception reaction time of an average driver is known to be 1.5-1.6 seconds [Gr00], [OISI96]. However, it is known that this value can decrease up to 0.6 seconds for experienced and careful drivers. Thus, any delay time higher than this value will not add any improvement to traditional human driver traffic condition. In this report, the delay time is assumed to be negligibly small by considering possible future improvements in wireless communication.

### Table 2.1: Categorization of ADAS

<table>
<thead>
<tr>
<th>Passive</th>
<th>Active</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lane Departure Warning</td>
<td>Brake Assist</td>
</tr>
<tr>
<td>Blind Spot Detection</td>
<td>Cruise Control (CC)</td>
</tr>
<tr>
<td>Collision Detection Warning</td>
<td>Adaptive CC (ACC)</td>
</tr>
<tr>
<td>Driver Alert Control/Attention Assist</td>
<td>Cooperative ACC (CACC)</td>
</tr>
<tr>
<td>Monitoring Systems</td>
<td>Adaptive Front Lighting Control</td>
</tr>
<tr>
<td>Pedestrian Detection Warning</td>
<td>Collision Detection</td>
</tr>
<tr>
<td>Rollover Warning</td>
<td>Lane Keeping</td>
</tr>
<tr>
<td></td>
<td>Electronic Stability Control</td>
</tr>
<tr>
<td></td>
<td>Anti-lock Braking System</td>
</tr>
</tbody>
</table>
As mentioned in the previous paragraph, types of wireless communication architectures can be narrowed down to two as being V2V and V2I&I2V. When an infrastructure exists with which vehicles can communicate (V2I&I2V), it is referred to as a centralized system or an Automated Highway System (AHS) [EsKh92], [Io97]. There is an analogy between this system and synchronization of robots since there is a centralized process which decides what actions should be performed. There have been a lot of studies about AHS starting from late 1960s in the USA, Europe and Japan [Io97]. According to the studies done by The California Partners for Advanced Transit and Highways (PATH) program, vehicles should be divided into subgroups called platoons, so that any failure in the structure can be compensated more easily and the number of vehicles that can be allocated in a specified road can be increased which will lead to a decrease in the traffic jams.

The other type of communication architecture is the vehicle to vehicle communication and it is called as decentralized system or Intelligent Vehicle-Highway System (IVHS). This communication architecture is more preferred due to its lower implementation costs. Many researches about possibilities in the wireless communication of vehicles can be found in the scope of PATH program starting from 1986 [Sh92]. Some decentralized (V2V) wireless communication structure examples in a platoon are:

- Everybody communicates with each other
- Everybody communicates with the ones ahead
- Everybody communicates with the leader and the one preceding
- Bidirectional communication
- Everybody communicates with the leader
- Unidirectional communication

The above communication structures are given in a decreasing complexity, i.e. the amount of communication links decreases. It is obvious that the communication of all vehicles with each other is the best choice when the information transmission is perfect. However, it should be realized that information transmission should be obtained with the least possible communication in order to prevent communication traffic. In that sense, the last four options are the ones that are commonly used in the literature. Basically, the unidirectional structure, which means that vehicles just communicate with the preceding one, is used to obtain the cooperative property for Adaptive Cruise Controlled (ACC) vehicles. However, there are more and more studies aiming to adopt other structures such as bidirectional control which means that vehicles communicates with the one ahead and the follower. Although [YaEy98] is not in favor of this structure due to the disturbance propagation in both directions, according to [NaAk04], bidirectional control is a better choice than unidirectional control since the simplest way to achieve a fully connected system is this structure.

Most of these communication structures are studied and conclusions are drawn about string stability in [SwHe99]. The importance of the presence of the reference vehicle information is shown in this paper and it has been concluded that only weak string stability can be achieved in the absence of this information. Therefore, as a solution, dividing the platoons into mini-platoons is suggested. In these mini-platoons, the last vehicle of a mini-platoon becomes the reference vehicle for the following mini-platoon. This structure type also prevents long distance wireless communications.
Other required systems are the sensors for absolute positioning and state measurements whether in longitudinal or 2D direction. Most of the (C)ACC equipped vehicles use sensors, e.g. radar, lidar, vision to gather the preceding vehicle’s relative position and velocity. Radar is the most commonly used state measurement sensor technology due to their ease of mounting and high angular resolutions. It is used for direct ranging and relative speed measuring. The follow-up technology is the lidar. It provides higher resolution and accuracy, yet shorter detection range. The newest technology is vision and it almost contains all of the positive qualities that are mentioned before. However, they need high computation power and are sensitive to environmental conditions. Although, sensors are the inevitable parts of (C)ACC equipped vehicles, in communicating vehicle platoons, this is not a must anymore. Vehicles can obtain this information via wireless communication. However, even in this case, sensors are needed to detect obstacles or merging vehicles which are not equipped with wireless communication. In addition to the state measurement, absolute positions should also be measured using sensors. In AHS, vehicles can get this information via wireless communication using some special layout. In normal roads, GPS devices can be used for this purpose. However, GPS technology is known to be very inaccurate, so it cannot be used as the only positioning device.

2.2.2 String stability

In literature, string stability is referred to as the prevention of the range error propagation in upstream direction in a string of interconnected vehicles [LiPe00], [SwHe96]. String stability is the primary evaluation criterion of a (C)ACC system among the other criteria such as stability, performance, etc. The range error of each vehicle starting from the platoon leader to the end follower should decrease so that the string stability is achieved. The attenuation of error or the oscillatory behavior is important since the amplification results in a decrease in road capacity, an increase in traffic congestion and even collisions.

Nowadays, although the “String stability” term is used by almost every researcher studying (C)ACC systems, it does not have a unique or concrete definition. Studies are carried out by first defining the string stability notion by itself. Examples can be found at [LiPe00], [SwHe96], [Sw97], [NaAk04], [KhDa04], [ShDe93], [KhDa04], [LeKi02], [GeFr98], [ZhIo04].

Two types of string stability can be found in the literature; weak string stability and strong string stability. In weak string stability, the whole platoon is considered as the system so that the local string instabilities are allowed; however, these kinds of platoons are not robust to singular perturbations such as parasitic actuator dynamics and signal processing lags [Sw97]. On the contrary, with strong string stability, the range errors are attenuated at every follower vehicle.

Another categorization on string stability is made about the type of the platoon. If all vehicles are identical to each other, string stability of this platoon is considered as homogenous string stability, else it is considered as heterogenous string stability. The latter is obviously more realistic since vehicles in normal traffic are certainly not identical. On the other hand, some special cases can be formed to realize homogenous string stability.

A significant factor that affects string stability is the inter-vehicle spacing policy. Since it is desired that the vehicles follow each other as close as possible in order to increase the road capacity, a policy should be found, which guarantees string stability, provides a certain level of safety and
reduces the inter-vehicle distance to a minimum. It also should be kept in mind that the policy should also be well defined for different velocities.

In this section, firstly string stability equations in the literature will be introduced and secondly possible inter-vehicle following policies will be explained.

### 2.2.2.1 Mathematics of string stability

There are two common string stability assessment techniques which are time and frequency domain approaches. Both give valuable results although they have some differences in some cases. Therefore, both approaches from various researches will be explained in this section.

Considering a vehicle platoon in the form of a string, i.e. one dimensional case, the interspacing error and its derivatives which give the velocity and acceleration errors are defined as:

\[
\begin{align*}
    e_i(t) &= x_{i-1}(t) - x_i(t) - d_{i,d}(t) \\
    \dot{e}_i(t) &= \dot{x}_{i-1}(t) - \dot{x}_i(t) - \dot{d}_{i,d}(t) \\
    \ddot{e}_i(t) &= \ddot{x}_{i-1}(t) - \ddot{x}_i(t) - \ddot{d}_{i,d}(t)
\end{align*}
\]

where \( i = 1, \ldots, \infty \) and vehicle 0 denotes the reference trajectory/vehicle, \( x_i(t) \) and \( d_{i,d}(t) \) are the absolute position and the desired inter-vehicle spacing of vehicle \( i \) with respect to vehicle \( i - 1 \), respectively. These notations are shown in Figure 2.1. The choice of \( d_{i,d}(t) \) which directly influences the string stability will be discussed in the next section.

![Figure 2.1: Notations used in a string](image)

According to [NaAk04], [LiMa01], string stability of \( N \) vehicles can be met with the requirement:

\[
\|e_1(t)\|_\infty \geq \|e_2(t)\|_\infty \geq \cdots \geq \|e_N(t)\|_\infty
\]

where index 1 refers to the leading vehicle and increases in upstream direction.

Here it should also be noted that for a homogenous platoon, it is sufficient to use only the first two terms since the others will automatically be satisfied. On the other hand, the full criterion should be worked out for a heterogeneous platoon.
Assuming linear vehicle models, the error propagation transfer function for two adjacent vehicles which is the ratio of range errors in frequency domain is defined as:

\[
G_i(s) = \frac{E_i(s)}{E_{i-1}(s)}
\]  

(2.5)

where \(E_i(s)\) is the Laplace transform of the interspacing error of vehicle \(i\).

By definition:

\[
G_i(s) = \int_0^\infty g_i(t) e^{-st} dt
\]  

(2.6)

which also indicates that:

\[
G_i(0) = \int_0^\infty g_i(t) \, dt
\]  

(2.7)

According to [NaAk04], [KhDa04], a necessary and sufficient criterion which guarantees string stability is:

\[
\|g_i(t)\|_1 \leq 1
\]  

(2.8)

which is derived from linear theory as it states[NaAk04]:

\[
\|e_i(t)\|_\infty \leq \|g(t)\|_1 \|e_{i-1}(t)\|_\infty
\]  

(2.9)

The 1-norm of \(g(t)\) is defined as \(\|g(t)\|_1 = \int_0^\infty |g(t)| \, dt\) and therefore (2.8) becomes:

\[
\int_0^\infty |g(t)| \, dt \leq 1
\]  

(2.10)

In order to use (2.7) the following relation is written as:

\[
\int_0^\infty |g(t)| \, dt \geq \left| \int_0^\infty g(t) \, dt \right| = ||G_i(0)||
\]  

(2.11)

After the derivations shown above, the equation which relates time domain to frequency domain is obtained using (2.8), (2.10) and (2.11):

\[
|G_i(0)| \leq \|g_i(t)\|_1 \leq 1
\]  

(2.12)

From the above equation a necessary, but not sufficient condition for string stability can be achieved, which is:

\[
|G_i(0)| \leq 1
\]  

(2.13)

since it does not guarantee (2.8).
However, if it is known that \( g_i(t) \) is positive in the entire time span, (2.13) becomes a necessary and sufficient condition since \( \int_0^\infty |g(t)| \, dt \) becomes equal to \( |\int_0^\infty g(t) \, dt| \) implying that \( |G_i(0)| = \|g_i(t)\|_1 \).

In addition to (2.13), [Sh91] states that \( |G_i(j\omega)| \) should be a strictly decreasing function of \( \omega \).

As a result, the condition proposed by [NaAk04] and [KhDa04] given in (2.8) can be met if:

\[
|G_i(s)| \leq 1 \text{ and } g_i(t) \geq 0 \quad (2.14)
\]

This result is also given in [ShDe93], [KhDa04], [LeKi02] as they underlines that \( |G_i(j\omega)| \) should be smaller than 1 for every \( \omega \).

Another useful information can be inferred from [Sw97] and [NaVu09] as they came up with the result that if the impulse response of \( g_i(t) \) does not change sign, then \( G_i(s) \) is string stable. This also indicates that string stability is satisfied with non-overshooting step response, since the step response is the integrated impulse response.

As a different approach, instead of using the position error propagation transfer function, the velocity propagation transfer function is proposed for heterogeneous platoons in [LiPe00] and [BoIo01]. Then the velocity propagation transfer function is defined as:

\[
H_i(s) = \frac{V_i(s)}{V_{i-1}(s)} \quad (2.15)
\]

where \( V_i(s) \) is the Laplace transform of the velocity of vehicle \( i \).

In addition, it is beneficial to show (2.15) in the following forms:

\[
H_i(s) = \frac{V_i(s)}{V_{i-1}(s)} = \frac{sX_i(s)}{sX_{i-1}(s)} = \frac{X_i(s)}{X_{i-1}(s)} \quad (2.16)
\]

\[
H_i(s) = \frac{V_i(s)}{V_{i-1}(s)} = \frac{A_i(s)/s}{A_{i-1}(s)/s} = \frac{A_i(s)}{A_{i-1}(s)} \quad (2.17)
\]

where \( X_i(s) \) is the Laplace transform of the position and \( A_i(s) \) is the Laplace transform of the acceleration of vehicle \( i \).

Then the following definition for heterogenouso string of vehicles is used to ensure the string stability:

\[
\|H_1(s) \cdot H_2(s) \cdot \ldots \cdot H_N(s)\|_\infty \leq 1 \quad (2.18)
\]

This proposed string stability definition also implies that if all vehicles are string stable, i.e. \( \|H_i(s)\|_\infty \leq 1 \) for \( i = 1, 2, \ldots, N \), then the vehicle platoon is also string stable. This is actually the main difference between heterogeneous and homogeneous strings since it is not sufficient to examine the first few vehicles’ string stability in heterogeneous strings.
[LiPe00] also defines “string stability margin” for heterogeneous vehicle strings. Since the tracking controllers are different for each vehicle, string stability margin which is the maximum number of vehicle ensuring the string stability is examined for different controller parameters for a specific headway time.

### 2.2.2.2 Inter-vehicle spacing policies

The inter-vehicle distance is an important parameter for ACC and CACC controlled vehicle platoons to satisfy string stability. Therefore, there are several definitions for spacing policies in the literature. In most cases, the challenge is to select the most appropriate spacing policy or the parameters of the spacing policy which will minimize the inter-vehicle distance while ensuring the string stability.

Every vehicle has to keep a safe distance between itself and the front vehicle. Human controlled vehicles obviously have to keep a longer distance compared to the automated ones. The reason of this is basically the delay and errors in human perception, reaction and actuation [RaTa00], [ZhIo04], [IoCh93]. There are two common definitions for the spacing policies in literature. One is being the constant time-headway and the other is constant distance.

The most famous and the preferred one is the constant time-headway policy [GeFr98], [LiPe00], [ZhIo04] which has the following general formulation:

$$d_{i,d}(t) = r_i + \tau v_i(t)$$

where $d_{i,d}(t)$ is the desired inter-vehicle distance, $r_i$ is the desired inter-vehicle distance at rest, $\tau$ is the headway time and $v_i(t)$ is the velocity of the vehicle $i$.

The headway time constant, $\tau$, is defined to be 0.255$L$ where $L$ is the length of a vehicle, known as the California constant time headway [IoCh93]. The idea behind this value is using spacing of one vehicle length for every 10 mph:

$$d_{i,d}(t) = \tau v_i = \frac{v_i}{10} \frac{3600}{1600} = 0.255L$$

where $r_i$ is neglected. (Note that $\frac{3600}{1600}$ is the conversion factor from miles per hour to meters per hour.)

However, this headway time constant is suggested for vehicles which are driven by humans involving the delays and reaction times. When the vehicle is driven autonomously including communication, this value can be decreased significantly. Using the suggested values of [Fe79] – the maximum allowable jerk during acceleration, $J_{\text{max}} = 76.2 \text{ m/s}^3$; the maximum acceleration of the follower vehicle, $a_{i,\text{max}} = 3.92 \text{ m/s}^2$; the maximum deceleration of the front vehicle, $d_{i-1,\text{max}} = 7.84 \text{ m/s}^2$- headway time constant is calculated as 0.12 seconds neglecting the communication delays. This value is found considering an extreme emergency situation where the lead vehicle is braking at its maximum deceleration and the follower is speeding up at its maximum acceleration. When the communication delays are included 0.3 seconds is proposed by [IoCh93] as a safe rule for vehicle following. It is shown that with this policy string stability can be obtained by choosing a proper value for $\tau$ and using radar. Communication is not a must for the achievement of string stability in this policy unlike to the constant distance policy since $\tau$ compensates for high speeds.
The other common spacing policy is the constant distance policy which is defined as:

$$d_{i,t}(t) = r_i$$ (2.21)

This policy can also be regarded as special condition for constant time-headway where $\tau$ is selected as 0. On the contrary to the constant time-headway policy, wireless communication is indispensable for the constant distance policy when the string stability is the point of concern [SwHe99] since the policy is not a function of velocity any more. Therefore, at high speeds it is impossible to track the leading car without knowing it is acceleration/deceleration behavior.

As stated before, many other spacing policies exist as well. For example, the difference of the square of velocities is used for wide vehicle followings in [IoCh93]:

$$d_{i,t}(t) = r_i + \tau v_i + \tau_2 (v_i^2 - v_{i-1}^2)$$ (2.22)

The term “$\tau_2(v_i^2 - v_{i-1}^2)$” is added to the ordinary constant time-headway policy and by means of that the relative velocities of the vehicles are used. Considering a two vehicle platoon situation, when both of the vehicles drive at the same speed, the newly added term has no effect. However, if the preceding vehicle is faster than the follow-up, the desired distance between the vehicles is decreased and vice versa. This inter-vehicle spacing policy is suggested to obtain safety vehicle following in an emergency situation where the follower accelerates while the lead vehicle brakes at maximum acceleration/deceleration values.

Another example is derived by curve fitting result of human driver behavior in [HeGo02] and presented as:

$$d_{i,t}(t) = r_i + \tau v_i^k = 2 + 6.33 v_i^{0.48}$$ (2.23)

It is observed that by using this policy the increasing rate of the desired range is smaller for high velocities, which makes it advantageous for high speed and high traffic density roads. On the contrary, this method offers less safe vehicle following as a consequence.

### 2.3 Synchronization

Synchronization has always been an attractive topic for robotic systems. In the first years of robots, they were used separately to accomplish a different task. On the other hand, today, synchronization, i.e. formation control, for multiple robots and vehicles are an active research area. And the application of formation control is increasing every day. Here are the some of the application fields of formation control:

- Aerospace [LeTa97], [ReBe02] ; aircrafts, spacecrafts, satellites,
- Robotics; box pushing robots [LeTa97], pizza robots, robots transporting large objects [Do95], [Ya94],
- Automotive; automated highway/vehicle systems [JiNi00],
- Marine; marine vessels for military purposes [KyPe96],
To give an example of the application fields, aircrafts can be considered [LeTa97]. Formation control can be achieved by controlling the aircrafts in order to keep V-shape formation which decreases the fuel consumption by means of aero-dynamical advantages. Moreover, by using this technique one pilot will be sufficient to fly a group of aircrafts which will decrease the labor needed or fatigue of the pilots will be less if more pilots are present. Another example can be given to the application in the field of robotics. Box pushing robots are good examples since synchronization and parallelism in robot operation are strictly needed for the accomplishment of a task [Do95], [Ya94]. By means of this formation, the need for more powerful or bigger robots is eliminated and the usage areas of the robots are extended.

Although the formation technique is used in some areas, there are still many challenges for its wide application. Sensing, communication and localization are the basic issues which make the implementation and application harder which are also mentioned in Section 2.2.1 for advanced driver assistance systems.

There are three main approaches to the formation problem in literature, namely master-slave, behavioral and virtual structure formations. These approaches will be discussed in the following sections.

2.3.1 Master-slave formation

The so-called Master-slave formation is also referred to as leader-follower approach in literature [ReBe02], [DoPa07]. As can be understood from the name of the approach, one of the robots (or any agent) is assigned to be the master or leader. The master tracks a path which was defined beforehand as desired. The other agents, referred to as slave or follower, tracks the leader’s state in a transformed relative manner. Therefore, in this approach, the leader is the most important element or even the only important element which needs special attention. The disadvantage of this technique is the absence of the feedback to the formation from the slaves. Since they do not contribute any information to the formation and just follow their own transformed track, the leader which actually defines the trajectory of the group has no information about the others. This means that any failure in the slaves is not known by the formation, which also results in a failure in the formation. In conclusion, the structure is not robust to perturbations on the slaves. Wireless communication is another disadvantage compared to behavioral formation (see next section) since it brings complexity and failure probability to the formation by means of disruptions or packet loss.

As mentioned, both master and slaves have predetermined reference trajectories to track, \((x_{ref,m}, y_{ref,m})\) and \((x_{ref,s}, y_{ref,s})\), respectively. For instance, the reference trajectory of the slaves for a mobile robot application (unicycle) can be defined as follows [DeOs01]:

\[
x_{ref,s} = x_m + l_x \quad \text{and} \quad y_{ref,s} = y_m + l_y
\]  

(2.24)

where \(l_x\) is the distance between the robots in \(x\) direction, \(l_y\) is the distance between the robots in \(y\) direction, \(x_m\) and \(y_m\) denote the position of the master. This relation bonds all of the slaves to the master and now, their trajectories are the transformed trajectory of the master.

If \(l_x\) and \(l_y\) are chosen as constant, it follows that:
CHAPTER 2

\[ \dot{x}_{\text{ref},s} = \dot{x}_m, \dot{y}_{\text{ref},s} = \dot{y}_m \text{ and } \ddot{x}_{\text{ref},s} = \ddot{x}_m, \ddot{y}_{\text{ref},s} = \ddot{y}_m \]  \tag{2.25}

The only remaining variable is the orientation angle \( \theta \) and its derivatives. It can also be proved for an ideal and undisturbed situation that:

\[ \theta_{\text{ref},s} = \arctan \left( \frac{\dot{y}_{\text{ref},s}}{\dot{x}_{\text{ref},s}} \right) = \arctan \left( \frac{\dot{y}_m}{\dot{x}_m} \right) = \theta_m \]  \tag{2.26}

which also means that the derivatives are equal. If this constant \( l_x \) and \( l_y \) is not preferred, then the variable \( \theta_{\text{ref},s} \) should also be defined according to \( \theta_{\text{ref},m} \).

As a result, the states of the slaves are written as the transformed states of the master, which makes the master only agent whose trajectory should be defined in the formation structure.

Much attention with this approach is given to the ground agents, i.e. robots, vehicles, etc., since it is easy to implement and construct an experimental environment. In addition, nowadays, there is a considerable amount of research on small unmanned flying objects using this technique. In the design of ground objects, it is seen that many researchers, [CoMo06], [DoPa07] and [DeOs01], use nonholonomic unicycle models which are discussed in Section 2.1 in detail. Using unicycle models gives the opportunity of concentrating mainly on the formation control instead of derivation of the dynamic model.

2.3.2 Behavioral formation

In the behavioral approach, several individual desired behaviors are prescribed for robots (or any agents) [LaBe00], [BaAr98], [LaYo00]. Then, all of these behaviors, which can be for instance collision avoidance, obstacle avoidance or formation keeping, are ordered in importance level by using weighting functions [ReBe02]. The advantages of the behavioral formation are the existence of a natural control strategy due to weighting when the robots have multiple tasks, the explicit feedback to the formation and the avoidance of the hazards at the same time. On the contrary, the disadvantages of the behavioral formation are the difficulty in the formation definition for the group behavior and mathematical analysis of stability. Therefore, exact definitions cannot be made for the stability of the approach.

There are three techniques for formation position determination in this approach, namely unit-center-referenced, leader-referenced and neighbor-referenced [BaAr98]. In the unit-center-referenced strategy a unit-center is calculated by all robots and each robot determines its own formation position relatively to that center. This technique makes the approach closer to the virtual structure formation (see next chapter). In the leader-referenced strategy, each robot determines its position relatively to the leading robot similar to the master-slave approach. In the last strategy which is neighbor-referenced, each robot maintains its position relative to the predetermined robot.

As a final remark, the above mentioned disadvantages of this approach make the behavioral formation less preferred among these three approaches.
2.3.4 Virtual structure formation

The last but the most advantageous approach is the virtual structure formation. This approach is an enhanced version of the leader-follower approach where the slaves can also communicate to the leader and with each other [IkJo06]. Since now all of the agents are mutually coupled to each other, the virtual structure is robust to the perturbation on any of the agents. In this approach, the entire system is threaded as a single a single structure, which makes the definition of the formation easier. The trajectory of the virtual structure is defined as it is done in the leader-follower approach for the leader. Then, the agents have their own following conditions, also referred to as the mutual coupling gains, which defines how much they should “stick” to the formation. In the definition of these conditions, again the task importance of each agent is considered as done in the behavioral formation.

Despite a lot of advantages, this approach also has some disadvantages. The obvious one is the need for communication between the agents, which is also mentioned in the leader-follower approach. The only approach without the need for wireless communication is the behavioral formation approach; however, it is also obvious that without communication it is not possible to get feedback from the formation. Another disadvantage is that the application of this approach is limited since defining a virtual structure in reality is not simple [ReBe02]. Therefore, no experiment has been carried out with real-sized objects. The last disadvantage that will be addressed in this report is the uncertainty of assigning the coupling parameters. A real-time control logic should be developed to judge whether the agent should stick to its own trajectory or the formation.

A proper approach for virtual structure formation is given in [RoNi04] where it is referred to as mutual synchronization of robots. The controller designed in [RoNi04] forms the basis of the controller that will be given in Chapter 5.

In this paper, frictionless rigid joint robots are considered. Applying the Euler-Lagrange formalism, the dynamic model of the $i^{th}$ robot, $i = 1, ..., N$, is described as:

$$M_i(q_i)\ddot{q}_i + C_i(q_i, \dot{q}_i)\dot{q}_i + g_i(q_i) = \tau_i \tag{2.27}$$

where $\tau_i$ denotes the vector of torques, $M_i(q_i)$ is the inertia matrix, $C_i(q_i, \dot{q}_i)$ denotes the Coriolis and centrifugal forces, $g_i(q_i)$ represents the gravity forces and $q_i$ is the joint coordinates.

The mutual synchronization controller $\tau_i$ for the $i^{th}$ robot of [RoNi04] is as follows:

$$\tau_i = M_i(q_i)\ddot{q}_i + C_i(q_i, \dot{q}_i)\dot{q}_i + g_i(q_i) - K_{d,i}\dot{s}_i - K_{p,i}s_i \tag{2.28}$$

where $K_{d,i}$ and $K_{p,i}$ are the positive definite gain matrices, $\dot{s}_i$ and $s_i$ are the synchronization errors defined by:

$$\dot{s}_i = \dot{q}_i - \dot{q}_{ri} \text{ and } s_i = q_i - q_{ri} \tag{2.29}$$

Here, the equations in (2.29) define the trade-off between tracking the reference trajectory and mutual synchronization.
A similar controller is also given in [YoNa08] in which multi-vehicle systems are studied for virtual structure formation. The kinematic model of $i^{th}$ vehicle is given by:

$$
\begin{bmatrix}
\dot{x}_i \\
\dot{y}_i \\
\dot{\theta}_i
\end{bmatrix} =
\begin{bmatrix}
\cos(\theta_i) & 0 \\
\sin(\theta_i) & 0 \\
0 & 1
\end{bmatrix}
\begin{bmatrix}
u_i \\
\omega_i
\end{bmatrix}
$$

(2.30)

where $x_i$ and $y_i$ are the positions of center of gravity of $i^{th}$ vehicle, $\theta_i$ is the heading angle of $i^{th}$ vehicle and $u_i$ and $\omega_i$ are the control inputs.

The virtual structure is considered using Virtual Vehicle (VV). The kinematics of $i^{th}$ VV is described using the relation between the vehicle and VV as:

$$
\begin{bmatrix}
x_{ri} \\
y_{ri} \\
\theta_{ri}
\end{bmatrix} =
\begin{bmatrix}
x_i + x_{di} \cos(\theta_i) - y_{di} \sin(\theta_i) \\
y_i + x_{di} \sin(\theta_i) + y_{di} \cos(\theta_i) \\
\theta_i
\end{bmatrix}
$$

(2.31)

where $x_{ri}$ and $y_{ri}$ are the positions of center of gravity of $i^{th}$ VV, $\theta_i$ is the heading angle of $i^{th}$ VV and $x_{di}$ and $y_{di}$ are the distance between VV and vehicle. The derivative of (2.31) is given by:

$$
\begin{bmatrix}
\dot{x}_{ri} \\
\dot{y}_{ri} \\
\dot{\theta}_{ri}
\end{bmatrix} =
\begin{bmatrix}
B_i & \nu_i \\
B_\theta & \omega_i
\end{bmatrix}
$$

(2.32)

where

$$
B_i =
\begin{bmatrix}
\cos(\theta_i) & -x_{di} \sin(\theta_i) - y_{di} \cos(\theta_i) \\
\sin(\theta_i) & x_{di} \cos(\theta_i) - y_{di} \sin(\theta_i)
\end{bmatrix}
\quad \text{and} \quad
B_\theta = [0 \quad 1]
$$

(2.33)

One of the formation controllers proposed in this paper is:

$$
u_i = B_i^{-1}(\sum_{j \in N_i} ((r_i - r_{ri}) - (r_j - r_{rj}) + \hat{r}_d)$$

(2.34)

where $u_i = [\nu_i \quad \omega_i]^T$, $r_i$ is the position of vehicle $i$, $N_i$ is the $i^{th}$ neighbor set, $\hat{r}_d$ is the constant reference velocity, $k$ is the positive definite controller gain and $r_{ri}$ is the reference relative position to $r_i$.

The controllers given in (2.28) and (2.30) both show the trade-off between reference tracking and keeping the formation. This trade-off is a basic concern of the virtual structure approach and manipulated by means of controller gains.

### 2.4 Summary

In this chapter, the overview of the literature is presented in three sections.

In Section 2.1, it is concluded that the posture kinematic, the configuration kinematic, the configuration dynamics and the posture dynamics models should be derived in order to obtain a
vehicle model. These models will be derived using the kinematic constrains and dynamics of the system. After deriving the vehicle model, a controller should be designed using input-output linearization which will enable the use of linear control techniques.

In Section 2.2, the technical issues such as wireless communication architectures and on-board sensors are described in detail considering their advantages and disadvantages. Furthermore, the primary evaluation criterion for vehicle following systems, string stability, is explained and mathematical relations are given. Finally, possible inter-vehicle spacing policies are discussed to be used later in the report.

Section 2.3 covers the literature about machine synchronization. Three formation techniques, master-slave, behavioral and virtual structure are explained. The type of the formation structure that will be used in this study is decided to be the virtual structure approach considering the advantages stated and its suitability to the vehicular application. The other reason is that the virtual structure formation is the most advanced formation technique, basically the enhanced version of others. Furthermore, it is the only formation structure that also accounts for the possible disturbances on each single agent.
CHAPTER 3

BICYCLE MODEL AND CONTROLLER DESIGN

In this chapter, the general procedure for the determination of the bicycle model and the design process of the controller for a single vehicle is explained. In the study, the bicycle model (1-track model) is used since the modeling of a complete car is unnecessary for the purpose of this study.

The dynamics of the model will be determined using the velocity constraints. Since the front and rear wheels are each lumped into one wheel, the lateral dynamics are derived in a simplified way.

In the controller design, input-output linearization by state feedback approach is used since the derived dynamic model is nonlinear [NoCa95], [BeAn02]. By means of that, linear control techniques are used.

3.1 Bicycle model development

In this section, the posture kinematic model, the configuration kinematic model, the configuration dynamic model and finally the posture dynamic model are developed, which are also mentioned in Section 2.1. The kinematics of the vehicle is investigated by means of the first two models which accounts for the constraints inducing the vehicle mobility. And with the last two models, the dynamics of the vehicle is explored by adding mass, inertia, etc. properties. Therefore, with these four models the behavior of the vehicle can be defined.

The first step in the derivation of the model is to define coordinate systems. The coordinate definitions are given in Figure 3.1. In the figure, there are two coordinates which are being absolute \((x, y)\) and local \((x_l, y_l)\) which is attached to the center of gravity. \(x_l\) denotes the longitudinal direction and \(y_l\) denotes the lateral direction of the vehicle.

To be able to switch from one coordinate system to another, a transformation matrix \(R(\theta)\) is introduced where \(\theta\) is the orientation angle of the car with respect to the global frame.

\[
R(\theta) = \begin{pmatrix}
\cos(\theta) & \sin(\theta) & 0 \\
-\sin(\theta) & \cos(\theta) & 0 \\
0 & 0 & 1
\end{pmatrix}
\] (3.1)
The transformation of the velocity from global frame to local frame is then defined as follows:

\[
\begin{pmatrix}
\dot{x}_1 \\
\dot{y}_1 \\
\dot{\theta}
\end{pmatrix}
= R(\theta)
\begin{pmatrix}
x \\
y \\
\theta
\end{pmatrix}
\] (3.2)

For the bicycle model shown in Figure 1.2, six generalized variables are used to describe the various motions of the model. These are the position variables as \(x\), \(y\) and \(\theta\); steering angle \(\beta\); and the rotation angles of the wheels \(\varphi_1\) and \(\varphi_2\). The sign convention of \(\varphi_1\) and \(\varphi_2\) is shown in Figure 2.3. These variables are shown in a vector, which will be met later:

\[
q = \begin{pmatrix}
x \\
y \\
\theta \\
\beta \\
\varphi_1 \\
\varphi_2
\end{pmatrix}
\] (3.3)

There are two holonomic constraints which describe the motion of the kinematic model. The first constraint is the slip constraint which is the restriction of the movement in lateral direction. The second constraint is again a slip constraint obtained by the assumption that the wheel rolls without slip. With this constraint relation between the forward velocity and the wheel rotation angle is derived.

The first slip constraint (lateral direction) equation for the model is:

\[
C_1 R(\theta)
\begin{pmatrix}
\dot{x} \\
\dot{y} \\
\dot{\theta}
\end{pmatrix}
= \begin{pmatrix}
\sin(\beta) & -\cos(\beta) & -L \cos(\beta) \\
0 & 1 & -L
\end{pmatrix}
R(\theta)
\begin{pmatrix}
\dot{x} \\
\dot{y} \\
\dot{\theta}
\end{pmatrix}
= 0
\] (3.4)

where \(L\) is the half distance between the wheels (also the distance between the wheel and the local coordinate frame origin). In (3.4), the first row of \(C_1\) describes the slip constraint in the front wheel, and the second row describes the one for the rear wheel.
The second slip constraint (forward direction) equation for the model is:

$$J_1 R(\theta) \begin{pmatrix} \dot{x} \\ \dot{y} \end{pmatrix} + J_2 \begin{pmatrix} \dot{\phi}_1 \\ \dot{\phi}_2 \end{pmatrix} = \\
\begin{pmatrix} \cos(\beta) & \sin(\beta) & L \sin(\beta) \\ 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} \dot{x} \\ \dot{y} \end{pmatrix} R(\theta) + \begin{pmatrix} r \\ 0 \end{pmatrix} \begin{pmatrix} \dot{\phi}_1 \\ \dot{\phi}_2 \end{pmatrix} = 0$$

(3.5)

where $r$ is the radius of the wheel. In (3.5), again the first row of $J_1$ and $J_2$ formulates the slip constraint for the front wheel, and the second row formulates the one for the rear wheel.

The degree of steerability and mobility should be defined before the derivation of the kinematic and dynamic models. The degree of steerability is the number of conventional steering wheels, in other words, number of the steering angles [CaBa96]. The degree of mobility is the degree of spatial freedom which can be obtained while keeping the steering angles constant.
The degree of steerability is given by the following equation:

\[
\delta_s = \text{rank} \left( C_1(\beta) \right) = \text{rank} \left( \begin{pmatrix} \sin(\beta) & -\cos(\beta) & -L \cos(\beta) \end{pmatrix} \right) \tag{3.6}
\]

where \( C_1 \) is the first row of the slip constraint (lateral) equation. Therefore, the degree of steerability of the model is 1 which is obvious since the car can be just steered from the front wheel.

The degree of mobility is given by the following equation:

\[
\delta_m = \dim(\text{null}(C_1)) = 3 - \text{rank}[C_1] \tag{3.7}
\]

where \( \dim(\text{null}(C_1)) \) is the dimension of the null space of \( C_1 \). Knowing the fact that the null space of \( C_1 \) includes all possible movements and by applying the rank-nullity theorem which is:

\[
\dim(N[A]) + \text{rank}[A] = n \tag{3.8}
\]

where \( A \) is an \((m,n)\) matrix, (3.7) is derived. When the mobility of the model is calculated, it is found as 1, since if the front wheel is steered to specific position and kept at there, the only possible movement is going forward and backward in a line or in a circle.

As a result, the bicycle model is a so-called \((\delta_m, \delta_s)\)=(1,1) type mobile robot where the first term in brackets represents the degree of mobility and the second term represents the degree of steerability.

The first model to be developed is the posture kinematic model which is just about the motion in the x-y plane. Therefore, wheel rotation is not included in this model which makes this model the simplest one. \( C_1 \) is the relevant matrix in finding the model. In order to find the model, the solution of (3.4) should be found. Howev

\[
\dot{\xi} = \begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{pmatrix} \tag{3.9}
\]

In (3.4), \( R(\theta)\dot{\xi} \) can be regarded as the null-space of \( C_1 \) since \( C_1[R(\theta)\dot{\xi}] = 0 \). For this reason, one should define a \( \Sigma(\beta) \) matrix whose columns span the null-space of \( C_1 \) and which will give the following equation; \( C_1 \Sigma(\beta) = 0 \) and define an input vector \( \eta_a \) (\( \eta_a \) is scalar (=dimension of \( \delta_m \)) for this specific case). Using this approach, one can derive the following relation:

\[
R(\theta)\dot{\xi} = \Sigma(\beta)\eta_a \tag{3.10}
\]

which gives:

\[
\dot{\xi} = R^T(\theta)\Sigma(\beta)\eta_a \tag{3.11}
\]

Using the fact that \( C_1 \Sigma(\beta) = 0 \) the following solution can be obtained:

\[
\Sigma(\beta) = \begin{pmatrix} 2L\cos(\beta) \\ L\sin(\beta) \\ \sin(\beta) \end{pmatrix} \tag{3.12}
\]
almost describes the posture kinematic model; however, due to the steering angle, the system is not linear with respect to the input. In order to linearize the system with respect to the inputs, \( \dot{\beta} = \zeta \) the relation is added to the model. So, the final posture kinematic model is:

\[
\begin{pmatrix}
\dot{\xi} \\
\dot{\beta}
\end{pmatrix} =
\begin{pmatrix}
R^T(\theta)\Sigma(\beta) & 0 \\
0 & 1
\end{pmatrix}
\begin{pmatrix}
\eta_a \\
\zeta
\end{pmatrix}
\]  

(3.13)

knowing \( R(\theta) \) and \( \Sigma(\beta) \) from (3.1) and (3.12), respectively:

\[
\begin{pmatrix}
\dot{\xi} \\
\dot{\beta}
\end{pmatrix} =
\begin{pmatrix}
0.5L\cos(\theta + \beta) + 0.5L\cos(-\theta + \beta) + L\cos(\theta + \beta) & 0 \\
0.5L\sin(\theta + \beta) - 0.5L\sin(-\theta + \beta) + L\sin(\theta + \beta) & 0 \\
\sin(\beta) & 0
\end{pmatrix}
\begin{pmatrix}
\eta_a \\
\zeta
\end{pmatrix}
\]  

(3.14)

For this case \( \eta_a \) represents the forward velocity in the direction of front wheel and \( \zeta \) represents the steering velocity.

Next, the configuration kinematic model is derived, which also includes the wheel rotation variable equations unlike to the posture kinematic model. Therefore, it forms a (6,1) vector.

The configuration kinematic model is:

\[
\begin{pmatrix}
\dot{\xi} \\
\dot{\beta} \\
\dot{\phi}
\end{pmatrix} =
\begin{pmatrix}
R^T(\theta)\Sigma(\beta) & 0 & 0 \\
0 & 1 & 0 \\
-J_2^{-1}I_1 & \Sigma(\beta) & 0
\end{pmatrix}
\begin{pmatrix}
\eta_a \\
\zeta
\end{pmatrix}
\]  

\[
= \begin{pmatrix}
0.5L\cos(\theta + \beta) + 0.5L\cos(-\theta + \beta) + L\cos(\theta + \beta) & 0 \\
0.5L\sin(\theta + \beta) - 0.5L\sin(-\theta + \beta) + L\sin(\theta + \beta) & 0 \\
\sin(\beta) & 0 & 0 \\
-2L & 1 & 0 \\
2L\cos(\beta) & 0 & 0 \\
r & 0 & 0
\end{pmatrix}
\begin{pmatrix}
\eta_a \\
\zeta
\end{pmatrix}
\]  

(3.15)

To make it compact, the equation can be expressed as:

\[
\dot{q} = S(q)\eta
\]  

(3.16)

where the notations express the matrices in the order that is in the previous equation.

In the next step, the configuration dynamic model is needed to express the already derived configuration parameters in a dynamical way. For this purpose, the following differential equation is used:
\[ M(q)\ddot{q} = C(q, \dot{q}) + F + B(q)\tau \]  

(3.17)

where \( M(q) \) is the mass matrix, \( C(q, \dot{q}) \) contains the centripetal and Coriolis terms and encompasses joint flexibility effects and gravitational effects, \( F \) is the total generalized force vector consisting of constraint forces, \( B(q) \) represents the generalized torque directions and \( \tau \) is the input torque vector [Wo07].

While constructing the matrixes above, the assumption of absence of friction forces is made. Since the car is a front driven, front steered type, there are two input torques, being the driving and the steering torque.

The matrices of the dynamical equation are:

\[
M(q) = \begin{pmatrix}
m & 0 & 0 & 0 & 0 & 0 \\
0 & m & 0 & 0 & 0 & 0 \\
0 & 0 & I_\theta & 0 & 0 & 0 \\
0 & 0 & 0 & I_s & 0 & 0 \\
0 & 0 & 0 & 0 & I_\varphi & 0 \\
0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}
\]  

(3.18)

\[ C(q, \dot{q}) = 0 \]  

(3.19)

Since \( F \) is the force vector containing the constraint forces, it can be formulated by the constraint equation matrix \( A \) and the so-called Lagrange multiplier vectors \( \lambda \).

\[ F = A\lambda \]  

(3.20)

where \( A = \begin{pmatrix} C_1R(\theta) & 0 \\ J_1R(\theta) & 0 \end{pmatrix}^T \) and \( \lambda_1 \) (2x1) is the friction force vector preventing the wheel from slipping laterally and \( \lambda_2 \) (2x1) is the friction force vector ensuring that the wheel turns without slip.

\[
B = \begin{pmatrix}
0 & 0 \\
0 & 0 \\
0 & 1 \\
1 & 0 \\
0 & 0
\end{pmatrix}
\]  

(3.21)

\[ \tau = \begin{pmatrix} \tau_s \\ \tau_d \end{pmatrix} \]  

(3.22)

where \( m \) is the mass of the car, \( I_\theta \) is the inertia of the car about the axis perpendicular to \( x \) and \( y \), \( I_s \) is the inertia of the wheel about its axis of steering, \( I_\varphi \) is the inertia of the wheel about the axis of rotation, \( A\lambda \) is the representation of the Lagrange forces ensuring the system obeys the constraints, \( \tau_s \) is the steering torque and \( \tau_d \) is the driving torque.

Now, the configuration dynamic model can be written by taking the derivative of the configuration kinematic model (3.16):

\[ \ddot{\eta} = S\eta + S\dot{\eta} \]  

(3.23)
From (3.4) and (3.5), it is known that $A^T \dot{q} = 0$. Using (3.16) it can be showed that:

$$R(\theta)A^T \dot{q} = A^T S(q)\eta = 0$$

(3.24)

(3.24) also states that:

$$A^T S(q) = 0$$

(3.25)

since $\eta$ cannot be zero (except for the stand-still position which is trivial).

(3.25) can also be written as:

$$\lambda A^T S(q) = 0$$

(3.26)

Using (3.20) in (3.26):

$$F^T S(q) = 0 \Rightarrow S^T(q) F = 0$$

(3.27)

Since the values of the Lagrange forces are not known, it is desired to eliminate them by substituting (3.23) in (3.17):

$$M\dot{\eta} + MS\dot{\eta} = C + F + B\tau$$

(3.28)

Multiplying (3.28) by $S^T$:

$$S^T M\dot{\eta} + S^T M S\dot{\eta} = S^T C + S^T F + S^T B\tau$$

(3.29)

which gives when used with (3.27):

$$\dot{\eta} = (S^T M S)^{-1} S^T (-M\dot{\eta} + C + B\tau)$$

(3.30)

The configuration dynamic model can be obtained when (3.20) is written in vector including $\dot{q}$:

$$\begin{pmatrix} \dot{q} \\ \dot{\eta} \end{pmatrix} = \begin{pmatrix} S \eta \\ (S^T M S)^{-1} S^T (-M\dot{\eta} + C + B\tau) \end{pmatrix}$$

(3.31)

where

$$\dot{S} = \begin{pmatrix} \beta \cos(\beta) & 0 \\ 2L(\theta \cos(\beta) \cos(\beta) - \beta \sin(\beta) \cos(\beta)) - L(\theta \cos(\theta) \sin(\beta) + \beta \sin(\theta) \cos(\beta)) & 0 \\ 2L(\theta \cos(\beta) \cos(\beta) - \beta \sin(\beta) \sin(\theta)) + L(-\theta \sin(\theta) \sin(\beta) + \beta \cos(\theta) \cos(\beta)) & 0 \\ \dot{\beta} \cos(\beta) & 0 \\ 0 & 0 \\ 0 & 0 \\ -\frac{2L}{r} \sin(\beta) \dot{\beta} & 0 \end{pmatrix}$$

(3.32)

which yields the model:
Regarding the configuration dynamic model in the previous step, the final step is to find a feedback equivalent of the configuration dynamic model and reduce it to a more useful form, namely the posture dynamic model. In order to achieve this, an arbitrary reference input, \( v = [v_1, v_2] \) is defined.

The torque input \( \tau \) can be found from the lower part of (3.31) as follows:

\[
\tau = (S^TB)^+ S^T M (Sv + S\eta)
\]

(3.34)

where \((S^TB)^+\) is the pseudo inverse of \(S^TB\). However, the pseudo inverse can be replaced by the exact inverse in this specific case.

The torque input can then easily be defined for the system as:

\[
\begin{pmatrix}
\tau_d \\
\tau_s
\end{pmatrix} = \begin{pmatrix}
v_1 (mr^2 + I_\phi) \\
v_2 I_s
\end{pmatrix}
\]

(3.35)

A feedback equivalent configuration dynamic model is:

\[
\begin{pmatrix}
\dot{x} \\
\dot{y} \\
\dot{\theta} \\
\dot{\phi}_1 \\
\dot{\phi}_2 \\
\dot{\eta}_a \\
\dot{\zeta}
\end{pmatrix} = \begin{pmatrix}
(2\cos(\theta)\cos(\beta) \cos(\beta) - L\sin(\theta) \sin(\beta))\eta_a \\
(2L\sin(\theta)\cos(\beta) + L\cos(\theta) \sin(\beta))\eta_a \\
\sin(\beta)\eta_a \\
\zeta \\
-\frac{2L}{r}\eta_a \\
\frac{2L\cos(\beta)}{r}\eta_a \\
\frac{r r_d}{mr^2 + I_\phi}
\end{pmatrix}
\]

(3.36)

Finally, the posture dynamic model is:

\[
\begin{pmatrix}
\dot{x} \\
\dot{y} \\
\dot{\theta} \\
\dot{\phi}_1 \\
\dot{\phi}_2 \\
\dot{\eta}_a \\
\dot{\zeta}
\end{pmatrix} = \begin{pmatrix}
(2\cos(\theta)\cos(\beta) \cos(\beta) - L\sin(\theta) \sin(\beta))\eta_a \\
(2L\sin(\theta)\cos(\beta) + L\cos(\theta) \sin(\beta))\eta_a \\
\sin(\beta)\eta_a \\
\zeta \\
v_1 \\
v_2
\end{pmatrix}
\]

(3.37)
Up to now, all equations are found using mathematical formulae without giving any physical interpretation. Instead of using the above mathematical formulae, one can also obtain the same model by using the mechanics. The physical meaning of the configuration dynamic model is elaborated in the following paragraphs.

Explaining the physical meaning of $\eta_a$ will be a good starting point. $\eta_a$ represents the forward velocity of the front wheel of the vehicle divided by $2L$. Using this fact, $\phi_1$, the rotational velocity of the front wheel, can be better understood:

$$\phi_1 = -\frac{V}{r} = -\frac{2L\eta_a}{r} = -\frac{2L}{r}\eta_a$$  \hspace{1cm} (3.38)

where $V$ is the forward velocity of the front wheel.

Knowing the fact that the rear wheel is the instantaneous centre of rotation, the equations of $\dot{\theta}$, $\dot{x}$ and $\dot{y}$ can be derived using the following physical relations.

In order to find $\dot{\theta}$, the principle of rotation around a fixed point (rear wheel) and the following relation is used. The perpendicular velocities are:

$$\dot{\theta}_L = \frac{\sin(\beta)V}{2} = \frac{\sin(\beta)(2L\eta_a)}{2} = \sin(\beta)L\eta_a$$  \hspace{1cm} (3.39)

So:

$$\dot{\theta} = \sin(\beta)\eta_a$$  \hspace{1cm} (3.40)

Similar to $\dot{\theta}$, derivations of $\dot{x}$ and $\dot{y}$ use the same principle, being the instantaneous point of zero velocity.

First the forward velocity is decomposed into its components (the corresponding angle is $\beta$), then each velocity vector is again decomposed into its components (the corresponding angle is $\theta$). Finally, they are added or subtracted according to their signs. The resulting equations are:

$$\dot{x} = V\cos(\beta)\cos(\theta) - 0.5V\sin(\beta)\sin(\theta) = (2L\cos(\theta)\cos(\beta) - L\sin(\theta)\sin(\beta))\eta_a$$  \hspace{1cm} (3.41)

$$\dot{y} = V\cos(\beta)\sin(\theta) + 0.5V\sin(\beta)\cos(\theta) = (2L\sin(\theta)\cos(\beta) + L\cos(\theta)\sin(\beta))\eta_a$$  \hspace{1cm} (3.42)

One should note that the division of the velocities by 2 is due to translating the velocity at the front wheel to the middle of the bicycle.

Furthermore, $\zeta$ represents the steering angle velocity and $\dot{\zeta}$ represents the steering angle acceleration.
CHAPTER 3

3.2 Controller design for the bicycle model

The aim of the study is to develop a controller for a car which can track a certain trajectory defined according to an arbitrary point of the car. This arbitrary point is assumed to be the center of the car at which the origin of the coordinate axes is defined, and the reference trajectory is in the form of \([x_{\text{ref}}, y_{\text{ref}}, \theta_{\text{ref}}]\) in this particular case. The first step in the development of the controller for the bicycle model is to define the controller point which is also called as the virtual control point. Due to the nonlinearities involved in the system, linearization is required in order to use linear control techniques. The output linearizing vector, \(z_1\), denotes the virtual control point and has dimension of \(\delta_s + \delta_m = 2\) [CaBa96]. Since the system is described by the posture variables \(q = [x, y, \theta]\) and the dimension of \(z_1\) is 2, there will be a point tracking problem, i.e. \(\theta\) will not be controlled if a point on the bicycle such as \(x\) and \(y\) is chosen, for example. Therefore, a point in front of the bicycle is selected which will make the variables equal to the reference signals in time in case of the presence of forward velocity. This control point is shown in Figure 3.4.

Before defining the control point, the dynamic model to be controlled using “input-output linearization by state feedback” [NoCa95], [PIVi06] is re-written, which is also mentioned in (3.37):

\[
\dot{q}_1 = \begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \\ \dot{\beta} \end{pmatrix} = S_1(q_1)\eta = \begin{pmatrix} (2L\cos(\theta)\cos(\beta) - L\sin(\theta)\sin(\beta)) \\ (2L\sin(\theta)\cos(\beta) + L\cos(\theta)\sin(\beta)) \\ \sin(\beta) \\ 0 \end{pmatrix} \begin{pmatrix} \eta_x \\ \eta_y \\ \eta_\theta \\ \eta_\beta \end{pmatrix}
\]

(3.43)

and

\[
\dot{\eta} = \nu
\]

(3.44)

The first step is to define the control point, \(z_1\) as:
where $\eta_a$ is an invertible function that maps from $\theta$ to $\alpha$. 

Since the inputs appear in the second derivative, the nonlinear system is said to have a relative degree of 2.

Although $z_1$ and $z_2$ and their derivatives are enough to track the reference inputs, one more vector, $z_3$, which will deal with the internal dynamics is needed. With the addition of $z_3$, full mapping between the controller parameters and the controlled parameters will be present. Even in the absence of $z_3$, the tracking could be achieved; however, the orientation of the vehicle would oscillate along the reference trajectory, which is obviously undesired. $z_3$ assures the internal stability for all cases. Therefore, $z_3$ is defined as:

$$z_3 = k(q_1)$$

where $k$ is an invertible function that maps from $q_1$ to $z_3$. 

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The following representations summarize the above explained procedure:

\[ z_1 = h(q_1) \]  
\[ z_2 = H(q_1)\eta \]  
\[ z_3 = k(q_1) \]  

Furthermore, \( \dot{z}_3 \) equals:

\[ \dot{z}_3 = \frac{\partial k}{\partial q_1} \dot{q}_1 = \frac{\partial k}{\partial q_1} S\eta = \frac{\partial k}{\partial q_1} SH^{-1}z_2 = Q(q_1)z_2 \]  

It should be noted that the previous equation has a solution if and only if \( H \) is a non-singular matrix. Therefore, \( z_1 \) should be chosen accordingly which makes \( H \) invertible for all cases. Fortunately, the determinant of \( H \) matrix is equal to \( 2e_cL \) which assures that \( H \) is not a singular matrix for all cases.

In order to linearize the system the following equality is needed:

\[ \dot{z}_2 = \dot{z}_1 = Hv + b(q_1, \eta) = w \]  

and so, the following equation is derived:

\[ v = H^{-1}(w - b(q_1, \eta)) \]  

The solution for the input \( w \) making the error in \( z_1 \) and \( z_2 \) zero is:

\[ w = \dot{z}_{1\text{ref}} - K_d(\dot{z}_1 - \dot{z}_{1\text{ref}}) - K_p(z_1 - z_{1\text{ref}}) \]  

where \( K_d \) and \( K_p \) are the positive definite gains of the PD controller.

The final system is:

\[ \dot{z}_1 = z_2 \]  
\[ \dot{z}_2 = w \]  
\[ \dot{z}_3 = Q(q_1)z_2 \]  

The final step is to examine whether the internal dynamics are stable or not since the control design must account for the whole dynamics. Studying the zero-dynamics will make possible to draw conclusions about the stability of the internal dynamics [SLi91]. A system can be written in the normal form as [SLi91]:

\[ \dot{z}_1 = \dot{z}_2 = 0 \]  
\[ \dot{z}_3 = P(0, z_3) \]  

where (3.65) denotes the zero-dynamics of a nonlinear system.
Considering our case, the zero-dynamics are given by (3.63). From this equation, it should be noticed that $\dot{z}_3$ is a function of $z_2$. Furthermore, $z_3 = [\theta \dot{\beta}]^T$ would be an appropriate choice [WiSi96]. The zero-dynamics obtained by setting $z_1 = z_2 = 0$, which results in:

$$\dot{z}_3 = [\dot{\Theta} \dot{\beta}]^T = 0$$

(3.66)

As a result, it is shown that the internal dynamics are marginally stable though not asymptotically stable. However, defining the control point at the front of the vehicle guarantees the stability for velocities larger than zero.

A schematic overview of the controlled system is given in Figure 3.5.

![Block scheme of the controlled system](image)

Figure 3.5: Block scheme of the controlled system

### 3.3 Summary

In this chapter, the bicycle model and its controller which will be used later in the report is designed. In the derivation of the model, kinematic constraints are used. By means of these constraints and dynamical equations, the posture dynamic model is derived. Furthermore, the physical interpretation of the model is given using the parameters in the posture dynamic model.

After obtaining the required model, a controller is designed using “input-output linearization by state feedback” which enables the use of linear controllers. Then a PD-controller is designed and the internal stability of the system is checked.
CHAPTER 4

SYSTEM DEFINITION AND STRING STABILITY

In this chapter, basically, the structure of the system and the notion of string stability are explained in detail. Firstly, in Section 4.1, the system structure and the assumptions are addressed. Then, two different concepts are proposed for a string stability approach in Section 4.2. In Section 4.3, the spacing policies that will be used in the study are described. Finally, in Section 4.4, the approach for string stability assessment is presented.

4.1 The system structure

The objective of this report is to obtain a cooperative driving system by means of motion synchronization (see Chapter 1). For the realization of this objective, the following topics should be addressed.

Vehicles in the formation system, measure their own absolute velocity and acceleration using sensors. Furthermore, it is possible to obtain the absolute position using GPS in the absence of infrastructure. However, this system is known to be very inaccurate; therefore, instead of absolute position, relative positions are used, which can be measured using radar. Since V2V wireless communication is a must for mutual coupling, vehicles can transmit velocity and acceleration information to each other. Besides obtaining from wireless communication, vehicles can also measure the relative velocity between themselves and the preceding vehicle using the radar. Time delays which may be present in the wireless communication and sensor measurements will not be considered in the study. System analysis in the presence of communication and measurement delays can be regarded as a future work.

The formation structure can be formed as either centralized or decentralized (see Section 2.2.1). If it is a centralized structure, then the reference trajectory of the virtual structure is determined by the infrastructure and the vehicles are informed about their reference trajectories which are the transformed versions of the reference trajectory of the virtual structure. If it is a decentralized structure, which makes the project more feasible as it does not require investment on roads, the reference trajectory of the virtual structure is defined by one of the vehicles, preferably the leading one.
4.2 String stability concepts

The basic information about string stability for the 1D applications which can be found in literature is presented in Chapter 2. The control strategy of this study will be evaluated by means of string stability using two different concepts; string stability with one sided coupling and string stability with two sided coupling. Therefore, platoons will be examined according to both of these notions and different conclusions will be given. In addition to that, the assessment of string stability will be performed slightly different for 1D and 2D cases. In the 1D cases, the only concern is to obtain a string stable platoon longitudinally, whereas, in the 2D cases, it is important to obtain a string stable platoon both longitudinally and laterally.

4.2.1 String stability—one sided coupling

In order to consider a vehicle platoon to be string stable in the presence of one sided coupling, the following requirement is sufficient which is also given in the Chapter 2:

\[ \|e_1(t)\|_{\infty} \geq \|e_2(t)\|_{\infty} \geq \cdots \geq \|e_N(t)\|_{\infty} \]  

(4.1)

From the above definition it is easy to understand that regardless of the vehicle that is disturbed, the distance error should get smaller upstream the platoon, i.e. subscript 1 always defines the leading vehicle of the platoon.

4.2.2 String Stability—two sided coupling

In this string stability concept, specific importance is given to the disturbed vehicle which is not important in the one sided coupling concept. Although they are different definitions, the condition given in (4.1) still can be used for this concept by modifying it with the above condition. Now, the subscript \(i\) defines the disturbed vehicle and by repeating (4.1) in both directions, the following requirement is obtained:

\[ \|e_1(t)\|_{\infty} \leq \cdots \leq \|e_{i-1}(t)\|_{\infty} \leq \|e_i(t)\|_{\infty} \geq \|e_{i+1}(t)\|_{\infty} \geq \cdots \geq \|e_N(t)\|_{\infty} \]  

(4.2)

4.2.3 String stability: 1D vs. 2D

In the literature, the string stability concept is used only for longitudinal vehicle followings. However, it is possible to extend this notion to two dimensions by modifying the concept. It is known that the main idea in longitudinal string stable platoon design is the attenuation of the errors or other signals such as velocity and acceleration (see Section 4.4.1). Thus, it is desired that the errors decrease upstream the platoon or when moving away from the disturbed vehicle. These errors propagate longitudinally, and these should be damped out along the platoon. If an analogy is made with the 2D application, the longitudinal propagation of errors now becomes circular propagation which should again be damped out along the platoon.

The 2D string stability notion can be better understood with the following example. In a multi-lane, busy road, vehicles usually drive side by side and very close to each other longitudinally. The longitudinal controller can handle the changes in acceleration or deceleration and cope with very close spacing distances between the vehicles. However, sometimes it is also necessary to have a
lateral controller which can deal with a sudden steering resulting from any unexpected condition. In those cases, the vehicle at the side should also steer in order to prevent a possible collision. If this sideways movement is not controlled, it can amplify in the successive lanes and finally result in a collision of the vehicle at the very side of the road. If a 2D (longitudinal + lateral) controller is used, the errors are considered in both longitudinal and lateral direction and so that the sideways movement error signals should also be attenuated in lateral direction. By means of this strategy, the vehicle at the very side of the road may not feel the lateral movement disturbance of the other vehicle and can drive safely. In the report until Chapter 5, all of the derivations are made for the 1D case. In Chapter 5, these derivations are generalized to 2D and corresponding simulations are performed.

4.3 Inter-vehicle following policy

Various vehicle following policies in the literature are presented in Section 2.2.2.2. As mentioned, the type of the policy is more important regarding string stability for ACC equipped vehicles which do not have the information of acceleration of the preceding vehicle or the trajectory which should be tracked. In the literature, it is stated that it is impossible to obtain a string stable platoon for ACC equipped vehicles using the constant distance policy. On the contrary, it is stated to be possible for CACC equipped vehicles, even though the type of the policy is still important in the string stability characteristics [SwHe99].

In the study, firstly, the control strategy is designed as vehicles have constant distance between each other. This desired distance, therefore, does not change with any factors such as velocity, relative velocity, etc.. For a 1D simulation, desired inter-vehicle distance is:

\[ d_{i,d}(t) = r_i \]  \hspace{1cm} (4.3)

where \( r_i \) is a constant value.

Secondly, the constant time-headway policy as referred to in the literature [GeFr98], [LiPe00] is used to improve string stability. The general formulation of the constant time-headway policy is:

\[ d_{i,d}(t) = r_i + \tau v_i(t) \]  \hspace{1cm} (4.4)

where \( r_i \) is the desired inter-vehicle distance at rest, \( \tau \) is the headway time and \( v_i \) is the velocity of the vehicle \( i \). Furthermore \( \tau \) equals 0.3 seconds which is proposed by [IoCh93] as a safe rule for vehicle following, and \( r_i \) will be equal to the one used in the constant distance policy for a fair comparison. Although \( \tau \) is set to the mentioned value, different values can also be used and these values can be compared in future studies.
4.4 Assessment of string stability

4.4.1 Motivation of using absolute position transfer functions for string stability

In the string stability concept, it is desired to avoid the signal amplification upstream the platoon, which can be caused by any aggressive or fluctuant driving behavior. It is known from the literature [NaVu09] that string stability can be examined by using input, output or error signal string stability functions. For homogenous platoons, it is possible to use any of them for the assessment of the string stability. However, for heterogeneous platoons which reflect the real vehicle conditions, it is important to use a string stability function that is independent from previous vehicle’s characteristics. Therefore, in this section, error and output string stability functions will be investigated.

The controller that will be used for formation is the PD controller designed in Section 3.2 at (3.60) with the addition of mutual coupling terms. Therefore, the same theory is used as given in Section 2.3 for mutual synchronization of robots. The basic formulation of the formation controller is given below and will be explained in detail in Chapter 5.

\[
w_i = \ddot{x}_{i,\text{ref}} - K_d,i(x_i - \dot{x}_{i,\text{ref}}) - K_p,i(x_i - x_{i,\text{ref}}) - \sum_{j=1,j \neq i}^{n} K_{p,ij}(e_i - e_j) - \sum_{j=1,j \neq i}^{n} K_{d,ij}(\dot{e}_i - \dot{e}_j)
\]

where \(w_i\) denotes the formation controller, \(x_i\) is the position of vehicle \(i\), \(x_{i,\text{ref}}\) is the reference position of vehicle \(i\), \(n\) is the total number of the vehicles in the formation, \(K_{p,ij}\) and \(K_{d,ij}\) are the positive definite gain matrices which denote the trade-off between tracking the trajectory and keeping the formation and \(e_i = x_i - x_{i,\text{ref}}\).

Firstly, the error string stability function will be considered as a candidate, which is given by the transfer functions from reference to position error of vehicle \(i\) and \(i - 1\):

\[
SS_{E,i}(s) = \frac{E_i(s)}{E_{i-1}(s)} = \frac{E_i(s)}{X_0(s)} \frac{X_0(s)}{E_{i-1}(s)}
\]

where \(E_i\) is the Laplace transform of the position error of vehicle \(i\) and \(X_0\) is the Laplace transform of the position of the reference vehicle.

A three vehicle platoon, one being the reference vehicle, is used for the evaluation of the error string stability function. It is hard to obtain a general equation for any number of vehicles in the platoon since the number of mutual coupling terms increase and makes the equations very complex. The platoon and the control scheme are shown in Figure 4.1.

The vehicle shown in this figure is modeled as an ideal vehicle including the feedback linearization block as \(P_i = k_{p,i}s^{-2}\) which is only valid for the 1D case and used for simplicity. Here, \(k_{p,i}\) denotes the dynamical characteristics of vehicle \(i\).

Furthermore, the path tracking controller is modeled as a PD feedback controller according to:
where $K_{p,i}$ and $K_{d,i}$ are the proportional and derivative gains, respectively; (4.7) stands for the representation of the mutual coupling controller.

Using the scheme in Figure 4.1, the transfer function of the first vehicle is derived (see Appendix A for the derivation):

\[
E_1 = \frac{1 - s^2 P_1 + (1 - s^2 P_2)C_{12}P_1}{1 + C_{c1}P_1 - \frac{C_{12}P_1 C_{23}P_2}{1 + C_{c2}P_2}}
\]  \hspace{1cm} (4.8)

where $P_i$ is the model of vehicle $i$ and $C_{ik}$ is the mutual coupling controller of vehicle $i$ to vehicle $k$. In addition, $C_{c,i}$ is:

\[
C_{c,i} = C_i + \sum_{k=1,i\neq k}^{n} C_{ik}
\]  \hspace{1cm} (4.9)

For this specific case, the transfer function of vehicle 2 will be the same as for vehicle 1, so the subscripts interchange since the three vehicle platoon shows a perfect symmetry. Therefore, the error string stability function is:
(4.10) shows that $SS_{E,2}(s)$ depends on the vehicle 1’s characteristics since they show up in the equation which is undesired. Another way to judge the string stability functions is to examine their steady state values. At least, it is desired to obtain a string stability function which is independent from previous vehicle’s characteristics at steady state.

When the low frequency asymptotic value of $SS_{E,2}(j\omega)$, i.e. $\lim_{\omega \to 0} |SS_{E,2}(j\omega)|$, is calculated, the following result is found, showing the dependency of $SS_{E,2}(j\omega)$ on the vehicle characteristics:

$$
\lim_{\omega \to 0} |SS_{E,2}(j\omega)| = \lim_{\omega \to 0} \left| \frac{C_{c2}(k_{p2} - k_{p1}k_{p2}) + (1 - k_{p2})C_{12}}{C_{c1}(k_{p1} - k_{p1}k_{p2}) + (1 - k_{p1})C_{21}} \right|
$$

(4.11)

It can be interpreted from (4.11) that error string stability is dependent on the previous vehicle’s characteristics even at steady state. This result can also be seen from Figure 4.2 that the low-frequency asymptotic value of the string stability function changes for different vehicle configurations.

Figure 4.2: Bode plot of error string stability function of a three vehicle platoon where the properties of vehicle 1 are changed for three simulations. The first shows the nominal vehicle, the second shows a vehicle with a higher bandwidth tracking controller $C_1$ and the third shows a vehicle with an increased gain $k_{p1}$. Vehicle 2 has the same configuration for all simulations.

Figure 4.2 shows that if the first vehicle has a higher bandwidth path tracking controller $C_1$, the string stability function is larger than 1 as the frequency goes to 0. Furthermore, if the first vehicle has a higher plant gain $k_{p1}$, the string stability function is smaller than 1 as the frequency goes to 0.
The other proposed evaluation candidate is the position string stability function which is given by the transfer functions from reference to position of vehicle \( i \):

\[
SS_{X,i}(s) = \frac{X_i(s)}{X_{i-1}(s)} = \frac{X_i(s)}{X_0(s)} \frac{X_0(s)}{X_{i-1}(s)}
\]  

(4.12)

Then the transfer function for vehicle 1 is found from Figure 4.1 as (see Appendix A for the derivation):

\[
\frac{X_1}{X_0} = \frac{C_1P_1 + s^2P_1 + \frac{C_{12}P_1(C_2P_2 + s^2P_2)}{1 + C_{c2}P_2}}{1 + C_{c1}P_1 - \frac{C_{12}P_1C_{21}P_2}{1 + C_{c2}P_2}}
\]  

(4.13)

Similar to (4.8), the transfer function of vehicle 2 will be the same as for vehicle 1, instead the subscripts interchange. Therefore, the output string stability function is:

\[
SS_{X,2}(s) = \frac{X_2(s)}{X_1(s)} = \frac{(C_2P_2 + s^2P_2)(1 + C_{c1}P_1) + (C_1P_1 + s^2P_1)C_{21}P_2}{(C_1P_1 + s^2P_1)(1 + C_{c2}P_2) + (C_2P_2 + s^2P_2)C_{12}P_1}
\]  

(4.14)

(4.14) also shows that the output string stability function depends on the vehicle 1’s characteristics. Therefore, the next step is to examine its steady state behavior.

The low frequency asymptotic value of the output string stability function will be considered as it was done for the error string stability function. After some simplifications and cancelations, the steady state value of output string stability function is calculated as:

\[
\lim_{\omega \to 0} |SS_{X,2}(j\omega)| = \lim_{\omega \to 0} \left| \frac{k_{p1}k_{p2}C_{12}C_{21}}{k_{p1}k_{p2}C_{21}C_{12}} \right| = 1
\]  

(4.15)

As an illustration, the same result can also be observed from Figure 4.3 which is performed with different vehicle configurations which are stated before.

Figure 4.3 shows that the steady state value of the output string stability function is equal to 1 regardless of the vehicle dynamics, tracking controllers and mutual coupling controllers, which concludes that output (position) string stability functions should be used for heterogeneous type of traffic conditions as well as homogenous ones.

The last string stability function proposed in [NaVu09], the input string stability function, is not investigated in this study since it is shown by [NaVu09] that it is highly dependent on the vehicle characteristics.
Figure 4.1: Bode plot of error string stability function of a three vehicle platoon where the properties of vehicle 1 are changed for three simulations. The first shows the nominal vehicle, the second shows a vehicle with a higher bandwidth tracking controller $C_1$ and the third shows a vehicle with an increased gain $k_{p1}$. Vehicle 2 has the same configuration for all simulations.

To conclude, it is seen that for the designed controller, there is no string stability function which is independent from the preceding vehicle’s characteristics. However, the output string stability function exhibits that it is independent from vehicle characteristics at steady state. Therefore, in the rest of the report, the output string stability function will be the criterion in assessment of string stability in frequency domain approaches. The time domain approach pointed out in (4.1) is still applicable.

4.4.2 String stability conditions for the formation structure

By means of a number of pre-simulations, it is seen that the usual way of the assessment of string stability, i.e. by defining the transfer functions from the reference to distance error or position for each vehicle, is not suitable for the virtual structure approach as the string stability given in (4.14) is always found to be equal to 1, i.e. marginally string stable, for any perturbations or initial errors in the reference trajectory. The reason is the fact that the reference trajectories and the controllers - except the mutual coupling terms - are the same for each vehicle, so that all of the vehicles are affected at the same level from the changes in the reference trajectory. The mutual coupling terms then will have no effect since the errors will cancel each other as they always remain the same ($e_i = e_j, \dot{e}_i = \dot{e}_j$ for any $i$ and $j$). This situation forms a special case for the virtual structure formation; therefore, the output string stability function should be obtained by another method.

Instead, the string stability can be examined by means of a fictional disturbance in one of the vehicles. In that case, the behavior of the vehicles will be different since the mutual coupling controllers will behave differently and the errors will not cancel each other. Then string stability function then becomes:
In Figure 4.4, a position disturbance on the first vehicle is given as an example and transfer functions are derived accordingly. The physical meaning of this disturbance can be regarded as a possible (continuous) off-track case of vehicle 1 where vehicle 0 is the reference. A 3+1 vehicle platoon is used since the propagation or the attenuation of the error can be seen more easily.

Using Figure 4.4, the transfer function from disturbance on the first vehicle to first vehicle’s position is found as:

\[
\frac{X_1}{d} = \frac{C_{c1}P_1 - K_2 C_{c2}P_1 - K_3 C_{c3}P_1}{1 + C_{c1}P_1 - K_2 C_{c2}P_1 - K_3 C_{c3}P_1}
\]

where

\[
C_{c,i} = C_i + \sum_{k=1, i \neq k}^{n} C_{ik}
\]

and

\[
K_2 = \frac{(1 + C_{c2}P_2)C_{c1}P_2 + C_{c1}P_3C_{c2}P_2}{(1 + C_{c2}P_2)(1 + C_{c3}P_3) - C_{c2}P_2C_{c3}P_2}
\]

\[
K_3 = \frac{(1 + C_{c2}P_2)C_{c1}P_3 + C_{c1}P_2C_{c3}P_3}{(1 + C_{c2}P_2)(1 + C_{c3}P_3) - C_{c2}P_2C_{c3}P_2}
\]

Accordingly the remaining vehicles’ transfer functions are found as:

\[
\frac{X_2}{d} = \frac{(1 + C_{c3}P_3)C_{c1}P_2 + C_{c1}P_3C_{c2}P_2}{(K_2 C_{c1}P_1 + K_3 C_{c3}P_1 - C_{c1}P_1 - 1) [(1 + C_{c2}P_2)(1 + C_{c3}P_3) - C_{c2}P_2C_{c3}P_2]}
\]

\[
\frac{X_3}{d} = \frac{(1 + C_{c2}P_2)C_{c1}P_3 + C_{c1}P_2C_{c3}P_3}{(K_2 C_{c1}P_1 + K_3 C_{c3}P_1 - C_{c1}P_1 - 1) [(1 + C_{c2}P_2)(1 + C_{c3}P_3) - C_{c2}P_2C_{c3}P_2]}
\]

It is obvious that these transfer functions are only valid for a vehicle platoon of 4 vehicles. When the number of the vehicles increases, additional terms are added to the transfer functions. However, the 4 vehicle platoon is sufficient to judge string stability since two string stability functions can be derived and therefore, their tendency for the platoons with increased number of vehicles can be interpreted.
For string stability, the following condition should be satisfied for every $\omega$:

$$
|SS_i(j\omega)| = \left| \frac{X_i(j\omega)}{X_{i-1}(j\omega)} \right| = \left| \frac{X_i(j\omega)}{d} \right| \left| \frac{d}{X_{i-1}(j\omega)} \right| \leq 1
$$

When (4.23) is used for the vehicles 2 and 3, which do not encounter the disturbance directly, the following relation is derived using (4.21) and (4.22):
 SYSTEM DEFINITION AND STRING STABILITY

\[ SS_3(s) = \frac{X_3(s)}{X_2(s)} = \frac{(1 + C_{c2}P_2)C_{31}P_3 + C_{21}P_2C_{32}P_3}{(1 + C_{c3}P_3)C_{21}P_2 + C_{31}P_3C_{23}P_2} \quad (4.24) \]

whose absolute value is desired to be smaller than or equal to 1 for signal suppression along the platoon.

The mutual controllers are defined according to their breakpoint frequencies for simplicity in derivations as:

\[ C_{i,k} = \omega_{br_{i,k}} \left( \omega_{br_{i,k}} + s \right) \quad (4.25) \]

where \( \omega_{br_{i,k}} \) denotes the breakpoint frequency of mutual coupling controller \( i,k \). Breakpoint frequency is defined as the value where the Bode plot of a filter or a system experiences the largest change in direction and it is defined as \( K_p/K_d \) for a PD controller. Furthermore, it should be noted that only positive definite gains are used in the controllers.

Recall that there are two string stability concepts given in Section 4.1, namely string stability-one sided coupling and string stability-two sided coupling. Therefore, the conditions which will be derived in the remainder of this section will be studied for these two different concepts.

According to the first string stability concept, string stability-one sided coupling (see Section 4.2.1), the string stability is important only in the upstream direction. Therefore, the idea that a vehicle responds identically to any other vehicle’s position or velocity error is used in the mutual coupling parameter assignment. In other words, the mutual coupling gains of one vehicle are assigned to be the same for all other vehicles. The reason for reducing all mutual coupling terms of a vehicle to a single term is to make the derivation of the condition about mutual coupling terms easier and be able to obtain a condition that can be used for any vehicle number platoon.

Therefore, according to the string stability-one sided coupling concept, it is stated that \( C_{12} = C_{13}, C_{21} = C_{23}, C_{32} = C_{31} \) and \( C_1 = C_2 = C_3 \). Furthermore, here it is assumed that the bandwidth of the path tracking controller is considerably larger than the bandwidth of the mutual coupling controller, which reduces \( C_{c1} \) to \( C_i \). By means of this assumption and the relations between the coupling terms, (4.23) and its condition can be simplified to:

\[ SS_3(s) = \frac{X_3(s)}{X_2(s)} = \frac{C_{21}P_2C_{31}P_3 + C_{21}P_2C_{32}P_3}{C_{31}P_3C_{21}P_2 + C_{31}P_3C_{23}P_2} \quad (4.26) \]

whose magnitude should be smaller than or equal to 1. Notice that the terms \( C_{21}P_2C_{32}P_3 \) and \( C_{31}P_3C_{23}P_2 \) are equal to each other. It is also know that:

\[ |SS_3(j\omega)| = \left| \frac{X_3(j\omega)}{X_2(j\omega)} \right| = \frac{|C_{21}P_2C_{31}P_3 + C_{21}P_2C_{32}P_3|}{|C_{31}P_3C_{21}P_2 + C_{31}P_3C_{23}P_2|} \leq 1 \quad (4.27) \]

Therefore for string stability, it is required that:

\[ |C_{21}P_2C_{31}P_3 + C_{21}P_2C_{32}P_3| \leq |C_{31}P_3C_{21}P_2 + C_{31}P_3C_{23}P_2| \quad (4.28) \]
Although \( C_{21} P_2 C_{32} P_3 \) and \( C_{31} P_3 C_{23} P_2 \) are equal to each other they cannot be cancelled due to the triangle inequality which indicates that:

\[
|a + b| \leq |a| + |b|
\]  

(4.29)

However, since it is assumed that the bandwidth of the path tracking controller is considerably larger than the bandwidth of the mutual coupling controller, the influence of the second terms are small with respect to the first terms in the fractions of (4.27). Therefore, the inequality shown in (4.29) can be regarded as equality. When \( C_{21} P_2 C_{32} P_3 \) and \( C_{31} P_3 C_{23} P_2 \) terms are cancelled from both sides of (4.28), the following relation is derived:

\[
\frac{|X_3|}{|X_2|} \equiv \left| \frac{C_{31}}{C_{21}} \right| \leq 1
\]  

(4.30)

resulting in:

\[
|C_{31}| \leq |C_{21}|
\]  

(4.31)

which implies in the frequency domain that:

\[
\sqrt{\omega_{br31}^4 + \omega_{br31}^2} \leq \sqrt{\omega_{br21}^4 + \omega_{br21}^2}
\]  

(4.32)

from which follows:

\[
\omega_{br31} \leq \omega_{br21}
\]  

(4.33)

To conclude and generalize, (4.33) states that if the breakpoint frequencies of the mutual coupling controllers are selected in a decreasing manner along the platoon, it is possible to achieve a one sided string stable platoon. This result is also supported by Bode plot analysis for \( SS_3(j\omega) \) and given in Figure 4.5.

The formulation of the coupling controllers can be organized and generalized as follows:

\[
\omega_{br_{ik}} = \omega_i \quad ; \quad i = 1, 2, \ldots, n \quad , \quad k = 1, 2, \ldots, n \quad \wedge \quad k \neq i
\]  

(4.34)

and the corresponding strategy is:

\[
\omega_1 > \omega_2 > \cdots > \omega_n
\]  

(4.35)

where \( \omega_i \) denotes the breakpoint frequency of the mutual coupling controller \( i, k \).
Figure 4.5: Magnitude plot of the output string stability transfer function, $SS_3$. Upper left figure shows $SS_3$ of a platoon where $|C_{31}| \geq |C_{21}|$. Upper right figure shows $SS_3$ of a platoon where $|C_{31}| = |C_{21}|$. Bottom figure shows $SS_3$ of a platoon where $|C_{31}| \leq |C_{21}|$. Identical tracking controllers and identical vehicle characteristics are used.

From this figure it is also seen that the output string stability function is smaller than 1 at every frequency when the condition derived in (4.35) is used. However, it should be noted that the low-frequency asymptotic value does not converge to 1 which was expected according to the derivations made in (4.15). However, recall that the string stability derivation without a disturbance forms a special case where it is always equal to 1. On the contrary, this system encounters a disturbance and the string stability equations are derived according to this disturbance.

Up to now, the vehicles which do not encounter the disturbance directly are analyzed and the condition for string stability is given. However, to prove the full string stability for the platoon, the remaining output string stability function, $SS_2(s)$, should also be analyzed although it is obvious that this function will be smaller than 1 since the vehicle 1 encounters the disturbance. The output string stability function between vehicle 1 and 2 is derived as:

$$SS_2(s) = \frac{X_2(s)}{X_1(s)} = \frac{(1 + C_{c3}P_3)C_{21}P_2 + C_{31}P_3C_{23}P_2}{(1 + C_{c2}P_2)(1 + C_{c3}P_3) - C_{32}P_3C_{23}P_2} \ast \left(1 - \frac{1 + C_{c1}P_1 - K_2C_{12}P_1 - K_3C_{13}P_1}{C_{c1}P_1 - K_2C_{12}P_1 - K_3C_{13}P_1}\right)$$ (4.36)
From (4.36), it is difficult to obtain an analytical relation as done for \( SS_3(s) \). Therefore, it is analyzed numerically using Bode plot analysis. The resulting plot is given in Figure 4.6.

![Bode Diagram](image.png)

**Figure 4.6**: Magnitude plot of the output string stability transfer function, \( SS_2 \). The mutual coupling gains are selected as they decrease along the platoon. Same tracking controllers and same vehicle characteristics are used.

From this figure it is also seen that the output string stability function is smaller than 1 for the entire frequency band. Furthermore, note that the magnitude of \( SS_2(s) \) becomes zero for low frequencies since the disturbance has no influence on other vehicles at low frequencies.

According to the second string stability concept, string stability-two sided coupling, it is known that the error attenuation in both directions from the disturbed vehicle is important; therefore, the mutual coupling parameter assignment should be changed with respect to the previous concept. Now, a vehicle will respond differently to other vehicle’s position or velocity errors according to the relative distance in the platoon. For instance, it will respond at the same level to the distance errors of the vehicle which is one vehicle ahead and the vehicle which is one vehicle behind. On the other hand, this response of a vehicle will be at a different level for the distance errors of the vehicle which is one vehicle ahead and the vehicle which is two vehicles ahead.

Therefore, according to the above definition, the relation between the parameters can be shown as \( C_{12} = C_{21} = C_{23} = C_{32} \) which are not equal to \( C_{31} = C_{13} \) and also \( C_1 = C_2 = C_3 \). The same assumption for the path tracking controller is also valid in this concept. As a result, for the platoon in concern, the condition derived in (4.33) still holds since condition (4.31) can also be reached by using new mutual coupling parameter definitions and knowing that small terms can be ignored compared to the high breakpoint frequency tracking controller. Except for the coupling parameter definitions, the string stability-two sided coupling concept also reflects the same requirement with the string stability-one sided coupling since the disturbance is in the leading vehicle (reference vehicle is considered to be virtual, not leading). If there are more vehicles in front of vehicle 1, i.e. leading vehicle is not the disturbed vehicle, the magnitude increase of the mutual coupling terms will be the mirror image on the other side of the platoon.

In order to use the same notation with the one sided coupling concept, vehicle \( i \) is called to be the vehicle 1 and starting from this vehicle the other vehicles are numbered in an increasing manner in
both directions. Therefore, string stability analysis is carried out in two cases according to (4.2). So, subscript 1 not anymore defines the leading vehicle, instead the disturbed vehicle.

A handy formulation for the mutual coupling gains in the two sided string stability concept can be formed as:

\[
\begin{align*}
\omega_{br_{ik}} &= \omega_1 \quad \text{with } |i - k| = 1 \\
\omega_{br_{ik}} &= \omega_2 \quad \text{with } |i - k| = 2 \\
&\vdots \\
\omega_{br_{ik}} &= \omega_{n-1} \quad \text{with } |i - k| = n - 1
\end{align*}
\]  

(4.37)

(4.37) can also be written in a compact form:

\[
\omega_{br_{ik}} = \omega_j \quad \text{with } |i - k| = j; i = 1,2,\ldots,n, k = 1,2,\ldots,n \land k \neq i, j > 0
\]  

(4.38)

and the corresponding strategy is:

\[
\omega_1 > \omega_2 > \cdots > \omega_{n-1}
\]  

(4.39)

where \(\omega_i\) denotes the breakpoint frequency of a mutual coupling controller which is defined by (4.37).

The simulations performed for one sided coupling are also applicable here since the leading vehicle is the disturbed vehicle. These conclusions on the mutual coupling terms will be examined and supported by the simulations in the next chapter.

To sum up, the most important elements are the mutual coupling terms in a virtual structure formation, which basically determines the string stability behavior of the platoon. Therefore, the conditions on mutual coupling terms for a string stable platoon are derived in this section and will be examined with a vehicle model in the next chapter. It is possible to achieve string stability using (4.34) and (4.35) for one sided coupling and (4.38) and (4.39) for two sided coupling.

### 4.5 Summary

In this chapter, the structure of the information system, string stability notions, vehicle following policy and the conditions for a string stable platoon with the designed controller are presented. Two string stability concepts are introduced and the basic definitions for 1D and 2D platoon cases are given. Then, two inter-vehicle spacing policies, constant distance and constant-headway time policies are defined to be used in the simulations. Finally, the output string stability function is concluded to be the suitable string stability function in the analysis of a heterogeneous platoon as well as a homogeneous platoon. Using it, conditions about the mutual coupling terms are given for the two string stability concepts stated in Section 4.2.
CHAPTER 5

EXPERIMENTAL EVALUATION OF THE VIRTUAL STRUCTURE CONTROLLER

Although there are several formation techniques, described in Section 2.3, the so-called virtual structure approach will be used for the formation of vehicles [RoNi04], [IkJo06]. In the other methods, namely master-slave formation and behavioral formation [ReBe02], [DoPa07], [LaBe00], the formation lacks in robustness with respect to the perturbations in one of the vehicles. On the other hand, by means of mutual coupling, the virtual structure can always keep the formation. Therefore, a virtual structure controller will be designed in this chapter using the controller designed in Chapter 3. Although the main controller will remain the same, the virtual structure will be defined differently for the one and the two dimensional cases and will be given in two different sections. Then, using the vehicle model designed in Chapter 3, the virtual structure controllers will be simulated with different trajectories and the assessment of string stability will be presented.

In Section 5.1, the design of the virtual structure controllers for the 1D and the 2D cases are explained. Then, these controllers are used for the assessment of string stability in Section 5.2. Finally, Section 5.3 presents summary of this chapter.

5.1 Formation controller design

In Chapter 3, a PD controller which is responsible for the path tracking is designed using input-output linearization by state feedback [NoCa95], [PIVi06]. In this section, using the path tracking controller, a virtual structure controller is designed, which is responsible for keeping the formation by means of mutual coupling between the vehicles.

The primary step is to define a reference trajectory for the formation structure. This reference trajectory is also referred to as the trajectory of the virtual center. Each vehicle has also its own reference trajectory which is defined with respect to the reference trajectory of the virtual structure, either using constant or time-varying relative distance with respect to the trajectory of the virtual center. The notion of virtual structure reference trajectory and the individual reference trajectory can be better understood from Figure 5.1, where they are denoted by $P_0$ and $P_1$, respectively.
Then, a controller should be designed which ensures that the formation structure can track the virtual structure trajectory as a cooperative system. By means of the mutual coupling between the vehicles, the formation is kept although any vehicle encounters perturbations. The main controller still remains to be (3.60) - which is the first three terms of (5.1) - where additional terms are added, which are referred to as the mutual coupling terms. So, the proposed controller is:

\[
    w_i = \dot{z}_{i,\text{ref}} - K_{d,i}(\dot{e}_i) - K_{p,i}(e_i) - \sum_{j=1, j \neq i}^{n} K_{p,ij}(e_i - e_j) - \sum_{j=1, j \neq i}^{n} K_{d,ij}(\dot{e}_i - \dot{e}_j)
\]  

(5.1)

where \(w_i\) denotes the formation controller output, \(z_i\) is the position of the control point of vehicle \(i\), \(z_{i,\text{ref}}\) is the reference position of the control point of vehicle \(i\), \(n\) is the total number of vehicles in the formation, \(K_{p,ij}\) and \(K_{d,ij}\) are positive definite gain matrices of the mutual coupling and finally \(e_i\) and \(\dot{e}_i\) are the formation errors defined by:

\[
    e_i = z_i - z_{i,\text{ref}}
\]

\[
    \dot{e}_i = \dot{z}_i - \dot{z}_{i,\text{ref}}
\]  

(5.2)

The most important notions in the virtual structure, obviously, are the gain matrices of the mutual coupling. The values of these gain matrices directly affect the formation. If these factors are chosen to be large enough, vehicles will not be able to track their individual trajectories perfectly in the presence of perturbations; however, the formation will remain almost the same. On the contrary, if this value is chosen to be small, vehicles can follow their individual trajectories; however, one cannot
talk about formation anymore. Due to these facts, a decision should be made beforehand, keeping in mind the trade-off between keeping the formation and tracking the reference trajectory.

After defining trajectories and the formation controller for the bicycle model designed in Chapter 3, the conditions on mutual coupling terms derived in Section 4.4 which will also be mentioned later in this chapter are used in the simulations performed in this chapter.

5.1.1. Trajectory design for the 1D case

In design of the virtual structure formation for the one dimensional case, firstly the reference trajectory, $P_0(t)$, where the centre of the virtual structure is attached should be defined:

$$P_0(t) = [x_0(t), y_0]^T$$  \hspace{1cm} (5.3)

where $x_0(t)$ and $y_0$ are the coordinates defined with respect to the global frame.

Then, according to the definition of $P_0(t)$, the reference trajectory of the $i^{th}$ vehicle for the 1D case is defined as:

$$P_i(t) = [x_{i,ref}(t), y_{i,ref}]^T = P_0(t) + [l_{x_i}(t), 0]^T$$  \hspace{1cm} (5.4)

where $l_{x_i}(t)$ is the x-coordinate of $i^{th}$ vehicle with respect to the local frame which is attached to the virtual centre. If $l_{x_i}(t)$ is chosen as a constant, the spacing policy becomes a constant distance policy which is defined in (4.3). The other option for this study is defining it as a function of velocity, resulting in so-called constant time-headway policy which is defined in (4.4).

If the constant distance policy (see (4.3)) is used, the reference trajectory of vehicle $i$ is defined as follows:

$$x_{i,ref} = x_0 + l_{x_i}$$  \hspace{1cm} (5.5)

where $x_0$ is the position of the virtual centre and $l_{x_i} = r_i$ where $r_i$ is a real number, so (5.5) becomes:

$$x_{i,ref} = x_0 + r_i$$  \hspace{1cm} (5.6)

Then, the derivatives of the reference trajectory of vehicle $i$ are:

$$\dot{x}_{i,ref} = \dot{x}_0$$  \hspace{1cm} (5.7)

$$\ddot{x}_{i,ref} = \ddot{x}_0$$  \hspace{1cm} (5.8)

since $l_{x_i}$ is constant.

Instead of the constant distance policy, if the constant time-headway policy (see (4.4)) is used and inter-vehicle spacing is defined as $l_{x_i}(t) = r_i + \tau \dot{x}_i$, then (5.6) turns into:

$$x_{i,ref} = x_0 + r_i + \tau \dot{x}_i$$  \hspace{1cm} (5.9)
CHAPTER 5

The derivatives of (5.9) are:

\[ \dot{x}_{i,\text{ref}} = \dot{x}_0 + \tau \ddot{x}_i \]

\[ \ddot{x}_{i,\text{ref}} = \ddot{x}_0 + \tau \dddot{x}_i \]

Although measuring \( \dot{x}_i \) and \( \ddot{x}_i \) is explained in previous chapters, nothing has mentioned about \( \dddot{x}_i \). Since measuring \( \dddot{x}_i \) via sensors will add complexity to the structure, direct measurement of \( \dddot{x}_i \) is not proposed in this study. Instead it will be found by taking the numerical derivative of \( \ddot{x}_i \), which is unfortunately not very reliable.

It is obvious that \( y_{i,\text{ref}} \) is always equal to \( y_0 \) which is a constant value, therefore the derivatives of \( y_{i,\text{ref}} \) are zero at any time.

The only remaining variable to be defined in the individual reference trajectories with respect to the reference trajectory of the virtual structure is the orientation angle, \( \theta_{i,\text{ref}} \). However, it is apparent that the orientation angle is always 0 or constant for a 1D application; therefore all orientation angles are always equal to each other and their derivatives are 0.

If the controller is modified for the 1D cases, (5.1) becomes:

\[ w_i = \ddot{x}_{i,\text{ref}} - K_{d,i}(\dot{e}_i) - K_{p,i}(e_i) - \sum_{j=1,j\neq i}^n K_{p,ij}(e_i - e_j) - \sum_{j=1,j\neq i}^n K_{d,ij}(\dot{e}_i - \dot{e}_j) \]

5.1.2 Trajectory design for the 2D case

The definition of the reference trajectory of the virtual structure is slightly different for two dimensional applications. The most important difference is apparently the orientation angle definition which makes the trajectory definition more difficult for 2D.

At the design of the virtual structure formation for the two dimensional case, firstly the reference trajectory where the centre of the virtual structure is attached should be defined:

\[ P_0(t) = [x_0(t), y_0(t)]^T \]

where \( x_0(t) \) and \( y_0(t) \) are the coordinates defined with respect to the global frame. Then the reference trajectory of the \( i^{th} \) vehicle is defined as:

\[ P_i(t) = P_0(t) + [lx_i(t), ly_i(t)]^T \]

where \( lx_i(t) \) is the x-coordinate and \( ly_i(t) \) is the y-coordinate of \( i^{th} \) vehicle with respect to the local frame which is attached to the virtual centre. Similar to 1D, \( lx_i \) and \( ly_i \) can be chosen as either constant or a function of time.
The equations derived for $x_{i,\text{ref}}$ and its derivatives in Section 5.1.1 are also applicable for the 2D case. Moreover, $y_{i,\text{ref}}$ and its derivatives also yield the same equations where $y$ is used instead of $x$.

The remaining variable to be defined in the individual reference trajectories with respect to the reference trajectory of the virtual structure is the orientation angle, $\theta_{i,\text{ref}}$, which is different from the 1D case. Since the individual reference trajectory, $P_i(t) = [x_{i,\text{ref}}(t), y_{i,\text{ref}}]^T$, relates to the center of gravity, the orientation angle should be known in order to translate this point to the virtual control point reference, $z_{i,\text{ref}}$. However, a transformation is needed in calculating the orientation angle since the ICR is at the rear wheel of the vehicle. In general the orientation angle of a vehicle is defined as:

$$
\theta = \arctan \left( \frac{\dot{y} - L \dot{\theta} \cos(\theta)}{\dot{x} + L \dot{\theta} \sin(\theta)} \right) \quad (5.15)
$$

(5.15) can be better understood from Figure 5.2. In the model, velocities are measured at the middle of the vehicle as $\dot{x}$ and $\dot{y}$. However, the orientation angle $\theta$ is equal to the arctangent of the velocities measured at the rear axle. Therefore, the velocities of the center of gravity should be transformed into the velocities at the rear axle of the vehicle before putting into the arctangent function, which results in (5.15).

(5.15) can be reduced to for small angular velocities, $\dot{\theta}$:

$$
\theta = \arctan \left( \frac{\dot{y}}{\dot{x}} \right) \quad (5.16)
$$

Then, using (5.16) the orientation angle of the virtual structure and the vehicle $i$ are defined as:

$$
\theta_0 = \arctan \left( \frac{\dot{y}_0}{\dot{x}_0} \right) \quad (5.17)
$$
Further derivations about the orientation angle will be made for two cases; constant distance policy and constant time-headway policy.

If the constant distance policy is the concern, using (5.7), the following simple derivation holds:

$$
\theta_{i,\text{ref}} = \arctan \left( \frac{\dot{y}_{i,\text{ref}}}{\dot{x}_{i,\text{ref}}} \right) = \arctan \left( \frac{\dot{y}_0}{\dot{x}_0} \right) = \theta_0
$$

(5.18)

On the contrary, if the constant-time headway policy is the concern, the derivation gets complicated. Using (5.10) and (5.17) the following relation is obtained which includes the states of the vehicle:

$$
\theta_{i,\text{ref}} = \arctan \left( \frac{\dot{y}_{i,\text{ref}}}{\dot{x}_{i,\text{ref}}} \right) = \arctan \left( \frac{\dot{y}_0 + \tau \dot{y}_i}{\dot{x}_0 + \tau \dot{x}_i} \right)
$$

(5.19)

Using these trajectory definitions, the formation controller given in (5.1) can be directly used for 2D applications.

### 5.2 Formation controller assessment

A theoretical approach using ideal vehicle dynamics is presented in Chapter 4 and by means of frequency domain analysis, conditions for the mutual coupling terms in order to achieve string stable platoons are presented. In this section, the formation controller is assessed using the designed vehicle model and by using the conditions derived in Section 4.4, string stable platoons are desired to be achieved. In other words it will be the verification of the results of Chapter 4 on mutual coupling terms. For this purpose, time domain analyses are carried out in two sections which are for the 1D and the 2D applications. In this approach, two different trajectories are used and the position errors are analyzed for determining the string stability.

#### 5.2.1 String stability assessment using the 1D tracks

String stability is evaluated in this section using time domain analyses and it is assumed that vehicles just drive in longitudinal direction. String stability is going to be tested by means of a start error and a position and velocity disturbance in a vehicle. The platoon consists of 4 vehicles. The following conditions which are also given in (4.1) and (4.2) are used:

$$
\|e_1(t)\|_\infty \geq \|e_2(t)\|_\infty \geq \cdots \geq \|e_N(t)\|_\infty
$$

(5.20)

$$
\|e_1(t)\|_\infty \leq \cdots \leq \|e_{i-1}(t)\|_\infty \leq \|e_i(t)\|_\infty \geq \|e_{i+1}(t)\|_\infty \geq \cdots \geq \|e_N(t)\|_\infty
$$

(5.21)

where (5.20) stands for the one sided coupling and (5.21) stands for the two-sided coupling (see Chapter 4 for details).

In the analysis, the model designed in Chapter 3 and the formation controller designed in this chapter are used, which are respectively:
The controller parameters for each simulation are expressed in the figure tags.

Four simulation results are given for the one sided string stability and the two-sided string stability concepts using constant distance and constant-time headway policies. In assigning the mutual coupling terms, the conditions derived in Chapter 4 are considered, which are as follows (see section 4.2.2 for further details).

For one sided coupling:

\[
\omega_1 > \omega_2 > \ldots > \omega_n
\]  \hspace{1cm} (5.24)

where:

\[
\omega_i = \omega_{br_{ik}} \text{ with } i = 1,2,\ldots,n \text{ and } k = 1,2,\ldots,n \text{ and } k \neq i \hspace{1cm} (5.25)
\]

and for two sided coupling:

\[
\omega_1 > \omega_2 > \ldots > \omega_{n-1}
\]  \hspace{1cm} (5.26)

where:

\[
\omega_j = \omega_{br_{jk}} \text{ with } j = |i - k|; i = 1,2,\ldots,n \text{ and } k = 1,2,\ldots,n \text{ and } k \neq i, j > 0 \hspace{1cm} (5.27)
\]

All simulations in the 1D case start with a starting error of 0.5 m in the third vehicle and at 6 seconds a step disturbance to the third vehicle’s reference position is applied as done in the frequency analysis in Chapter 4. The reference trajectory profile used in the simulation is given in Appendix C.

The first simulation is presented for the one sided string stability concept and constant distance policy in Figure 5.3. Therefore, the breakpoint frequencies of the mutual coupling terms are assigned to decrease upstream the platoon (see (5.24) and (5.25)). The exact values are stated in the corresponding figure.

Figure 5.4 shows that the error in both cases decrease in the upstream direction, i.e. the error of vehicle 2 is less than the error of vehicle 1, which is a direct consequence of the mutual coupling term assignment as \(\omega_1 > \omega_2\). Since \(\omega_1\) is larger than \(\omega_2\), vehicle 1 reacts more to the error in vehicle 3 than vehicle 2 does. Due to the mentioned mutual coupling assignment, vehicle 2 gives more importance to trajectory tracking than keeping the formation, which results in a smaller position error.
error. Therefore, the error resulting from the starting error and disturbance can be attenuated upstream the platoon, which is the requirement of a string stable platoon.

![Figure 5.3: Position error due to a starting error and a disturbance in vehicle 3 (1D, constant distance policy, one sided coupling). The controller parameters are: the path tracking controller with a breakpoint frequency of 10 rad/s and the mutual coupling terms with breakpoint frequencies of $\{\omega_1 = 7.07, \omega_2 = 6.33, \omega_3 = 5.48, \omega_4 = 4.47\}$ rad/s where $\omega_{br,k,l} = \omega_l$; $l = 1, 2, ..., n$, $k = 1, 2, ..., n \land k \neq l$

![Figure 5.4: Zoomed view in the interesting areas of Figure 5.3.]

Further simulations revealing the effect of the mutual coupling term breakpoint frequencies and therefore their tuning are given in Appendix B in order to limit the number of simulation plots given in this chapter. Appendix B shows that the increase in the mutual coupling terms enhances the formation; however, results in a poor individual trajectory tracking. Therefore, general optimum values for the mutual coupling terms do not exist; they should be altered from application to application according to the objectives.
In the second simulation, the two-sided coupling concept using the constant distance policy is investigated and the results are given in Figure 5.5. Since the vehicle which is disturbed or starts with an initial error is vehicle 3, the breakpoint frequencies of the mutual coupling terms are assigned according to the relative distance between vehicle 3 and the vehicle of interest (see also (5.26) and (5.27)). The exact values are given in the corresponding figure.

Figure 5.5: Position error due to a starting error and a disturbance in vehicle 3 (1D, constant distance policy, two-side coupling). The controller parameters are: the path tracking controller with a breakpoint frequency of 10 rad/s and the mutual coupling terms with breakpoint frequencies of \(\omega_1 = 7.07, \omega_2 = 6.33, \omega_3 = 5.48\) rad/s where \(\omega_{br,j,k} = \omega_j\) with \(|i-k| = j; i = 1, 2, ..., n, k = 1, 2, ..., n, k \neq i\).

It can be interpreted from Figure 5.6 that the position errors of the vehicles decrease while moving away from vehicle 3, i.e. the position error of vehicle 2 and 4 are equal to each other and these are larger than the position error of vehicle 1. So, the string stability condition for the two-sided coupling concept is met with the designed formation control strategy.
In addition to the previous two simulations, two more simulations are presented for the 1D case. The only difference of these simulations from the previous ones is that constant-time headway policy is used instead of constant distance policy. The result for the one sided coupling is given in Figure 5.7.

![Figure 5.7: Position error due to a starting error and a disturbance in vehicle 3 (1D, constant-time headway policy, one sided coupling). The controller parameters are: the path tracking controller with a breakpoint frequency of 10 rad/s and the mutual coupling terms with breakpoint frequencies of \( \omega_1 = 7.07, \omega_2 = 6.33, \omega_3 = 5.48, \omega_4 = 4.47 \) rad/s where \( \omega_{br,ik} = \omega_i; i = 1, 2, \ldots, n, k = 1, 2, \ldots, n \wedge k \neq i \).](image)

![Figure 5.8: Zoomed view in the interesting areas of Figure 5.7.](image)

A similar result to the one with constant distance policy can be seen in Figure 5.8 since the error attenuates upstream the platoon. The only difference is that the magnitude of errors decreases with this policy. However, the main idea remains the same; the vehicle at the end (vehicle 4) feels the starting error or the disturbance less than the others. Therefore, the platoon is string stable and the two policies do not differ for formation control strategy.
The two-sided coupling concept for the 1D with a constant-time headway time is simulated and the results are given in Figure 5.9.

The discussions given in the previous simulation about constant-headway time are also applicable for this simulation. Also in Figure 5.10, it can be interpreted that the position error of vehicle 1 is smaller than the position errors of vehicle 2 and 4 which is the expected result.
CHAPTER 5

however, since it affects all vehicles, the result in terms of string stability remains the same. Therefore, unlike to CACC, headway time does not have much influence on virtual structure controlled platoons in terms of string stability.

5.2.3 String stability assessment using the 2D tracks

In this section, the string stability is evaluated using time domain analysis for 2D applications and it is done using the so-called o-track trajectory (see Appendix C for further details). Again a 4 vehicle platoon is used for the simulation and the starting error and position disturbance are the testing cases. With this section, it is desired to show that the results of the 1D case derived in Chapter 4 can also be generalized to the 2D case. The same vehicle model and formation controller as for the 1D case are used in the 2D simulations.

Since for a 2D case, it is rather difficult to refer to a vehicle as the leading vehicle or the follower, two-sided coupling approach is used in the assignment of the mutual coupling terms which is explained in the previous section. All cars are assumed to be driving in different lanes and their behavior is observed for the cases mentioned in the previous paragraph. The vehicle in the outer lane is denoted with vehicle 1 and the vehicle in the inner lane is denoted with vehicle 4 (see Figure 5.11 for the position figure).

In order to obtain a vehicle platoon where vehicles drive side to side in different lanes, \( l_x(t) \) and \( l_y(t) \) should not be taken constant, instead they should be defined as follows, which will result in vehicle trajectories shown in Figure 5.11:

\[
\begin{align*}
l_x(t) & = -a_l \sin(\theta(t)) \\
l_y(t) & = a_l \cos(\theta(t))
\end{align*}
\]  

(5.28)  

(5.29)

where \( a_l \) defines the distance between vehicles measured along the radius line. When the above values for \( l_x(t) \) and \( l_y(t) \) are used, the inter-vehicle spacing is always equal to \( a_l \) when measured along the radius line of the o-track and in the absence of a disturbance.

![Figure 5.11: Vehicle position graph of o-track](image-url)
Therefore, the individual vehicle reference trajectories become:

\[
\dot{x}_{i,\text{ref}} = \dot{x}_0 - a_i \cos(\theta_0) \dot{\theta}_0
\]  
\[\text{(5.30)}\]

\[
\dot{y}_{i,\text{ref}} = \dot{y}_0 - a_i \sin(\theta_0) \dot{\theta}_0
\]  
\[\text{(5.31)}\]

\[
\ddot{x}_{i,\text{ref}} = \ddot{x}_0 + a_i \sin(\theta_0) \dot{\theta}_0^2 - a \cos(\theta_0) \ddot{\theta}_0
\]  
\[\text{(5.32)}\]

\[
\ddot{y}_{i,\text{ref}} = \ddot{y}_0 - a \cos(\theta_0) \dot{\theta}_0^2 - a \sin(\theta_0) \ddot{\theta}_0
\]  
\[\text{(5.33)}\]

The analysis of string stability has some difficulties in 2D cases since there exist small errors in the reference tracking due to the path tracking controller design. The reason for these position errors is the control technique, since the controlled point is not the center of gravity of the vehicle, instead the virtual control point. However, all of the measurements are made according to the vehicles’ center of gravities. Also it should be noted that these errors can be ignored compared to the size of the track. The errors are shown in Figure 5.12.

Due to the above discussion, the disturbance is applied in two cases where \(x\) and \(y\) position errors are approximately zero in the interval of application time, which makes the analysis reliable.

In the simulation, vehicle 3 again starts on its trajectory with a starting error in both directions, a disturbance applied in \(y\) direction at approximately 7 seconds and in \(x\) direction at approximately 11 seconds.

![Figure 5.12: Position error in reference tracking in the o-track without any disturbance or starting error. Left one is the error in \(x\) position and right is the error in \(y\) position.](image)

Firstly, the position error response of the vehicles in \(x\) direction is given in Figure 5.13.
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Figure 5.13: Position error in x direction due to a starting error and a disturbance in vehicle 3 (2D, constant distance policy, two-sided coupling). The controller parameters are: the path tracking controller with a breakpoint frequency of 10 rad/s and the mutual coupling terms with breakpoint frequencies of \( \{\omega_1 = 7.07, \omega_2 = 6.33, \omega_3 = 5.48\} \text{ rad/s} \), where \( \omega_{br,i,k} = \omega_j \) with \( |i - k| = j \); \( i = 1, 2, ..., n \), \( k = 1, 2, ..., n \land k \neq i \).

Figure 5.14: Zoomed view in the interesting areas of Figure 5.13.

Figure 5.14 shows that the vehicles in the neighborhood of vehicle 3, i.e. vehicle 2 and vehicle 4, have larger error than vehicle 1, which makes the platoon string stable according to the two-sided coupling concept. In the response to the disturbance graph, it is seen that the error of vehicle 4 gets larger than the error of vehicle 2 in time although it should be entirely the same. The reason of it is the already existing errors in the path tracking controller which is shown in Figure 5.12. However, at the time of disturbance application, the responses are less affected from these pre-existing errors since they are approximately zero.

Secondly, the position error response of the vehicles in y direction is given in Figure 5.15.
Figure 5.15: Position error in y direction due to a starting error and a disturbance in vehicle 3 (2D, constant distance policy, two-sided coupling). The controller parameters are: the path tracking controller with a breakpoint frequency of 10 rad/s and the mutual coupling terms with breakpoint frequencies of \(\{\omega_1 = 7.07, \omega_2 = 6.33, \omega_3 = 5.48\}\) rad/s where \(\omega_{br,i,k} = \omega_j\) with \(|i - k| = j\); \(i = 1, 2, ..., n\), \(k = 1, 2, ..., n\) and \(k \neq i\).

Figure 5.16: Zoomed view in the interesting areas of Figure 5.15.

In the analysis of error response in y direction, unfortunately, it is not possible to judge about string stability using the starting error since the pre-existing error due to the path tracking controller starts to increase immediately after the start and dominates it, although it can be still possible but the errors are too small for the analysis. On the other hand, it is still possible to end up with a conclusion using the disturbance. In Figure 5.16, it is seen that the errors of vehicle 2 and 4 are equal to each other and smaller than the one of vehicle 1, as expected. Therefore, with the conditions about the mutual coupling terms given in Section 4.2.2, a string stable platoon is obtained also in 2D.
5.3 Summary

In this chapter, the detailed design of the formation controller is explained and then the formation structure is assessed by means of string stability. In addition to the controller design, the first section also includes the trajectory design for one and two dimensional applications. In the second section, the conditions derived for the mutual coupling terms in Chapter 4 are applied for cooperative driving and the string stability is examined by time domain analyses. The time domain analysis is made in two cases; 1D and 2D, where starting errors and position and velocity disturbances are used for the testing. All of these analyses showed that the formation controller is capable of obtaining string stable platoons both in 1D and 2D.
CHAPTER 6

CONCLUSIONS AND RECOMMENDATIONS

The notion of string stability is the main topic of this report. Unlike the studies published on this subject, the control strategy cannot only be used for one dimensional applications but also for two dimensional applications and therefore, string stability is analyzed for both application types. In Section 6.1, the main results of this thesis are presented and recommendations for future studies are stated in Section 6.2.

6.1 Conclusions

A new vehicle following system which is expected to be the next step of CACC is designed in this study. The basic synchronization principles used in the field of robotics are implemented to the vehicular application to obtain so-called string stable vehicle platoons in longitudinal and lateral directions. Similar to CACC, on-board sensors and wireless communication (V2V or V2V+I2V or I2V+V2I) are the required technologies. The assessment of this new cooperative driving system is made by means of frequency and time domain string stability analyses, which is new for a 2D application.

Firstly, a model of a front steered vehicle is designed and used for controller design, simulations and string stability analysis.

Secondly, a single vehicle path tracking controller is designed for the model using “input-output linearization by state feedback”. For this purpose, a virtual control point is defined and the controller is designed to control this point. This controller is capable of trajectory tracking and formed by a PD controller with an acceleration feedforward setting. Note that this controller is itself not suitable for formation control.

Before designing the formation controller for the system, the control objectives are stated. Accordingly, two concepts for string stability notion are defined; one sided coupling and two-sided coupling. In the first string stability concept, one sided coupling, the attenuation of the position errors is desired upstream the platoon regardless of a possible disturbance or a starting error. However, in the second string stability concept, two sided coupling, the notion is strictly dependent on the vehicle experiencing the disturbance. It is desired that the position errors attenuate in both directions while moving away from the disturbed vehicle. This new concept, therefore, is more suitable for the two dimensional behavior since it is usually not possible to assign the vehicles as the leader or follower in 2D.
Then, the appropriate string stability function is selected among others to be used in the frequency domain assessment. Although all of the string stability functions are dependent on the preceding vehicle characteristics for the designed formation structure, the output string stability function achieves to be independent at low frequencies. Therefore, the results obtained in the string stability function analysis show that output string stability function is the most suitable one that can be used for any kind of platoon, i.e. homogeneous or heterogeneous.

String stability of a platoon is systematically analyzed in frequency domain using transfer functions for ideal vehicle dynamics in the 1D case. The conditions for the mutual coupling terms of the formation controller are derived separately for the two string stability concepts. For the one sided string stability concept, it is concluded that the breakpoint frequencies of the mutual coupling terms should decrease upstream the platoon. Similarly, for the two-sided string stability concept, it is concluded that the breakpoint frequencies of the mutual coupling terms should decrease while moving away from the disturbed vehicle in both directions. The higher the difference between the breakpoint frequencies, the better is the string stability of the platoon for both concepts. Unfortunately, it is not possible to declare optimum values for the mutual coupling terms since they determine the trade-off between keeping the formation and tracking the reference trajectory. Therefore, they should be assigned considering the application.

After deriving the conditions for ideal vehicle dynamics, the formation controller is designed and presented in detail. The formation controller is basically formed by the path tracking controller where mutual coupling terms are added for the formation of vehicles. Using the formation controller and the conditions derived for ideal vehicle dynamics and the 1D case, analyses are done using the vehicle model in time domain. Apart from the frequency domain analysis for ideal vehicle dynamics, time domain analysis is carried out for the 1D and 2D cases. The effect of spacing policy is examined and concluded that the type of the spacing policy does not have much influence on the virtual structure controlled platoons. All simulations in the time domain analyses show that string stability can be achieved with the conditions derived using ideal vehicle dynamics, which validates the theoretical approach. Therefore, with the developments of the technology in the future, the virtual structure technique can be implemented in vehicular applications using the conclusions presented in this study.

To sum up, a new string stable vehicle following system for 2D vehicular applications and, consequently, a new 2D string stability definition for the assessment of the system are presented in this study.

### 6.2 Recommendations

Several points that need to be improved or further studied are presented in this section.

**Large-scale deployment and mixed traffic condition:** This study only focuses on the application of the virtual structure technique for the ideal case, i.e. every vehicle is equipped with the necessary technologies and can communicate with all other vehicles, at least via the infrastructure. However, with the current technology, it is impossible to form platoons where every vehicle has the virtual structure control system, in practice. Therefore, the cases involving the unequipped vehicles should be considered and measures should be taken. On-board sensors which are also mentioned in the
CONCLUSIONS AND RECOMMENDATIONS

report can be used in the detection of this type of vehicles. Yet, the system developed in this study can be used for special applications or in isolated and structured roads for public transportation.

Although the analysis shows that the designed system is feasible, when the required technology and the safety issues are considered, it can be concluded that large-scale deployment of the control method seems to require many years.

Communication delays: The effects of latencies or the maximum load of the communication structure is not examined in this study. However, in practice, their effects should be considered. On the other hand, since the system currently is far away from large-scale deployment, it is predicted that in the future the communication delays will not cause severe problems with the developed technology for small platoons, i.e. string of vehicles in close vicinity. Small platoons should also be preferred considering the communication load. Introduction of infrastructure or forming small platoons where the vehicle at the end of the preceding platoon and the leader of the follower platoon communicate can be the solution. Furthermore, by this problem the question: ‘what is the optimum or maximum number of vehicles in a platoon?’ also arises, which can be a subject of future studies.

Online update for the mutual coupling parameters: The choice of mutual coupling terms is an important issue in the formation. By adjusting their values, it is possible to decide whether the vehicles should give more importance on trajectory tracking or on maintaining the formation structure. In the presented study, the values of the mutual coupling terms are determined beforehand and they are not changed during the simulations. In practice, however, there can be certain moments in which a change of these values is required. Therefore, an online updating system for the mutual coupling parameters should be developed, which responds to the changing requirements.

Improvement of the vehicle model and controller, i.e. addition of tire slip: The point tracking controller designed in this study does not include tire slip compensation. Although this phenomenon does not have an important influence on the 1D case; for the 2D case, the controller should be improved for tire slip. Tire slip should not be underestimated for high speed, high acceleration cornering situations. Therefore, it is better to make the assessment of string stability using a vehicle model and controller which take tire slip into account.

Real-time experiment: This study provides a theoretical approach and its practical validation using a Simulink model. However, it is obvious that the results of simulations made in a computer environment may deviate from the results of real-time simulations. Therefore, as a further study, realization of a real-time experiment of the proposed control technique and its comparison with the results presented in this study is suggested.

Reference trajectory of the virtual center: In the study, no special interest is devoted to the choice of the reference trajectory of the virtual center; vehicles are assumed to follow a pre-defined trajectory. However, in practice, one of the most problematic issues is the choice of reference trajectory. One possible option for the solution is to define the trajectory of the leading vehicle as the reference trajectory of the virtual center and define the following vehicles individual reference trajectories according to it. Yet, this can only be realized for the 1D platoons and also it is not a reliable solution. As stated in the previous discussions, it is impossible to refer to a vehicle as the
leading vehicle in the 2D platoons. Therefore, existence of an infrastructure determining the optimum reference trajectory makes the implementation of this study easier. The infrastructure determines the most efficient and safest trajectory considering the current situation on the traffic and updates this information in every time interval. However, the method of the definition of the reference trajectory will be a subject of further research.
REFERENCES


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APPENDIX A

TRANSFER FUNCTION DERIVATIONS

In this thesis, numerous transfer functions and some calculation results about these transfer functions are presented directly. In this chapter, the derivations of them are addressed.

In Section A1, the derivation of (4.8) and (4.10); in Section A2, the derivation of (4.11); in Section A3, the derivation of (4.13) and (4.14); in Section A4, the derivation of (4.15) are given.

A1. Derivation of (4.8) and (4.10)

The derivation procedure for the error string stability transfer function is described in this section.

The first step is to find the relation between \( X_0 \) and \( E_1 \):

\[
X_0 - E_1(C_1 + C_{12})P_1 + E_2C_{12}P_1 - X_0s^2P_1 = E_1
\]  

(A.1)

which can be written as:

\[
E_1 = X_0 \frac{(1 - s^2P_1)}{(1 + C_{c1}P_1)} + E_2 \frac{C_{12}P_1}{(1 + C_{c1}P_1)}
\]  

(A.2)

where \( C_{c,1} = C_1 + \sum_{k=1,k\neq 1}^{n} C_{1,k} \).

Then, the relation between \( X_0 \) and \( E_2 \) should be found:

\[
X_0 - E_2(C_2 + C_{21})P_2 + E_1C_{21}P_2 - X_0s^2P_2 = E_2
\]  

(A.3)

which can be written as:

\[
E_2 = X_0 \frac{(1 - s^2P_2)}{(1 + C_{c2}P_1)} + E_1 \frac{C_{21}P_2}{(1 + C_{c2}P_2)}
\]  

(A.4)

Solving together (A.2) and (A.4), the following transfer functions are derived:

\[
\frac{E_1}{X_0} = \frac{1 - s^2P_1 + \frac{(1 - s^2P_2)C_{12}P_1}{1 + C_{c2}P_2}}{1 + C_{c1}P_1 - \frac{C_{12}P_1C_{21}P_2}{1 + C_{c2}P_2}}
\]  

(A.5)
Since the error string stability function, $SS_{E,2}(j\omega)$, is defined as:

$$SS_{E,2}(j\omega) = \frac{E_2}{X_0} \frac{X_0}{E_1}$$  \hspace{1cm} (A.7)

$SS_{E,2}(j\omega)$ can be found using (A.5) and (A.6):

$$SS_{E,2}(j\omega) = \frac{1 + C_{c2}P_2 - s^2P_1 - s^2P_1C_{c2}P_2 + (1 - s^2P_2)C_{12}P_1}{1 + C_{c1}P_1 + C_{c1}P_1C_{c2}P_2 + C_{c2}P_2 - C_{12}P_1C_{21}P_2}$$ \hspace{1cm} (A.8)

It is apparent that the denominator of the first factor and the numerator of the second factor cancel each other. Then, the following relation is found after simplification of (A.8):

$$SS_{E,2}(j\omega) = \frac{1 + C_{c2}P_2 - s^2P_1 - s^2P_1C_{c2}P_2 + (1 - s^2P_2)C_{12}P_1}{1 + C_{c1}P_1 - s^2P_2 - s^2P_2C_{c1}P_1 + (1 - s^2P_1)C_{21}P_2}$$ \hspace{1cm} (A.9)

**A2. Derivation of (4.11)**

The objective is to calculate the low frequency asymptotic value of $SS_{E,2}(j\omega)$, i.e. $\lim_{\omega \to 0}|SS_{E,2}(j\omega)|$.

Since the terms $P_i$ include $1/s^2$, the first step is to eliminate them and let $\omega \to 0$. Using (A.9):

$$\lim_{\omega \to 0}|SS_{E,2}(j\omega)| = \lim_{\omega \to 0} \left| \frac{1 - k_{p1}}{s^2} \frac{k_{c2}(k_{p2} - k_{p1}k_{p2}) + (1 - k_{p2})k_{p1}C_{12}}{1 - k_{p2}} \right|$$ \hspace{1cm} (A.10)

$$= \lim_{\omega \to 0} \left| \frac{k_{c2}(k_{p2} - k_{p1}k_{p2}) + (1 - k_{p2})k_{p1}C_{12}}{k_{c1}(k_{p1} - k_{p1}k_{p2}) + (1 - k_{p1})k_{p2}C_{21}} \right|$$

where $k_{p,i}$ is the plant gain.
A3. Derivation of (4.13) and (4.14)

The derivation procedure for the output string stability transfer function is described in this section.

The first step is to find the relation between $X_0$ and $X_1$:

$$(X_0 - X_1)C_1P_1 + (X_0 - X_1 - (X_0 - X_2))C_{12}P_1 + X_0s^2P_1 = X_1$$

which can be written as:

$$X_1 = X_0 \frac{(C_1P_1 + s^2P_1)}{(1 + C_{c1}P_1)} + X_2 \frac{C_{12}P_1}{(1 + C_{c1}P_1)}$$

(A.12)

Then, the relation between $X_0$ and $X_2$ should be found:

$$(X_0 - X_1)C_1P_1 + (X_0 - X_1 - (X_0 - X_2))C_{12}P_1 + X_0s^2P_1 = X_1$$

which can be written as:

$$X_2 = X_0 \frac{(C_2P_2 + s^2P_2)}{(1 + C_{c2}P_2)} + X_2 \frac{C_{21}P_2}{(1 + C_{c2}P_2)}$$

(A.14)

Solving together (A.12) and (A.14), the following transfer functions are derived:

$$\frac{X_1}{X_0} = \frac{C_1P_1 + s^2P_1 + C_{12}P_1(C_2P_2 + s^2P_2)}{1 + C_{c1}P_1 - C_{12}P_1C_{21}P_2}$$

(A.15)

$$\frac{X_2}{X_0} = \frac{C_2P_2 + s^2P_2 + C_{21}P_2(C_1P_1 + s^2P_1)}{1 + C_{c2}P_2 - C_{21}P_2C_{12}P_1}$$

(A.16)

Since the output string stability function, $SS_{X_2}(j\omega)$, is defined as:

$$SS_{X_2}(j\omega) = \frac{X_2}{X_0} \frac{X_0}{X_1}$$

(A.17)

$SS_{X_2}(j\omega)$ can be found using (A.15) and (A.16):

$$SS_{X_2}(j\omega) =$$

$$\frac{(C_2P_2 + s^2P_2)(1 + C_{c1}P_1) + (C_1P_1 + s^2P_1)C_{21}P_2}{1 + C_{c2}P_2 + C_{c1}P_1C_{c2}P_2 + C_{c1}P_1 - C_{12}P_1C_{21}P_2}$$

$$\frac{1 + C_{c1}P_1 + C_{c1}P_1C_{c2}P_2 + C_{c2}P_2 - C_{12}P_1C_{21}P_2}{(C_1P_1 + s^2P_1)(1 + C_{c2}P_2) + (C_2P_2 + s^2P_2)C_{12}P_1}$$

(A.18)
It is apparent that the denominator of the first fraction term and the numerator of the second fraction term cancel each other. Then, the following relation is found after simplification of (A.18):

\[
SS_{X_2}(j\omega) = \frac{(C_2 P_2 + s^2 P_2)(1 + C_{c1} P_1) + (C_1 P_1 + s^2 P_1) C_{21} P_2}{(C_1 P_1 + s^2 P_1)(1 + C_{c2} P_2) + (C_2 P_2 + s^2 P_2) C_{12} P_1}
\]  \hspace{1cm} (A.19)

**A4. Derivation of (4.15)**

The objective is to calculate the zero frequency asymptotic value of \(SS_{X_2}(j\omega)\), i.e. \(\lim_{\omega \to 0} |SS_{X_2}(j\omega)|\).

Since the terms \(P_i\) include \(1/s^2\), the first step is to eliminate them and let \(\omega \to 0\). Using (A.19):

\[
\lim_{\omega \to 0} |SS_{X_2}(j\omega)| = \lim_{\omega \to 0} \left| \frac{(C_2 P_2 + s^2 P_2)(1 + C_{c1} P_1) + (C_1 P_1 + s^2 P_1) C_{21} P_2}{(C_1 P_1 + s^2 P_1)(1 + C_{c2} P_2) + (C_2 P_2 + s^2 P_2) C_{12} P_1} \right|
\]

\[
= \lim_{\omega \to 0} \left| \frac{C_2 P_2 C_{c1} P_1 + C_1 P_1 C_{21} P_2}{C_1 P_1 C_{c2} P_2 + C_2 P_2 C_{12} P_1} \right| = \lim_{\omega \to 0} \left| \frac{k_{p1} k_{p2} C_{21} C_{21}}{k_{p1} k_{p2} C_{12} C_{12}} \right| = 1
\]  \hspace{1cm} (A.20)

where \(k_{p,i}\) is the plant gain and knowing that \(C_1 = C_2\).
APPENDIX B

THE EFFECT OF MUTUAL COUPLING TERMS

The breakpoint frequencies of the mutual coupling terms directly influence the formation. This influence is demonstrated by means of several simulations using different breakpoint frequency values for the one sided coupling concept.

In the following figure, the same coupling parameters with the ones in Chapter 5 are used. Two more figures will be given for the comparison of the results.

Figure B.1: Position error due to a starting error and a velocity disturbance in vehicle 3 (1D, constant distance policy, one sided coupling). The controller parameters are: the path tracking controller with a breakpoint frequency of 10 rad/s and the mutual coupling terms with breakpoint frequencies of \( \{ \omega_1 = 7.07, \omega_2 = 6.33, \omega_3 = 5.48, \omega_4 = 4.47 \} \) rad/s

where \( \omega_{br, i,k} = \omega_i; i = 1, 2, ..., n, k = 1, 2, ..., n \land k \neq i \)
In the next figure, simulation result performed with higher breakpoint frequency coupling terms is presented.

Figure B.3: Position error due to a starting error and a velocity disturbance in vehicle 3 (1D, constant distance policy, one sided coupling). The controller parameters are: the path tracking controller with a breakpoint frequency of 10 rad/s and the mutual coupling terms with breakpoint frequencies of \( \{ \omega_1 = 8.94, \omega_2 = 8.37, \omega_3 = 7.75, \omega_4 = 7.07 \} \) rad/s where \( \omega_{br,ik} = \omega_i; i = 1, 2, \ldots, n, k = 1, 2, \ldots, n \land k \neq i \)

Figure B.4: Zoomed view in the interesting areas of Figure B.3.
When Figure B.2 and Figure B.4 are compared, it is perceived that the position errors of vehicles 1, 2 and 4 increase with the increased mutual coupling breakpoint frequencies. Therefore, this increase shows that the cooperative driving becomes better since the vehicles react more to the errors in vehicle 3 and tend to keep the formation even by getting away from their own trajectories. As a conclusion, if the formation is much more important than being on track, this mutual coupling term assignment can be preferred.

As a last simulation about the effects of mutual coupling terms, Figure B.5 is presented. In this simulation, only vehicle 1’s breakpoint frequency of the mutual coupling term is increased with respect to the first simulation.

![Figure B.5: Position error due to a starting error and a velocity disturbance in vehicle 3 (1D, constant distance policy, one sided coupling). The controller parameters are: the path tracking controller with a breakpoint frequency of 10 rad/s and the mutual coupling terms with breakpoint frequencies of \( \{\omega_1 = 8.94, \omega_2 = 6.33, \omega_3 = 5.48, \omega_4 = 4.47\} \) rad/s where \( \omega_{br,i,k} = \omega_i; i = 1, 2, ..., n, k = 1, 2, ..., n \land k \neq i \).](image)

![Figure B.6: Zoomed view in the interesting areas of Figure B.5.](image)

In Figure B.6, it is seen that all of the position errors increase. Although only vehicle 1’s breakpoint frequency of the mutual coupling term is increased, this increase also effects vehicle 2 and 4 since
there is also mutual coupling between them. As a result, in the simulation, the position error of vehicle 1 increases at most as expected, but also the position errors of vehicle 2 and 4 also increase slightly compared to Figure B.2. This shows that even increasing the breakpoint frequency of one of the mutual coupling terms effect the whole formation structure and improves it.
APPENDIX C

REFERENCE TRAJECTORIES

C.1. The longitudinal track and o-track properties

The longitudinal reference trajectory used in the simulations of 5.2.2 is presented in Figure C.1 and its properties are given in Table C.1.

Figure C.1: Position, velocity and acceleration profile of longitudinal track
APPENDIX C

Table C.1: Properties of longitudinal track

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sampling Frequency</td>
<td>0.002 [s]</td>
</tr>
<tr>
<td>Length</td>
<td>400 [m]</td>
</tr>
<tr>
<td>Maximum Velocity</td>
<td>60 [km/s]</td>
</tr>
<tr>
<td>Maximum Acceleration</td>
<td>4 [m/s]</td>
</tr>
</tbody>
</table>

The o-track used as a reference trajectory in the simulations of 5.2.3 is presented in Figure C.2 and its properties are given in Table C.2.

![Graphs of position, velocity, and acceleration of o-track](image-url)
Table C.2: Properties of o-track

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sampling Frequency</td>
<td>0.002 [s]</td>
</tr>
<tr>
<td>Circle radius</td>
<td>40 [m]</td>
</tr>
<tr>
<td>Maximum Velocity in x&amp;y direction:</td>
<td>60 [km/s]</td>
</tr>
<tr>
<td>Maximum Acceleration in x&amp;y direction:</td>
<td>7 [m/s]</td>
</tr>
</tbody>
</table>

C.2. Position of vehicles in o-track

Vehicles’ individual reference trajectories are shown in Figure C.3.

**Figure C.3: Position graph of o-track**