Process zone and cohesive element size in numerical simulations of delamination in bi-layers
Master Thesis

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MT 10.21

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Abstract

Brittle interfacial failure is one of the major sources of failure in devices that consist of multiple thin and stacked layers, manufactured using different materials. Insight in the delamination process is therefore of fundamental importance in the development of such devices. One method to gain insight in the failure mechanism is the usage of finite element analysis. A widely used numerical tool to simulate interfacial failure is cohesive zone modelling. One of the main issues in applying the CZM in a finite element model is the size of the cohesive zone elements. When the mesh of the discretized problem is too coarse, the so-called solution jump problem may occur. This results in oscillations in the global load-displacement behaviour of the structure or even a diverging analysis. Prevention of convergence issues is provided by mesh refinement. However, a too fine mesh may lead to excessive computational times.

The aim of the present project is to define cohesive zone element mesh size requirements that allow stable numerical simulations of interface delamination. An estimation of the process length is provided by means of an analytical derivation using linear fracture mechanics and simplified equilibrium equations. Three typical bi-layers tests are examined to this end. Their corresponding estimated process zone lengths are successfully numerically validated across a range of material, interface and structural properties with the use of the Van den Bosch cohesive zone model [24]. In addition, the minimum number of required cohesive elements within the process zone is numerically determined. The resulting mesh design rules are applied to two practical applications which are found in electronic devices: a 90° fixed arm peel test and a buckling-driven delamination of a bi-layer. Summarising, the presented work provides guidelines for a minimum required mesh size that ensures numerical stability in interface delamination simulations.
# Abbreviations

<table>
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<tr>
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<tbody>
<tr>
<td>CZM</td>
<td>Cohesive Zone Modeling</td>
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<td>CZE</td>
<td>Cohesive Zone Element</td>
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<td>DCB</td>
<td>Double Cantilever Beam</td>
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<tr>
<td>ELS</td>
<td>End Load Split</td>
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<tr>
<td>FE</td>
<td>Finite Element</td>
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<tr>
<td>FEA</td>
<td>Finite Element Analysis</td>
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<td>FEM</td>
<td>Finite Element Method</td>
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<td>LEFM</td>
<td>Linear Elastic Fracture Mechanics</td>
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<td>MEMS</td>
<td>Micro Electro Mechanical Systems</td>
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<tr>
<td>SIP</td>
<td>System In Package</td>
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<tr>
<td>STELLA</td>
<td>STretchable ELectronics for Large Area applications</td>
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<tr>
<td>TSL</td>
<td>Traction Separation Law</td>
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<td>VCCT</td>
<td>Virtual Crack Closure Technique</td>
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List of Symbols

$\alpha$ load angle

$\beta_1$ Ratio between shear and normal traction

$\beta_2$ Ratio Mode II/III critical work of separation and Mode I critical work of separation

$\delta$ Size of separation

$\delta_n$ Size of normal separation

$\delta_s$ Size of shear separation

$\nu$ Poisson’s ratio

$\lambda$ Separation

$\lambda_c$ Characteristic separation

$\phi$ Angle between the cohesive zone midline and the separation vector

$\psi$ Load angle

$\sigma_Y$ Yield stress

$\theta$ Angle between the cohesive zone midline and the top or bottom surface of the interface

$a$ Crack length

$a_0$ Initial crack length

$d$ Mode mixity parameter

$\vec{e}$ Unit vector

$E$ Young’s modulus

$F$ Reaction force

$G$ Work of separation

$G_c$ Critical work of separation, fracture toughness

$G_i$ Mode $i$ work of separation

$h$ Height of beam

$I$ Second moment of inertia

$K$ Stress intensity factor

$l$ Length of beam

$l_a$ Length of the zone in the crack plane in which $T_{max}$ is exceeded

$l_{pz}$ Length of process zone

$M$ Moment

$P$ Applied force

$q$ Exponential decay factor

$q_i$ Generalized displacement

$Q_i$ Generalized force

$T$ Traction

$T_n$ Normal traction

$T_s$ Shear traction
\begin{itemize}
\item $T_{n,max}$ Maximum normal traction
\item $T_{s,max}$ Maximum shear traction
\item $u$ Beam tip opening displacement
\item $U_{pot}$ Potential energy
\item $U_{strain}$ Strain energy
\item $w$ Width of beam
\item $W$ Damage variable
\end{itemize}


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Chapter 1

Introduction

1.1 Background

1.1.1 Delamination

Delamination, also referred to as interfacial cracking, is a failure phenomenon that occurs frequently in layered structures [25]. Flexible displays, Micro Electro Mechanical Systems (SEMS), System In Package (SiP), stretchable electronics, etc. are examples of devices that consist of (multiple) stacked layers. Delamination in these devices may arise under various circumstances, such as low velocity impacts, mechanical loading, temperature fluctuations, etc. Insufficient adhesion at the interface, which is often brittle, is likely to prevent such devices from functioning properly. Insight in the delamination process is therefore of fundamental importance in the development of such devices.

1.1.2 Cohesive zone models

One method to gain insight in the failure mechanism is the usage of finite element analysis. Finite element analysis provides a way to predict the failure behaviour of materials, in order to optimise the structure of the electronic devices. A widely used numerical tool to simulate interfacial failure is cohesive zone modelling [7, 13, 18, 24, 28]. The basis for the cohesive zone model (CZM) can be traced back as far as to the works of Dugdale and Barenblatt [2]. Figure 1.1 schematically shows cohesive zone elements in a finite element model. At the interface, cohesive zone elements are placed which describe the interface traction as a function of the opening displacement, by means of the so-called traction-separation law (TSL). Energy is dissipated when these elements are opened until complete loss of traction occurs. The zone wherein energy is dissipated is called the process zone or sometimes the cohesive zone.

One of the main issues in applying the CZM in a finite element model is the size of the cohesive zone elements [22]. When the mesh of the discretized problem is too coarse, the so-called solution jump problem may occur. This results in oscillations in the global load-displacement behaviour of the structure or even a diverging analysis [7, 18, 23, 28]. In figure 1.2(b) a typical example of an oscillating global response of a double cantilever beam (figure 1.2(a)) is
shown. Refining the mesh eliminates this problem, but may lead to excessive computational times. The solution jump problem is directly related to the number of elements within the process zone \([22, 23]\). Therefore, mesh optimisation requires an accurate prediction of the process zone length and the minimum number of cohesive zone elements required within this zone. In this way, the most suitable mesh can be generated \textit{a priori}, which ensures a proper description of the process zone and consequently, prevents undesired numerical instabilities.

Yang and Cox \([29]\) successfully used analytical derivations for the length of the process zone of mode I and II loaded slender geometries as presented by Massabò and Cox \([15]\) for generating their mesh of a laminate with a central hole and slit. Only one type of interface toughness with different geometries was analysed. More investigation is necessary to determine and validate appropriate mesh size requirements for the analysis of different devices with different properties.

Apart from refining the mesh, other methods are used to reduce the undesired oscillatory behaviour. Turon et al. \([23]\) suggest to lower the maximum traction of the interface, which
increases the process length. The drawback of using a reduced interfacial strength is that the stress concentrations in the bulk material near the crack tip are less accurate and also the accuracy of the global response is reduced. Samimi is currently developing an enriched cohesive zone model [18] for brittle interfaces. In this formulation, the displacement field is enriched such that the crack tip can be located at an arbitrary position within the cohesive zone element, thereby reducing the number of elements required in the process zone. Promising results have been achieved in two dimensions, but three-dimensional analyses are not yet possible. Another method is the local arc-length control method [28]. This method does not reduce the oscillations observed in the global response during delamination, but simply prevents numerical issues that are caused by the oscillations. It however requires small increments and thus long computing times. The focus in this project is on the solution of mesh refinement with a conventional cohesive zone model, partly motivated by the drawbacks of the alternative methods. Furthermore, even if one would choose one of the alternatives, in any numerical delamination analysis it is advantageous to know the size of the process zone before generating the mesh.

1.2 Aim

The aim of the project is to define cohesive zone element mesh size requirements that allow stable numerical simulations of interface delamination.

1.3 Strategy

In order to accomplish the above aim, an analytical study and numerical simulations will be performed.

Firstly an accurate prediction of the process length is necessary. To this end, an estimate of the process zone length will be derived for specimens with simple geometries and loading conditions. An infinite geometry loaded mode I and II, a T-peel test, Double Cantilever Beam (DCB) test and End Load Split (ELS) test will be examined. The three tests are often used to determine the mode I (T-peel and DCB test) or mode II (ELS test) interface toughness of laminates. The analytical predictions will be validated by a numerical study across a range of interface, material and geometry properties.

The next step is the quantification of the minimum number of elements required within the process zone to obtain a convergent and accurate solution. The number of elements within the process zone will be varied in the models of the three tests to examine its effect on convergence and on the global response of the model. By combining the required number of elements within the process zone and the prediction of the process zone length, a mesh size guideline for the various specimens can be constructed.

The final step is to assess the applicability and validity of the gained mesh design rules for more practical applications which are found in electronic devices. A 90° fixed arm peel test and a buckling-driven delamination of a bi-layer are examined for this purpose.
1.4 Outline of the report

In chapter 2, the CZM which will be used to perform the numerical simulations is described. In chapter 3, an estimate of the process zone length for various specimens will be derived and validated. The minimum number of elements required within the process zone is determined in chapter 4. The obtained mesh guideline is applied to two practical examples in chapter 5. Finally the general conclusions will be summarised and discussed in chapter 6.
Chapter 2

Cohesive zone modelling

Various methods exist to simulate delamination in numerical models. Among the various damage models, the cohesive zone model seems particularly attractive for practical application since it is effective and requires only two parameters, which can be determined in experiments [1]. Compared to methods based on Linear Elastic Fracture Mechanics (LEFM), like the Virtual Crack Closure Technique (VCCT) and $J$-integral, it offers some important advantages. The methods within the LEFM framework require an initial delaminated area and the direction of crack propagation to be known in advance, while in a CZM the crack can propagate along any path where the cohesive zone elements (CZE) are placed. In addition, the CZM is more versatile, because it can model both ductile and brittle interfaces well, and is easy to implement in the mesh. Therefore, cohesive zone modelling is considered to be most obvious choice to model a wide range of delamination problems.

In the course of the years several cohesive zone models have been developed. In this project the CZM developed by Van den Bosch [24] is used as it is particularly suitable for the laminates of interest to Philips. The CZM was developed to model the delamination of a polymer coating from a steel substrate. This involves large displacements and deformations at the interface and in the bulk materials, which also occur in several electronic applications. Two other models, the default CZM included in MSC.Marc© 2008 and a CZM with a local arc length method developed by Van Hal [28], are also described and compared with the model of Van den Bosch in appendix A. This comparison supports the use of the Van den Bosch CZM. In the remainder of this report the use of cohesive zone model refers to the cohesive zone model as developed by Van den Bosch, which is described in more detail below.

2.1 Traction separation law

The cohesive zone model uses an exponential traction separation law (TSL) to describe the mechanical behaviour of an interface. The TSL provides a single relation between the traction vector $\vec{T} = T\vec{e}$ and the separation vector $\vec{\lambda} = \lambda\vec{e}$ of the cohesive zone. The vector $\vec{e}$ is a unit vector along the line between two material points which coincide before delamination occurs. Accordingly $\vec{T}$ and $\vec{\lambda}$ have the same direction. The length of the traction vector does not only depend on the TSL and the corresponding interface parameters, but also on the orientation
(δ) of $\vec{e}$ and the angle between the top and bottom surface of the interface (θ) (see figure 2.2). This is in contrast to many other formulations which use two distinct relations for the traction in normal and shear direction [24]. The implementation of the TSL is shown in figure 2.1 and is described as follows,

$$T = \frac{G_{Ic} \lambda}{\lambda_c^2} \exp \left( \frac{-\lambda}{\lambda_c} \right) \exp \left( \ln(\beta_2) \frac{d}{2} \right) \tag{2.1}$$

$$= \frac{G_{Ic} \lambda}{\lambda_c^2} \exp \left( \frac{-\lambda}{\lambda_c} \right) \beta_2^\frac{d}{2}, \tag{2.2}$$

in which $G_{Ic}$ is the critical work of separation in normal direction (mode I opening), which is equal to the area below the traction - separation curve, $\lambda$ the separation and $\lambda_c$ the characteristic separation at which the maximum traction, $T_{max}$, is experienced. The parameter $d$ describes the opening mode and enables one to include the influence of mode mixity. The parameter $\beta_2$ relates the energy dissipation in the normal and shear direction as follows,

$$\beta_2 = \frac{G_{IIc}}{G_{Ic}} \tag{2.3}$$

where $G_{IIc}$ is the critical work of separation in mode II. This form of mode mixity results in a path dependency [24].

The maximum traction for the exponential law can be derived from equation (2.2) and has the following form

$$T_{max} = \frac{G_{Ic}}{\lambda_c \exp(1) \beta_2^\frac{d}{2}}. \tag{2.4}$$
2.2 Mode mixity

The mode mixity parameter $d$ can be related to the angles $\phi$ and $\theta$, which are shown in figure 2.2. In this figure a 2D-representation of a cohesive zone is depicted. The vectors $\vec{d}_1$ and $\vec{d}_2$ are the components of the unit normals $\vec{n}_1$ and $\vec{n}_2$ of the two cohesive edges perpendicular to $\lambda \vec{e}$. The mode-mixity parameter $d$ is defined as

$$d = |\vec{d}_1 - \vec{d}_2|.$$  \hspace{1cm} (2.5)

Geometrical considerations lead to the following expression of $d$ for a 2D cohesive zone

$$d = |\sin(\phi + \theta) + \sin(\phi - \theta)|$$

$$= 2 |\sin(\phi)||\cos(\theta)|.$$  \hspace{1cm} (2.6)

$$= 2 |\sin(\phi)||\cos(\theta)|.$$  \hspace{1cm} (2.7)

Consequently, the parameter $d$ has a value between 0 (mode I) and 2 (mode II). To examine the influence of $d$ on the behaviour of the CZ model, a mixed mode condition is considered whereby the two faces are kept parallel to each other ($\theta = 0$). The critical work of separation in mixed mode is, with the use of equations (2.2) and (2.7), given by

$$G_c = \int_0^\infty T(\lambda) d\lambda = G_{Ic} \beta_2^{\sin(\phi)} \beta_2^{\sin(\phi)}, \text{ if } \theta = 0.$$  \hspace{1cm} (2.8)

In figure 2.3 the work of separation is shown as a function of angle $\phi$. The energy dissipated in mode I ($d = 0$) is independent of $\beta_2$ and equals $G_{Ic}$; the dissipation in mode II ($d = 2$)
equals $G_{Ic}\beta_2$. In practice a value of $\beta_2 \geq 1$ is most common, because in mode II more work of separation is measured due to additional dissipation mechanisms at the interface.

In figure 2.3, equation (2.2) is plotted for three values of $\beta_2$ in mode II ($d = 2$). One can see that both $T_{max}$ and $G_c$, the area below the traction - separation curve, depend on the parameter $\beta_2$, only if $d \neq 0$.

In figure 2.5 the work of separation is shown for different values of $\theta$: 0, 45 and 90°. An angle of 0° implies that the top and bottom of the interface are parallel to each other, 45° that the top is perpendicular to the bottom and 90° that the top of the interface is rotated by 180° with regard to the bottom of the interface. The influence on the amount of dissipated energy of $\beta_2$, which has been set to 1.5 in the figure, decreases with an increasing value of $\theta$. This is best shown by the curve with $\theta = 90^\circ$. In this case the value of $d$ is always 0, such that the amount of dissipated energy is independent of the value of $\beta_2$. One must keep in mind that a situation in which $\theta = 90^\circ$ is not likely to occur in practice, as well as various other combinations of $\theta$ and $\phi$. 

![Figure 2.3: Dissipated energy as a function of $\phi$ for different values of $\beta_2$. Mode I corresponds to $\phi = 0^\circ$ and mode II to $\phi = 90^\circ$. $\theta = 0^\circ$ for all curves.](image)

![Figure 2.4: Traction - separation curves of a cohesive zone element opened by a mode II loading ($d = 2$) for different values of $\beta_2$.](image)

![Figure 2.5: The work of separation is shown for different values of $\theta$: 0, 45 and 90°.](image)
Figure 2.5: Dissipated energy, for $\beta_2 = 1.5$, as function of $\phi$ for various angles between the top and bottom of the interface ($\theta$).

2.3 Damage

An important feature of the CZM is its capability to model irreversible behaviour. It is assumed that in order to achieve irreversible behaviour, unloading occurs in a linear way to the origin [13], cf. elasticity based damage. The amount of irreversible cohesive energy is indicated by a damage variable $W[-]$ as follows,

$$W = \frac{\lambda_{\text{max},t}}{\lambda_c},$$

(2.9)

where $\lambda_{\text{max},t}$ is the maximum separation value reached until time history $t$ and satisfies the following Kuhn-Tucker conditions

$$(\lambda_{\text{max},t} - \lambda) \geq 0, \quad \dot{\lambda}_{\text{max},t} \geq 0, \quad \dot{\lambda}_{\text{max},t}(\lambda_{\text{max},t} - \lambda) = 0.$$  

(2.10)

The damage variable increases linearly from 0 for the undamaged case to $\infty$ for the completely damaged case as shown in figure 2.6(a). A separation larger than 6 times $\lambda_c$ ($W = 6$) is often considered as completely damaged. At this point 98.3 % of the energy within the cohesive zone has been dissipated.

As a result of the above relations, unloading follows the secant stiffness until a fully unloaded and undeformed state is reached, as can be seen in figure 2.6(b). When reloaded, the elastic unloading path is followed again until the traction-separation curve is reached, which is then followed upon further loading.
2.4 Contact

Compression of the cohesive zone, which would lead to a negative value of $\lambda$ (which corresponds with penetration of the bulk material) and $T$, is prevented by the contact algorithms in MSC.Marc©. Note that Van den Bosch originally implemented a contact formulation in his CZM. However, Van den Bosch concluded it results in a less robust CZM due to the possibility of inconsistent contact conditions [24].

2.5 Numerical implementation

The geometrically nonlinear version of the CZM has been implemented in a 3D eight-node user defined element with four Gauss points located in its mid-plane. Figure 2.7 shows the element conventions. The element is shown in a deformed state, as the undeformed element is collapsed, i.e., it has zero thickness. In the deformed state $\vec{T}$ connects associated points $L$ and $K$ (which coincided before deformation) located on the top surface $S_1$ and bottom surface $S_2$ respectively. The integration points are located on the mid-plane $ABCD$. In contrast to many other (small deformation) cohesive zone formulations, traction equilibrium at the interface is always maintained even when large surface changes occur in either side of the cohesive zone. This is a result of the fact that the tractions are resolved as first Piola-Kirchhoff tractions defined on the original, undeformed configuration $S_0$. 
Figure 2.7: An eight-node three-dimensional cohesive zone element with four Gauss integration points [24].
Chapter 3

Prediction of the process zone length

Having an estimate of the process zone length, \( l_{pz} \) is crucial in determining an appropriate cohesive element size \([7, 17, 23]\). Inexperienced users often base their prediction of the process zone size solely on the interface properties. This prediction is accurate only when a peel test specimen is modelled, as will be revealed in section 3.2. Other specimen geometries, however, need a more extensive approach to estimate the process zone length. An infinite geometry loaded mode I and II, a T-peel test, double cantilever beam test and end load split test will be examined to this end.

In the first section a uniformly loaded infinite medium is considered. The estimation of the plastic zone, from which a process zone can be derived, of an infinitely large geometry has been determined by many with different models. Irwin published such method as early as 1958 \([19]\) and many others followed (Dugdale \([19]\), Hillerborg \([9]\), Hui \textit{et al.} \([12]\), Falk \textit{et al.} \([3]\)). A process zone derivation analog to the Irwin analysis of the plastic zone is examined in the first section for both mode I and II loading for initial insight.

The estimation of the process zone of the T-peel test is relatively straightforward and presented in section 3.2.

The specimen height does not influence the process zone of both an infinite geometry and T-peel test. Suo \textit{et al.} \([20, 21]\) derived an expression for the process zone length for thin laminates, wherein the height of the laminate does influence the length of the process zone. Therefore the process zone is also considered to be a structural property. In section 3.3 and section 3.4 an expression is derived for the process zone length of a DCB (mode I loaded slender geometry) and an ELS (mode II loaded slender geometry) respectively, based on the work of Suo \textit{et al.} \([20, 21]\).

Yang and Cox \([29]\) used the analytical derivations for the length of the process zone of mode I and II loaded slender geometries as presented by Massabò and Cox \([15]\), which is similar to the derivation of Suo \textit{et al.} \([20, 21]\), for guidance in generating their mesh. Three models were analysed which contained typical carbon/epoxy laminate properties. The first model is a laminate consisting of four plies with a centre hole, the second model is a variant with an asymmetrical ply lay-up and the third is a laminate with a sharp slit in the centre instead of a
hole. Therefore only one type of interface with different geometries were analysed. To properly validate the proposed predictions of the process zone length a numerical study performed across a range of material, interface and geometry properties is considered necessary and described in section 3.5.

3.1 Infinite geometry

3.1.1 Mode I

The first type of crack propagation considered is a uniformly mode I loaded infinite geometry with an initial crack in the interface and linear elastic isotropic material behaviour, see figure 3.1. The properties of the bulk material on both sides of the interface are equal. In addition plane strain condition is assumed. Notice that because of the interface, we are strictly speaking no longer dealing with a continuum. However, this affects only the toughness and not the stress field or energy release rate.

\[
\begin{align*}
\sigma_{xx} &= \frac{K_I}{\sqrt{2\pi r}} \left[ \cos\left(\frac{1}{2}\varphi\right) \left(1 - \sin\left(\frac{1}{2}\varphi\right) \sin\left(\frac{3}{2}\varphi\right)\right) \right] \\
\sigma_{zz} &= \frac{K_I}{\sqrt{2\pi r}} \left[ \cos\left(\frac{1}{2}\varphi\right) \left(1 + \sin\left(\frac{1}{2}\varphi\right) \sin\left(\frac{3}{2}\varphi\right)\right) \right] \\
\sigma_{xz} &= \frac{K_I}{\sqrt{2\pi r}} \left[ \cos\left(\frac{1}{2}\varphi\right) \sin\left(\frac{1}{2}\varphi\right) \cos\left(\frac{3}{2}\varphi\right) \right] \\
\sigma_{yy} &= \nu(\sigma_{xx} + \sigma_{zz})
\end{align*}
\]

Here \( K_I \) is the stress intensity factor and \( r \) is the distance from the crack tip. The stresses
near the crack tip, as determined for linear elastic behaviour, go to infinite: \( \lim_{r \to 0} \sigma = \infty \). This is unrealistic, because above a certain stress level the crack starts to propagate and/or the bulk material yields. In the Irwin analysis the Von Mises criterion is used to determine the size of the plastic zone in absence of an interface. In the present derivation it is assumed that the crack will propagate if \( T = \sigma_{zz} \geq T_{\text{max}} \).

In this analysis of the process zone only the stress components in the plane of the crack \( (\varphi = 0) \) are considered. Therefore the relevant stress component near the crack tip become,

\[
\sigma_{zz} = \frac{K_I}{\sqrt{2\pi r}} \tag{3.5}
\]

The assumption of \( T_{\text{max}} \) being the maximum allowed stress in the crack plane leads to

\[
T_{\text{max}} = \frac{K_I}{\sqrt{2\pi l_a}}. \tag{3.6}
\]

Here \( r \) has been replaced by \( l_a \), the length of the zone in the crack plane in which \( T_{\text{max}} \) is exceeded. The length \( l_a \) can be found by rewriting (3.6)

\[
l_a = \frac{K_I^2}{2\pi T_{\text{max}}^2}. \tag{3.7}
\]

In figure 3.2 one can see that by limiting the stress components, which decreases the stress over length \( l_a \), the global equilibrium is no longer fulfilled. Irwin suggested to correct the stress distribution to restore the equilibrium. This is achieved by extending the process zone \( l_a \) to \( l_{pz} \), by which the curve of the stress of the elastic zone shifts along. This is shown in figure 3.3.

The corrected size of the process zone is determined such that the total force, transmitted in z-direction, remains as large as for fully elastic material behaviour and no interface at the crack plane. Therefore, the length \( l_{pz} \) is determined such that the hatched surfaces of figure 3.3(a) and figure 3.3(b) are equal. This implies

\[
In Figure 3.2: Crack with corresponding stresses in the crack plane [19].
The corrected length of the process zone is found by combining (3.7) and (3.8) as

$$l_{pz} = K^2_{I} \pi l_{a}. \quad (3.9)$$

To apply the analytically derived process zone to cohesive zone modelling a connection must be made to the (critical) energy release rate. Irwin derived a relation between the stress intensity factor $K$ and the energy release rate $G$ as \[14\],

$$K^2 = GE', \quad (3.10)$$

with $E' = E$ in plane stress and $E' = E/(1 - \nu^2)$ in plane strain. In addition, only the mode I critical work of separation is relevant, because of the uniform mode I loading. Therefore equation (3.9) can be rewritten as

$$l_{pz} = \frac{E'G_{lc}}{\pi T_{max}^2}. \quad (3.11)$$

In this final result the process zone length is expressed in terms of material and interface properties, which are the necessary properties to construct a finite element model with cohesive zone elements.

### 3.1.2 Mode II

Similar to the mode I derivation, the crack propagation of a uniformly mode II loaded infinite geometry with an initial crack and linear elastic isotropic material behaviour is considered in
this subsection. The stress components near the crack tip are given by

\[
\begin{align*}
\sigma_{xx} &= \frac{K_{II}}{\sqrt{2\pi r}} \left[ -\sin\left(\frac{1}{2}\phi\right) \left( 2 + \cos\left(\frac{1}{2}\phi\right) \cos\left(\frac{3}{2}\phi\right) \right) \right] (3.12) \\
\sigma_{zz} &= \frac{K_{II}}{\sqrt{2\pi r}} \left[ \sin\left(\frac{1}{2}\phi\right) \cos\left(\frac{1}{2}\phi\right) \cos\left(\frac{3}{2}\phi\right) \right] (3.13) \\
\sigma_{xz} &= \frac{K_{II}}{\sqrt{2\pi r}} \left[ \cos\left(\frac{1}{2}\phi\right) \left( 1 - \sin\left(\frac{1}{2}\phi\right) \sin\left(\frac{3}{2}\phi\right) \right) \right] (3.14) \\
\sigma_{yy} &= \nu (\sigma_{xx} + \sigma_{zz}). (3.15)
\end{align*}
\]

In the mode specimen, the shear stress \(\sigma_{xz}\) in the crack plane ((3.14) with \(\phi = 0\)) is critical with respect to crack propagation. An expression for the maximum stress in the interface is therefore given by,

\[
T_{\text{max}} = \frac{K_{II}}{\sqrt{2\pi l_a}}, (3.16)
\]

in which \(l_a\) has been substituted for \(r\). After correction and using (3.10), one can find the process zone of a mode II uniform loaded specimen with infinite geometry

\[
l_{pz} = \frac{E'G_{IIc}}{\pi T_{\text{max}}^2}. (3.17)
\]

One can notice that the expression of \(l_{pz}\) for Mode II (3.17) is identical to the one for mode I (3.9), with the exception that the mode II critical work of separation is used.

In conclusion, the expressions for the process zone length of a uniform mode I and II loaded infinite geometry show a dependency on the material property \(E'\) and the interface properties \(G_c\) and \(T_{\text{max}}\).

### 3.2 T-peel test

The T-peel test method is primarily intended for determining the relative peel resistance of adhesive bonds between flexible films. It uses a T-type specimen in a tensile testing machine [6]. In figure 3.4(a) a schematic overview of a T-peel test specimen is shown. The unbonded ends of the specimen are clamped in grippers and loaded with a constant head speed (quasi-static). The peel force \(P\) versus the head displacement \(u\) is measured; a typical response is depicted in figure 3.4(b). The critical work of separation of the adhesive bonds \(G_c\) can be determined with,

\[
G_c = \frac{P_f}{w}, (3.18)
\]

where \(w\) is the width of the specimen (size of the specimen in \(y\)-direction) and \(P_f\) the plateau-force during delamination. The energy dissipation by plastic bending energy in the films, \(G_p\),
is neglected. If significant plasticity occurs, the work of separation of the adhesive bond is
found by subtracting the plastic energy from the measured energy.

\[ P, \frac{u}{2} \]

\[ \frac{P, u}{2} \]

\[ x \]

\[ y \]

\[ z \]

\[ (a) \]

Figure 3.4: T-peel test: (a) schematic overview of the specimen; (b) typical force - displacement response.

The prediction of the process zone of the T-peel test is relatively straightforward. Homogeneous elastic material properties are assumed in this derivation. In figure 3.5 the numerically
determined normal stress distribution in z-direction at the crack plane of a T-peel test is
shown. The process zone is fully developed and is situated from \( x \approx 1 \, mm \) to \( x \approx 2.5 \, mm \).
The peak stress value of \( 1 \, Nmm^{-2} \) occurs at \( x \approx 2 \, mm \) and is equal to the maximum interface traction used in the model.

\[ \sigma_{zz} \]

\[ \frac{N}{mm^{2}} \]

\[ x \, [mm] \]

0 1 2 3 4 5 6 7 8 9

1 0.8 0.6 0.4 0.2 0 -0.2

Figure 3.5: The numerically determined normal stress in z-direction at the crack plane of a T-peel test specimen (with an initial crack length of 0.5 \( mm \) and a specimen length of 9 \( mm \)),
when the process zone is fully developed.

To arrive at an expression for the process zone, a free body diagram with the forces in
z-direction on the top film is constructed as shown in figure 3.6(a). A rectangular stress
distribution in the process zone is assumed (figure 3.6(b)), similarly to the stress distribution
of the uniformly loaded infinite geometry. Although figure 3.5 shows this is a coarse estimation of the stress distribution when the exponential TSL is used, the rectangular shaped stress distribution is assumed, as it can be seen as a limit. This particular shape leads to the smallest length of the process zone predicted, because there is a maximum traction at every point in the process zone. Generally, the process zone size is also dependent on the shape of the traction separation law besides the two parameters $T_{\text{max}}$ and $G_c$, because it influences the stress distribution in the process zone. The equilibrium of forces in $z$-direction acting on the film is given by

$$P - \int_0^{l_{pz}} T(x) w dx = G_c w - l_{pz} T_{\text{max}} w = 0 ,$$

so that

$$l_{pz} = \frac{G_c}{T_{\text{max}}} .$$  \hspace{1cm} (3.20)

Rewriting this relation in a dimensionless form gives

$$\frac{l_{pz}}{h} = \frac{G_c}{T_{\text{max}} h} = D_1 ,$$

where $D_1 = \frac{G_c}{T_{\text{max}} h}$ is introduced as a dimensionless parameter.

Striking in the prediction of the process zone length according to (3.20) is that it equals a constant times the characteristic separation $\lambda_c$ of the CZM, and that it is independent of the elastic modulus $E'$ and the height $h$ of the layers. This contrasts with the prediction for the interface crack in an infinite medium ((3.11) or (3.17)), which does depend on the elastic modulus of the bulk material in addition to the interface properties. This difference arises from the large flexibility of the films in the T-peel test.

Notice that only the force equilibrium in $z$-direction is considered in (3.19), because the moment, which arises from the bending of the film, can be neglected due to the small radius of the flexible films. This moment results in a small negative stress near $x \approx 2.5 \text{ mm}$ (in figure 3.5) ahead of the crack. In the next section a similar, but less flexible, specimen is considered, such that this moment can no longer be neglected.
3.3 Double cantilever beam test

The double cantilever beam (DCB) test, as defined in ASTM 3433, is generally used to determine the mode I fracture toughness of a test specimen. In figure 3.7(a) a schematic overview of a DCB specimen is shown. The crack, with initial length \( a_0 \), is initiated by inserting a wedge or by inserting a Teflon film at the mid-plane of the laminate layup prior to curing, depending on the material of the test specimen [7]. After the preparation, the two beams are pulled apart at a quasi-static rate. The force-displacement response can be derived analytically as will be shown in this section and a typical example of such a response can be seen in figure 3.7(b). By doing so, also the energy release rate during delamination is determined, which is necessary to derive the process zone length.

Figure 3.7: Double cantilever beam: (a) schematic overview of the specimen; (b) typical force-displacement response.
The analytical solution is based on the concepts of elastic bending theory and linear elastic fracture mechanics. The potential energy of the DCB specimen is given by [7]

\[ U_{pot} = U_{strain} - Pu, \]  

(3.22)

where \( U_{strain} \) is the strain energy of the specimen, \( P \) is the load applied in vertical direction on the upper and lower parts of the DCB specimen, and \( u \) is the corresponding tip opening displacement. The strain energy of a beam due to bending is known to be [4]

\[ U_{strain} = \int_0^l \frac{M^2}{2E'I} \, dx, \]  

(3.23)

with length \( l \), applied moment \( M \), elastic modulus \( E' \) and the second moment of inertia \( I \) of the beam.

![Free body diagram of the DCB configuration.](image)

Figure 3.8: Free body diagram of the DCB configuration.

The material is assumed to be elastically isotropic and homogeneous. With the help of a simple free body diagram as shown in figure 3.8, one can find the following expression for the strain energy due to bending of the DCB specimen
\[ U_{\text{strain}} = \int_0^a \frac{(P_x)^2}{2EI_1} \, dx + \int_0^a \frac{(P_x)^2}{2EI_2} \, dx, \quad (3.24) \]

where \( I_i \) is the second moment of inertia in part \( i \) of the specimen. The height of the beam is given by \( 2h \) and its width by \( w \). The second moments of inertia of the three parts of the beam are therefore as follows

\[ I_1 = I_2 = \frac{wh^3}{12} = I, \quad I_3 = \frac{w(2h)^3}{12} = 8I. \quad (3.25) \]

Substitution of (3.25) into (3.24) and performing the integrations gives the strain energy of the DCB configuration as

\[ U_{\text{strain}} = \frac{P^2a^3}{3EI}. \quad (3.26) \]

Substituting (3.26) into (3.22) and using Castigliano’s second theorem\(^1\) the tip opening displacement is found to be given by

\[ u = \frac{\partial U_{\text{strain}}}{\partial P} = \frac{2Pa^3}{3EI}. \quad (3.27) \]

This relation can be inverted to give the force-displacement relation

\[ P = \frac{3EI}{2a^3}u. \quad (3.28) \]

Hence, the energy release rate during crack growth is given by

\[ G_I = -\frac{1}{w} \frac{\partial U_{\text{pot}}}{\partial a} = \frac{P^2a^2}{EIw}. \quad (3.29) \]

The crack remains stationary, \( a = a_0 \), as long as the condition \( G_I < G_{IC} \) is fulfilled. Therefore, the initial DCB response is given by substituting \( a_0 \) for \( a \) in (3.28),

\[ P = \frac{3EI}{2a_0^3}u. \quad (3.30) \]

If the critical mode I energy release rate \( G_{IC} \) equals \( G_I \), then the crack propagates with \( a > a_0 \). The propagating crack response is obtained by solving (3.29) for the crack length \( a \), which is then substituted into (3.28),

\[ G_{IC} \]

\(^1\)If the strain energy of a linear elastic structure can be expressed as a function of the generalized force \( Q_i \); then the partial derivative of the strain energy with respect to generalized force gives the generalized displacement \( q_i \) in the direction of \( Q_i \).
\[ P = \sqrt{\frac{2}{3u}} (wG_{IC})^{3/4} (E'I)^{1/4}. \]  

(3.31)

After delamination is completed, the DCB response is given by substituting \( l \) for \( a \) in (3.28),

\[ P = \frac{3E'I}{2J^3} u. \]  

(3.32)

To relate \( G_I \) to \( l_{pz} \) the moment equilibrium of the upper beam near the crack tip is considered, as shown in figure 3.9. Here \( Pa \) is the externally applied moment, \( T \) the traction at the interface and \( \sigma_{comp} \) is the compressive stress that exists just in front of the process zone, which is assumed to have a similar rectangular stress distribution and amplitude as the cohesive stress to simplify the derivation. The assumption of the existence of \( \sigma_{comp} \) is supported by a numerical result of a DCB simulation shown in figure 3.10, which shows the normal stress in \( z \)-direction at the crack plane of the reference DCB specimen \( (a_0 = 20 \text{ mm}, T_{max} = 2.5 \text{ Nmm}^{-2}) \) when the process zone is fully developed. The physical process zone is situated from \( x \approx 20 \text{ mm} \) to \( x \approx 34 \text{ mm} \) and the compressive stresses just in front of the process zone \( (34 \text{ mm} \lesssim x \lesssim 37 \text{ mm}) \) are clearly visible. However, the cohesive and compressive stresses in the numerical analysis do not have the same stress distribution as used in the derivation, which therefore will yield only an estimate of the process zone length.

![Free body diagram of part of the upper beam of a DCB loaded by moment near the crack tip.](image)

Figure 3.9: Free body diagram of part of the upper beam of a DCB loaded by moment near the crack tip.

The equilibrium of moments about point A is given by,

\[ Pa - \int_0^{l_{pz}} \sigma_{comp}(x) w \, dx - \int_0^{l_{pz}} T(x) w \, dx = Pa - w l_{pz}^2 T_{max} = 0. \]  

(3.33)

Substituting (3.33) into (3.29) gives,

\[ G_I = \frac{12l_{pz}^4 T_{max}^2}{E' h^3}. \]  

(3.34)

During delamination it holds that \( G_I = G_{IC} \), which leads to an expression for the process zone length of a DCB specimen which reads
Rewriting to a dimensionless form while introducing the dimensionless parameters $D_1$, $D_2$ and $D$ gives,

$$\frac{l_{pz}}{h} = \left(\frac{1}{12}\right)^{1/4} D^{1/4}, \quad \text{with} \quad D = D_1 D_2, \quad D_1 = \frac{G_{lc}}{T_{max} h}, \quad D_2 = \frac{E'}{T_{max}}. \quad (3.36)$$

Using a different cohesive and compressive stress distribution will only influence the dimensionless constant, which is $\left(\frac{1}{12}\right)^{1/4}$ in (3.36).

The parameter $D_1$ describes the influence of the interface properties on the process zone length. An increase of the brittleness of the interface, will decrease $D_1$ and therefore also the estimated process length. The parameter $D_2$ describes the ratio of the bulk material stiffness and the interface strength and gives an indication of the deformation of the bulk material as a result of the interface tractions.

In comparison with the estimated process length for the T-peel test (3.20), which has flexible films, the bending stiffness influences the process zone length of a DCB specimen. The influence of the geometry characteristic $h$ on the process zone length of a DCB specimen, is the difference with the estimated process zone length of an infinite medium (3.11).

Finally, the power $1/4$ arises through the moment equilibrium at the crack plane (3.33).

### 3.4 End load split test

In this section an analytical prediction of the force-displacement response, the energy release rate and the process zone length of the end load split (ELS) configuration is derived. The
analysis partially parallels that of to the DCB specimen of the previous section. The ELS test is generally used to determine the mode II fracture toughness of a test specimen. An example of an ELS specimen and a typical force-displacement response are given in figure 3.11. The mode II loading is promoted by the relative sliding of the upper surface of the lower beam with respect to the lower surface of the upper beam. The relative sliding, which is assumed frictionless, occurs due to the rotation of the upper and lower parts of the ELS specimen.

Figure 3.11: End load split: (a) schematic overview of the specimen; (b) typical force-displacement response.

Figure 3.12: Freebody diagram of the ELS configuration.

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Using the free body diagram (figure 3.12) and (3.23) one can find the following expression for the strain energy due to bending of the ELS specimen

\[ U_{\text{strain}} = \int_{0}^{a} \frac{(P_x)^2}{8EI_1} \, dx + \int_{0}^{a} \frac{(P_x)^2}{8EI_2} \, dx + \int_{a}^{l} \frac{(P_x)^2}{2EI_3} \, dx , \quad (3.37) \]

Performing the integrations gives the strain energy of the ELS configuration as

\[ U_{\text{strain}} = \frac{P^2(l^3 + 3a^3)}{48E'I} . \quad (3.38) \]

To determine the relation between the tip opening displacement and force \( P \) Castigliano’s second theorem is applied, resulting in

\[ P = \frac{24E'I}{l^3 + 3a^3}u . \quad (3.39) \]

Hence, the energy release rate can be found as

\[ G_{II} = -\frac{1}{w} \frac{\partial U_{\text{pot}}}{\partial a} = \frac{3P^2a^2}{16E'Iw} . \quad (3.40) \]

The crack remains stationary as long as the condition \( G_{II} < G_{IIc} \) is fulfilled. Therefore, the initial ELS response is given by

\[ P = \frac{24E'I}{l^3 + 3a_0^3}u . \quad (3.41) \]

Crack propagation occurs when \( G_{II} \) equals \( G_{IIc} \). The response during crack propagation is given by,

\[ u = \frac{P l^3 + 3P \left( \frac{16wG_{IIc}E'I}{3P^2} \right)^{1.5}}{24E'I} . \quad (3.42) \]

The large expression for \( P \) can be found by rearranging (3.42). The ELS response after complete delamination is given by substituting \( a = l \) in (3.39) as

\[ P = \frac{6E'I}{l^3}u . \quad (3.43) \]

To derive a relation between the process zone length and the energy release rate (3.40) the force equilibrium in \( x \)-direction of the upper beam near the crack tip is considered, as shown by figure 3.13. The stresses \( \sigma_{b1} \) and \( \sigma_{b2} \) are the normal stresses due to bending and \( T \) is the shear traction due to the interface cohesion. To verify the existence of the shear stress at the crack plane a simulation with the reference ELS specimen, with \( T_{\text{max}} = 50 \text{Nmm}^{-2} \) has been performed, see figure 3.14. This figure shows the stresses in \( zx \)-direction at the crack plane.
of the reference ELS specimen \((a_0 = 35 \text{ mm})\) when the process zone is fully developed. The process zone is situated from \(x \approx 43 \text{ mm}\) to \(x \approx 54 \text{ mm}\). In front of the process zone also a small amount of shear traction is observed. This is due to the fact that the shear stress opens the interface over its full length, because of the exponential traction law used in the numerical model. This effect is neglected in the derivation.

![Free body diagram of part of the upper beam of an ELS specimen near the crack tip.](image)

Figure 3.13: Free body diagram of part of the upper beam of an ELS specimen near the crack tip.

![Stresses in \(zx\)-direction at the crack plane of the reference ELS specimen when the process zone is fully developed.](image)

Figure 3.14: Stresses in \(zx\)-direction at the crack plane of the reference ELS specimen when the process zone is fully developed.

The traction distribution in the process zone is assumed rectangular (see figure 3.6(b)), such that equilibrium of forces in \(x\)-direction is given by

\[
\int_0^h \sigma_{b1}(z)w \, dz - \int_0^h \sigma_{b2}(z)w \, dz + \int_{l_{pz}} T(x)w \, dx = \frac{3Pa}{4h} - wl_{pz}T_{max} = 0. \tag{3.44}
\]

The product \(Pa\) then becomes

\[
Pa = \frac{4}{3}hT_{max}l_{pz}w. \tag{3.45}
\]

Substituting (3.45) into (3.40) and setting \(G_{II} = G_{IIc}\) leads to

\[
l_{pz} = \frac{1}{2} \left[ \frac{G_{IIc}}{T_{max}h} \frac{E'}{T_{max}} \right]^{1/2} h. \tag{3.46}
\]
Rewriting to a dimensionless form gives

\[ \frac{l_{pz}}{h} = \frac{1}{2} D^{1/2}, \]  

(3.47)

where \( D = D_1 D_2 \) is defined identically to the DCB specimen, thus the process zone length of the ELS specimen depends on the interface \((G_c \text{ and } T_{\text{max}})\), material \((E')\) and structural \((h)\) properties. The most striking difference between the analytical estimation of the process zone length for a mode I and mode II slender geometry (DCB and ELS test), is the different powers, 1/4 for mode I and 1/2 for mode II. Because the process zone length of the ELS test follows from a force equilibrium (3.44) instead of a moment equilibrium (3.33) (DCB test) this difference in powers arises. This leads to the mode II process zone length being more sensitive to the shape of the traction separation law than the mode I process length.

### 3.5 Validation

In the previous sections of this chapter, 3.1 - 3.4, predictions for the process zone length of different geometries and loading modes have been derived. In the current section the accuracy of these predictions is validated by means of a numerical model with the CZM described in chapter 2. The numerical process zone length has been defined as the length where the cohesive zone is separated by \( \lambda_c \leq \lambda \leq 6\lambda_c \), which is the softening part of the TSL as shown in figure 3.15. This choice is explained in appendix B.

![Figure 3.15: Traction - separation curve with the exponential law. Hatched area defines the numerical process zone.](image)

Firstly the T-peel test is examined, followed by the DCB and ELS tests, whereby the infinity geometry is mentioned aside.

#### 3.5.1 T-peel test

A reference T-peel specimen is constructed with properties that are listed in table 3.1. A plane strain condition is used and the numerical process zone is determined when the process
zone in the model is fully developed for the first time during the T-peel test. In other words it is the first moment where any cohesive zone element has exceeded the value of $6\lambda_c$ and is considered to be fully damaged.

$$\lambda_c = 0.008 \text{ mm}$$

<table>
<thead>
<tr>
<th>$l$ [mm]</th>
<th>$w$ [mm]</th>
<th>$a_0$ [mm]</th>
<th>$h$ [mm]</th>
<th>$E$ [N mm$^{-2}$]</th>
<th>$\nu$ [-]</th>
<th>$T_{\text{max}}$ [N mm$^{-2}$]</th>
<th>$G_c$ [N mm$^{-1}$]</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>0.008</td>
<td>2</td>
<td>0.001</td>
<td>$10^5$</td>
<td>0.3</td>
<td>2.5</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 3.1: Reference geometric, material and interface properties of the T-peel test model.

The parameters $G_c$ and $T_{\text{max}}$ are varied to determine their influence on the numerical process zone length, which is shown in figure 3.16. In both plots the numerical results (red dots interconnected by a dashed line) and the predicted lengths (3.21) (blue line) resemble each other closely, proving the accuracy of (3.21) in the domain considered to predict the process zone length of a T-peel test.

### 3.5.2 Double cantilever beam test

The DCB specimen can be regarded as a T-peel test with beams that behave less flexible. Therefore the reference parameters of the geometry have been altered as can be seen in table 3.2. The increase of height $h$ of the beams ensures less flexibility, such that the specimen deforms similar to a typical DCB specimen.

The finite element model is a point loaded DCB which does not have a steady state process zone length (3.36). When the initial crack length is sufficiently large, this effect can be neglected as shown in appendix C. Besides, the process zone is larger when the (initial) crack length is small, therefore more elements will span the process zone, which will only benefit convergence. The influence of the parameters is examined by using the reference set and varying on of them ($E$, $G_c$, $T_{\text{max}}$ or $h$).
Table 3.2: Reference geometric properties of the DCB specimen.

<table>
<thead>
<tr>
<th>$l$ [mm]</th>
<th>$w$ [mm]</th>
<th>$a_0$ [mm]</th>
<th>$h$ [mm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>60</td>
<td>30</td>
<td>20</td>
<td>1</td>
</tr>
</tbody>
</table>

In figure 3.17 the predicted process zone of the DCB (blue line) (3.36), is compared to the numerical process zone length (red dots interconnected by a dashed line).

In all plots the slopes of the curves resemble each other closely, regardless of which parameter is changed. This indicates that in the domain considered, $10^3 \leq D \leq 10^6$, the found relation between the parameters $E'$, $G_c$, $T_{max}$, $h$ and the length of the process zone is valid. The fact that the curves are almost on top of each other is mainly coincidence, as previously it was concluded that the factor $(\frac{1}{12})^{1/4}$ in (3.36) depends on assumptions made in the derivation with respect to the stress distribution at the interface. Also it is no surprise that although the factor $(\frac{1}{12})^{1/4}$ is a lower bound, the numerical process zone length is sometimes overestimated by the prediction. This is due to the definition of the numerical process zone length ($\lambda_c \leq \lambda \leq 6\lambda_c$), which is smaller than the physical process zone length ($0 \leq \lambda \leq 6\lambda_c$).

In figure 3.18 the effect of a further reduction of the $l_{pz}/h$ ratio is examined. When comparing the predictions of the process zone length for the DCB (3.36) and the uniformly loaded infinite geometry (3.11) one can see that there is a transition point between the two predictions near $l_{pz}/h = 1$. To reach this point, the value of $D$ is reduced down to $10^{-2}$ by increasing $T_{max}$. The results are compared with the prediction for the DCB specimen (blue line) and infinite geometry (cyan line) process zone length in figure 3.18. The numerical results clearly show a change in slope near $l_{pz}/h \approx 1$, indicating that the DCB prediction is indeed not valid for values of $l_{pz}/h \ll 1$ (or $D \ll 1$). For values of $D$ below 0.5, the infinite geometry prediction seems more or less to follow the same slope as the results. In this domain the height of the beams of the DCB test are relatively large compared to the process zone length, such that the free surfaces have no influence on what occurs in the process zone. In addition, the assumption of a central crack in an infinite geometry, instead of an edge crack (which is more similar to a DCB specimen), does not seem to harm the prediction of the process length in this domain. This is due the fact that compared to the infinite geometry, the stress intensity factor $K_1$ is multiplied by a factor of approximately 1.12 [10]. This increases the prediction of the process zone length (3.11) with the factor 1.12, if a derivation of the process zone length for an edge crack is performed similar to the derivation for the infinite geometry as performed in section 3.1.1.
(a) $E$ varied: $2 \times 10^3 - 2.5 \times 10^6 \text{Nmm}^{-2}$.

(b) $G_c$ varied: $0.2 - 25 \text{Nmm}^{-1}$.

(c) $T_{\text{max}}$ varied: $0.5 - 10 \text{Nmm}^{-2}$.

(d) $h$ varied: $0.0625 - 8 \text{mm}$.

Figure 3.17: Comparison of the DCB prediction (3.36) and numerical process zone as function of the dimensionless parameter $D$. 
3.5.3 End load split test

To validate the ELS theory it is necessary to alter $T_{\text{max}}$. The maximum traction is increased to $50\,N\,mm^{-2}$ to reduce the process zone length. To ensure a stable crack growth and prevent numerical difficulties the initial crack length $a_0$ is also increased to $35\,mm$. A comparison between the ELS response with both initial crack lengths is shown in figure 3.19. The figure shows that the ELS model with a small initial crack length exhibits a global snapback, which requires a more complicated (arc length) procedure to obtain a converged solution.

![Figure 3.19: Comparison of the ELS response (determined with (3.41) - (3.43)) for $a_0 = 20\,mm$ (green curve) and $a_0 = 35\,mm$ (red curve).](image)

The parameters $E, G_c, T_{\text{max}}$ and $h$ are varied to determine their influence on the numerical process zone length, which is shown in figure 3.20. In all plots the numerical results (red dashed line) and the predicted lengths (3.47) (blue line) resemble each other closely in the considered domain.

In contrast to the mode I results there does not seem to be the expected change in slope when the ratio of $l_{pz}/h$ is decreased to a value (far) below 1 as shown in figure 3.21. In the domain
Figure 3.20: Comparison of the ELS prediction (3.47) and numerical process zone as function of the dimensionless parameter $D$.

(a) $E$ varied: $10^4 - 10^6 \text{Nmm}^{-2}$.

(b) $G_c$ varied: $0.01 - 10 \text{Nmm}^{-1}$.

(c) $T_{max}$ varied: $25 - 100 \text{Nmm}^{-2}$.

(d) $h$ varied: $0.05 - 8 \text{mm}$.
considered, $10^{-2} \leq D \leq 10^2$, only the ELS prediction is able to describe the process zone length in an accurate manner. Therefore only (3.47) will be used to determine the required number of elements in the process zone of a mode II loaded specimen independent of the value of $D$.

Figure 3.21: Comparison of the adapted ELS prediction (3.47) and infinite geometry theory (3.17) with the numerical determined process zone of an ELS specimen.
Chapter 4

Minimum number of elements in the process zone

The determination of an appropriate mesh size \textit{a priori} depends on the number of elements required within the process zone. In the previous chapter the process zone length is predicted and in this chapter the minimum number of elements required within it is determined. In literature no consensus exists with respect to the necessary number of elements for a successful simulation. Turon [23] provided a brief overview of numerical studies in which the minimum number of elements in the process zone are determined. Mi et al. [16] suggested to use at least 2 elements, Falk et al. [3] 2 to 5 elements, while Moës and Belytschko [17] used no less than 10 elements. Note that differences in used traction separation laws, solution control settings, and definitions of the process zone length influence the defined minimum number of elements, which hampers a comparison between studies on this topic. Therefore the minimum number of elements required with the use of the current CZM will be determined in this chapter, as a result of which an appropriate mesh size can be determined \textit{a priori}. The number of elements, \( N_e \), in the process zone is given by,

\[
N_e = \frac{l_{pz}}{l_e},
\]

where \( l_e \) is the mesh size in the direction of crack propagation.

4.1 T-peel test

To examine the influence of \( N_e \) on the structural response of a T-peel test, two different element sizes, 0.002 mm and 0.008 mm, are applied to the T-peel test. The process zone length is varied by means of varying \( T_{max} \). The resulting force - displacement responses are plotted in figure 4.1. Increasing oscillations are visible with increasing \( T_{max} \) for both element sizes. If \( T_{max} \) is increased to a point where less than one element spans the numerical process zone, convergence is unlikely to occur. Oscillations in the global force - displacement responses become very small when \( N_e \geq 4 \). In figure 4.1 the response for \( N_e = 4 \) is shown
by the red curves ($T_{\text{max}} = 30 \text{ Nmm}^{-2}$ when $l_e = 0.008 \text{ mm}$ and $T_{\text{max}} = 120 \text{ Nmm}^{-2}$ when $l_e = 0.002 \text{ mm}$). These findings are summarised in tables in appendix D.

Using the criterion of $N_e = 4$ leads to the following normalised maximum element length,

$$\frac{l_e}{h} = \frac{1}{4}D_1.$$  \hspace{1cm} (4.2)

Using this expression for the normalised maximum element length, a convergence plot can be constructed as shown in figure 4.2, in which a red cross indicates a non-convergent solution (simulation aborted during delamination) and a green plus a convergent solution. The figure shows that when an element length is chosen that is smaller than or equal to the element length predicted by (4.2) the simulation is successful (indicated by the hatched area). It is clear that a safe margin exists with respect to the maximum element size, because $N_e = 4$ is applied to reduce oscillations in the global force - displacement response. Therefore the recommendation can be used as a secure guideline for constructing a cohesive zone mesh for T-peel test specimens.

Figure 4.1: Influence of process zone length and element size on force - displacement response of T-peel test.
Figure 4.2: Comparison of the prediction for the normalised maximum element size $\frac{l}{h}$ for a T-peel test specimen (4.2) with numerical outcomes.

4.2 Double cantilever beam test

Similar to the T-peel test, a DCB specimen with three different element sizes, 0.25 mm, 1 mm and 4 mm, is examined. The resulting force - displacement responses are plotted in figure 4.3. The portion of the response with negative slope corresponds to a propagating crack. With increasing $T_{\text{max}}$ the interface is more brittle, the peak forces are higher and the response during delamination is more oscillatory. When small elements are applied the oscillation is less visible in the global response, due to the little impact of the opening of one (small) element with regard to the global structural response. The DCB delaminates over a length of 40 mm $(l - a_0)$, therefore opening an element with the size of 4 mm dissipates 10 % of the total delamination energy, while opening an element of 0.25 mm only contributes 0.625 %.

If $T_{\text{max}}$ is increased to a point where less than 2 elements span the numerical process zone, convergence is unlikely to occur. The T-peel test requires only 1 element in the process zone to converge. The increase of $N_e$ to obtain a converged solution is possibly attributed to the sharp limit point in the global force - displacement response when delamination starts. In addition, the (physical) global force - displacement response of the T-peel test remains constant during steady state delamination, while the response of the DCB test decreases. Because of this, the local failure of one cohesive zone element in the DCB specimen can possibly cause a snap-back, while the local failure of one cohesive zone element in the T-peel test generally only causes a snap-through, which harms the convergence less than snap-back.

Oscillations in the global force - displacement responses become small when $N_e \gtrsim 4$. In figure 4.3 the response with $N_e = 4$ is shown by the red curves for all element sizes. These findings are summarised in tables in appendix D.
Figure 4.3: Influence of process zone length and element size on force-displacement response of the reference DCB.
The prediction of the required element length can be found by the use of equations (4.1) and (3.36) (for \( D \leq 1 \)) and (3.11) (for \( D \geq 1 \)). With \( N_e = 4 \) this leads to the following normalised maximum element length,

\[
\frac{l_e}{h} = 0.13D^{1/4}, \quad \text{for } D \ll 1, \quad \text{and} \quad \frac{l_e}{h} = \frac{1}{4\pi}D, \quad \text{for } D \gg 1.
\] (4.3)

Based on these two relations, combining them with the data depicted in figure 4.3 and using the convergence information obtained from the simulations in the \( l_{pz}/h \ll 1 \) domain, a plot can be constructed which shows how the prediction of an appropriate element size compares with the convergence of the actual numerical simulations. This is shown in figure 4.4, in which a red cross indicates a non-convergent solution (simulation aborted) and a green plus a convergent solution. The figure shows that when an element length is chosen that is smaller than or equal to the element length recommended by (4.3) the simulation is successful (indicated by the hatched area). A safe margin exists with respect to the maximum element size recommended, because \( N_e = 4 \) is applied to reduce oscillations in the global force-displacement response. Especially for larger values of \( D \), the recommended maximum element length is on the safe side and significantly larger element lengths can be used. Therefore the recommendation can be used as a secure guideline for constructing a cohesive zone mesh for DCB specimens.

![Figure 4.4: Comparison of the prediction for the normalised maximum element size \( \frac{l_e}{h} \) for a DCB specimen (4.3) with numerical outcomes.](image)

### 4.3 End load split test

The influence of the mesh size on convergence has been determined for the ELS test similarly to the DCB and T-peel test specimen. The force-displacement response with the three different element sizes and varying maximum interface tractions are plotted in figure 4.5. The same phenomena as for the DCB specimen are visible, e.g. the peak forces increase with increasing \( T_{\text{max}} \). On the contrary, there is hardly any oscillation visible when a few cohesive zone elements are within the process zone. If \( T_{\text{max}} \) is increased to a point where less than about 2.5 elements span the numerical process zone, convergence is unlikely to occur. This is slightly more than for the DCB specimen. Possibly this is caused by the sharper limit point
Figure 4.5: Influence of process zone length and element size on force - displacement response of the reference ELS. The solution with the highest interface strength for all the element sizes considered, aborted before delamination started (black curves). Therefore no $N_e$ could be obtained and $T_{max}$ is mentioned in the legend.

in the global force - displacement response of the ELS specimen. The results are summarised in tables in appendix D.

Consistent with the T-peel test and DCB specimen, $N_e = 4$ is chosen as a limit. This leads to the following normalised maximum element length,

$$\frac{l_e}{h} = \frac{1}{8} D^{1/2}. \quad (4.4)$$

Using this expression a convergence plot can be constructed as shown in figure 4.6 in which a red cross indicates a non-convergent solution and a green plus a convergent solution. The figure shows that when an element length is chosen that is smaller than or equal to the element length recommended by (4.4) the simulation is successful (indicated by the hatched area). Because of the safe margin the recommendation can be used as a secure guideline for
constructing a cohesive zone mesh for ELS specimens.

Figure 4.6: Comparison of the prediction for the normalised maximum element size $\frac{l_e}{R}$ for an ELS specimen (4.4) with numerical outcomes.

4.4 Conclusions

In this chapter the influence of the number of elements, $N_e$, in the process zone is examined for the T-peel, DCB en ELS test. A minimum number of 4 elements gives a satisfactory result with respect to the convergence of the simulation and oscillations in the global force-displacement response, such that an appropriate cohesive element size can be determined \textit{a priori}. Fewer cohesive elements can be used, up to a minimum of 1 element for the T-peel test, if convergence is the only criterion and significant oscillations are taken for granted. Note that deformation of the bulk material, and particularly deformation at the surface of the interface, is relatively small in these examples. Large deformation could possibly reduce the convergence, which may compromise the margin that exists when $N_e = 4$ is used.
Chapter 5

Applications

In the previous chapters the process zone length and appropriate cohesive zone element size have been derived and numerically examined for typical test such as the T-peel, DCB and ELS specimen. In this chapter the obtained knowledge is applied to two practical examples, geometries considered more complex and consist of multiple materials. The purpose of these two applications is to assess the validity of the mesh design rules determined in the previous chapter for more realistic cases.

The first example consists of a 90° fixed arm peel test with an asymmetric geometry, aimed to gain insight in the delamination process of stretchable electronics. Herein a thin copper film is peeled off a soft rubber substrate. The second example is an analysis of layer buckling and delamination of a typical thin bi-layer structure that is used in flexible display applications.

5.1 Stretchable electronics

5.1.1 Introduction

Stretchable electronics are electronic circuits deposited on stretchable substrates or embedded completely in a stretchable material such as silicone or polyurethane. The stretchability of the electronics provides possibilities for use in healthcare, wellness and functional clothes [11]. This type of electronics remains functioning properly after various modes of deformation like bending, twisting and stretching and is, depending on its application, able to deal with the influence of the mechanical environment (e.g. machine washability, vibration) and chemical environment (e.g. water, salt, organic acids, sweat). To develop and manufacture these kind of products the European STELLA project was started in 2006, with Philips Applied Technologies as one of the partners. The abbreviation STELLA stands for STretchable ELectronics for Large Area applications, where large area covers the various areas mentioned previously in which stretchable electronics can be applied.

An example of a stretchable electronic application is shown in figure 5.1(a), which shows a deformable thermometer that can be used to measure the local temperature of a body part while exercising. Herein two rigid semiconductors are interconnected with thin metal
conductor lines that are placed on a rubber substrate (figure 5.1(b)), which supplies the stretchability of the product.

One of the issues encountered in the development of stretchable electronics is adhesive fracture between the metal lines and the substrate. After delamination the wiring is no longer constrained by the substrate, such that shorts can originate (figure 5.2). Insight in the delamination process is therefore of fundamental importance in predicting failure of a stretchable electronic device.

Figure 5.2: Delamination of metal lines from the elastomeric substrate leads to possible shorts [27].

5.1.2 Model

A 90° fixed arm peel test with a copper-rubber interface is examined, to gain insight in the delamination process of stretchable electronics. This test was performed by Van der Zanden [27] with the main aim to determine the interface properties of the copper-rubber interface. Herein a thin copper film is peeled off a soft rubber substrate. In figure 5.3 a picture of the experimental peel test setup is shown and schematically depicted.
After initial delamination the peel force becomes more or less constant as the specimen reaches a steady state. Under the assumption that the film and substrate deform elastically and that the substrate stiffness is large compared to the film stiffness, the peel force per unit of sample width becomes a direct measure for the adhesion energy. Clearly both criteria are not fulfilled for the copper-rubber interface investigated in the work of Van der Zanden [27]. However it was shown that the errors with regard to these assumptions can be neglected for this specific case [27]. As a result, the measured peel force determines the critical work of separation of the interface as defined by equation (3.18). To predict the process zone length of this peel test, relation (3.20) for the T-peel test is used, because of its similarities with the $90^\circ$ fixed arm peel test.

To examine its applicability to this particular peel test a numerical parameter study is performed. The geometry of the 3D finite element model is shown in figure 5.4. A plane strain condition is imposed by applying corresponding boundary conditions and ties. At the bottom plane of the substrate all degrees of freedom are fixed, corresponding to the experiment where the sample is glued to the test setup. The peeling of the copper film is simulated by prescribing an 8 mm displacement in $z$-direction at the upper left corner point of the copper film, which is denoted by $u$ in figure 5.4. The reference properties of the model are similar to the values measured by Van der Zanden. The copper film has a Young’s modulus of 84.5 GPa and a Poisson’s ratio of 0.3. The plasticity of the copper is modelled by a table fit as shown in figure 5.5. The incompressible rubber substrate is modelled by a Neo-Hookean model with a $C_{10}$-value of 0.165 Nmm$^{-2}$ using Herrmann elements. The height of the film, $h_1$, is 0.017 mm and $h_2$ represents the substrate height, 1.1 mm. The size of the specimen in $y$-dimension, $w$,
is 18 mm and the length \( l \) is 15 mm. The length of the test specimen is actually 84 mm, but it is reduced in the model to 15 mm to reduce the necessary CPU time. This is possible because the steady state peel force is reached soon after the start of the delamination process. The initial crack length, \( a_0 \), is 2.5 mm and represents the manually delaminated area in the peel experiment. The interface properties are given by \( T_{\text{max}} = 2.5 N \text{mm}^{-2} \) and \( G_c = 1.35 N \text{mm}^{-1} \). The predicted process zone length (3.20) then becomes 0.54 mm. Although only a minimum of one element in the process zone is required for convergence in the T-peel test, an element size \( l_e \) of 0.025 mm is used to determine the process zone length accurately.

![Figure 5.5: True stress-strain curve-fit of the copper. Yield stress \( \sigma_Y = 150 N \text{mm}^{-2} \).](image)

5.1.3 Results

A parameter study is performed to examine the validity of (3.20) to predict the process zone length of this peel test. The results of the parameter study are shown in figures 5.6 and 5.7. Figure 5.6 shows the parameters which should not affect the size of the process zone according to (3.20), figure 5.7 shows the two parameters, \( T_{\text{max}} \) and \( G_c \), that should affect the process zone length. The red dashed line shows the numerical process length as a function of the considered parameter and the blue line the length predicted by (3.20). It is important to note that the numerical process zone lengths are determined using the undeformed geometry of the peel test model. Specifying a mesh size of a finite element model is by definition dependent on the undeformed geometry and therefore predicting the deformed length of the process zone is irrelevant in this respect.

In figures 5.6(a), 5.6(b) and 5.6(c) the influence of different parameter values is indeed negligible in the considered value range. Nevertheless, the absolute value of the process length is underestimated, which is not expected based on the experience of the T-peel test. This underestimation can be explained by the amount of substrate deformation (for the reference parameters of \( T_{\text{max}} \) and \( C_{10} \)), which depends on the ratio of the interface strength \( T_{\text{max}} \) and rubber substrate stiffness \( C_{10} \). If the stiffness of the substrate is increased from the reference value \( C_{10} = 0.165 N \text{mm}^{-2} \) to a value \( C_{10} \gtrsim 1 N \text{mm}^{-2} \), the numerically determined process zone length is no longer overestimated, but underestimated by the prediction, as shown in
Figure 5.6: Influence material and geometric properties of both rubber and copper on the process zone length of the numerical peel test model. Prediction (3.20) is used.
Increasing the substrate stiffness increases the process zone length until the saturation-point at $C_{10} \approx 100 \, Nmm^{-2}$. The substrate does not show deformation anymore when the substrate is this stiff, and therefore increasing its stiffness further will not influence the process zone length. Changing the ratio of the interface strength $T_{\text{max}}$ and substrate stiffness $C_{10}$ by decreasing $T_{\text{max}}$ has a similar effect, as can be seen in figure 5.7(a). The two smallest values of $T_{\text{max}}$, 0.3125 and 0.625 $Nmm^{-2}$, also give a process zone length that is underestimated by the prediction, as expected due to the decreased substrate deformation. The dependence on the interface toughness $G_c$ has the same slope as predicted, with an overestimation of its absolute value (figure 5.7(b)).

![Figure 5.6(d)](image)

Figure 5.7: Influence interface properties on the process zone length of the numerical peel test model with a soft substrate. $C_{10} = 0.165 \, Nmm^{-2}$. Prediction (3.20) is used.

The convergence of the model is examined in figure 5.8 by varying $T_{\text{max}}$ of the reference model from 0.3125 to 10 $Nmm^{-2}$. About two elements to span the process zone ($l_{pz} = 0.175 \, mm$ with the reference properties) are chosen, which is expected to be nearly critical with respect to convergence. Therefore, an element size of 0.075 $mm$ is applied. A red cross indicates a diverged solution in the figure and a green plus a converged solution. The figure shows that when an element length is chosen that is smaller than the element length predicted by (4.2) the simulation is successful. The margin that is used in the element size prediction prevents that too few elements are within the process zone for convergence, because as previously mentioned, the process zone length is underestimated due to the soft substrate. An oscillating response is therefore expected for the highest values of $T_{\text{max}}$ that converge. Concluding, (4.2) is considered to successfully predict the element size required for convergence for this peel test.
Figure 5.8: Comparison of the prediction for the normalised maximum element size $\frac{L_{e}}{l_{1}}$ (4.2) for the peel test specimen with its numerical outcome.

## 5.2 Flexible displays

### 5.2.1 Introduction

Flexible displays are essentially very thin display screens that can be produced on flexible or stretchable material in a variety of shapes. Flexible displays offer many benefits over the currently available display technologies, such as reduction in thickness and weight, improved durability and freedom form. In figure 5.9(a) an example of an flexible display is shown. One critical failure mode that is frequently observed in flexible electronics applications is examined in this section. From an initial region of poor adhesion between film and substrate, buckling-driven delamination of the film occurs due to the compressive stresses [26], as shown in figure 5.9(b). This failure mode possibly causes reliability and functional issues and should be avoided.

Figure 5.9: (a) Example of a flexible display from Polymer Vision; (b) observed buckling driven delamination within pixels [25].
5.2.2 Model

An example of a typical coupled buckling and delamination failure mode is examined in this section. In figure 5.10 the bi-layered buckling-driven delamination model is shown. It is a model which matches closely the model of Van der Sluis [26]. The main modification is the use of elastic instead of non-linear elasto-plastic material behaviour for the substrate to reduce CPU time. This model was designed to quantify the interface properties.

![Figure 5.10: Buckling-driven delamination model of the two-layer system (picture is not to scale) [26].](image)

The bi-layer model is compressed by applying an $x$-displacement $u$ of $-10 \, \mu m$ on the right-hand side. Only half of the geometry is modelled by using the symmetry of the geometry. A 3D-model is used because of the used CZM, but it is effectively turned into a 2D-model by using the appropriate boundary conditions and ties. In order to trigger buckling, an initial geometric imperfection is inserted, with length $a_0 = 1 \, \mu m$ and height $h_2 = 0.0015 \, \mu m$. The height of the Arylite substrate $h_3$ is $200 \, \mu m$ and its length $l$ is $150 \, \mu m$. The indium tin oxide (ITO) layer has an height $h_1$ of $0.25 \, \mu m$. Linear elastic material behaviour is used for both materials. The stiffness of the ITO-layer is 120 $GPa$ with a Poisson’s ratio of 0.3. The stiffness of the substrate is 3 $GPa$ with a Poisson’s ratio of 0.38. The reference interface properties are given by $T_{\max} = 116 \, \text{Nmm}^{-2}$ and $G_c = 0.032 \, \text{Nmm}^{-1}$. The stiffness of the interface is therefore much lower than the stiffness of the substrate.

5.2.3 Results

Based on the previous example of the peel test one can already predict that the substrate is unlikely to deform largely, due to the high stiffness of the ITO layer in comparison with the interface strength, which simplifies the prediction. In addition one expects a mixed loading mode in the process zone, with mode I loading dominating when the buckle starts to form, because of the buckle movement in $z$-direction. When more compression is applied, more shearing will occur at the interface, because of the growing difference in length of the layers. To estimate the process zone length, the predictions of the DCB (3.36) and ELS test (3.47)
are used. The reference process zone length of the DCB prediction is 0.80 $\mu m$ and of the ELS prediction 4.41 $\mu m$. The cohesive zone element size is set to 0.25 $\mu m$, such that a minimum of 3 elements span the process length even for a pure mode I condition. However, in reality probably more elements will span the process zone, because a mixed mode condition will occur. In figure 5.11 the process zone length is shown at both the start of the delamination (mode I dominant) and at the end of the delamination (mode II dominant) and compared with the corresponding predictions for variations of various material, geometry and interface properties.

Figure 5.11: Influence of various material and geometric properties of the ITO layer and interface properties on the dimensionless numerical process zone size at both the start (red dashed line) and end of the delamination (green dotted line). Predictions (3.36) (blue solid line) and (3.47) (cyan dashed line) are used.
Figure 5.12: The mixed mode conditions in the process zone, which is denoted by the red line on the substrate, at both the start (a) and the end (b) of the delamination. The start of the delamination is mainly mode I loading, in contrast to the end of the delamination which contains mainly mode II loading in the process zone.

The figures show that the process zone length is well predicted for both the start as well as the end of the delamination, regardless the parameter that is varied. Similarly to the DCB and ELS test, the numerical process zone length is usually underestimated by the prediction. In particular, this applies to the start of the delamination, which does result in a pure mode I loading in the process zone, as shown in figure 5.12(a). Because of the mixed mode loading, the process zone length increases compared to a pure mode I loaded case. Figure 5.12(b) shows the process zone (red line) at the end of the delamination, which contains mainly mode II loading as expected.

The convergence of the model with $l_e = 0.25\mu m$ is examined in figure 5.13 by varying $T_{max}$ of the reference model from 60 to 400 $Nm^{-2}$. Only the start of the delamination is considered, because the process zone is smallest at that point during delamination and therefore critical with respect to the choice of the cohesive zone element size. A red cross indicates a non-converged solution in the figure and a green plus a converged solution. The figure shows that when an element length is chosen that is smaller than the element length predicted by (3.36) the simulation is successful. Again, a larger element size than recommended by (3.36) can be used, especially as there is no pure mode I loading at the start of the delamination, which increases the process zone length. An important observation is a multiple number of buckles occurred during simulations, as schematically shown in figure 5.14, which are not explainable. Therefore it is not sure whether the delamination of the cohesive zone elements is the cause of the aborted simulations or the buckling itself.
Figure 5.13: Comparison of the prediction for the normalised maximum element size $l_n/l_1$ (4.3) for the buckling-driven delamination model with its numerical outcome.

Figure 5.14: Example of buckling instabilities in a diverged buckling-driven delamination simulation (not to scale).
Chapter 6

Conclusions

The aim of the project is to define cohesive zone element mesh size requirements that allow stable numerical simulations of interface delamination. To this end, two steps are taken subsequently; the estimation of the process zone length and the determination of the number of elements required within the process zone.

The present study shows that the process zone length can be estimated analytically, for three typical bi-layer problems: A T-peel test, bending in mode I and bending in mode II are examined. A numerical validation across a range of material, interface and geometric properties shows that the estimate is accurate in predicting the process zone length for these specimens, despite the fact that several assumptions have been made in the derivation.

Interestingly, the process length shows a dependency on a combination of the properties of the bulk material ($E'$), interface ($T_{max}$ and $G_c$) and geometry ($h$). This dependency is captured by the dimensionless parameter $D = D_1 D_2$. The dimensionless parameter $D_1$ describes the influence of the interface properties on the process zone length. An increase of the brittleness of the interface, will decrease $D_1$ and therefore also the estimated process length. The dimensionless parameter $D_2$ describes the ratio of the bulk material stiffness and the interface strength and gives an indication of the deformation of the bulk material as a result of the interface tractions.

For peel like specimens, which contain at least one thin film, the bending stiffness of the specimen is too small to have an influence. Therefore the material and geometric properties are not relevant in estimating its process zone length, and only the interface properties (captured by $D_1$) determine its process zone length. On the other hand, the geometry of the specimen can be relatively large compared to the process zone length ($D \ll 1$), such that the geometric properties no longer influence the process zone length. Remarkably, in the numerical validation this phenomenon is only visible for mode I loading and not for mode II loading. In addition, the dependency of the process zone length on $D$ differs for a slender mode I and II loaded geometry ($D \gg 1$), by a difference in powers.

The second step to determine the relation between the process zone size and required element size, is performed by means of numerical study for the three types of bi-layer problems. A minimum of four elements within the numerical process zone provides a stable, converged solution with minimal oscillating global response.
Mesh design rules are constructed by combining above results. The validity of the derived mesh design rules is successfully examined by two practical applications which are found in electronic devices: A 90° fixed arm peel test and a buckling-driven delamination of a bi-layer. In the fixed arm peel test, large deformations at the interface occur, resulting in an underestimation of the process zone length. However, convergence is still obtained when the mesh design rules are applied, although an oscillatory response is possible. A lower bound for the process zone length is obtained by the mode I cohesive zone element criterion, which is used to determine an appropriate cohesive zone element size. This analysis of the buckling-driven delaminating bi-layer shows the value of the mesh design rules with respect to mixed mode.

The mesh design rules resulting from this study are limited to rate-independent interface delamination of elastic, homogeneous, isotropic bi-layers. The size of the cohesive zone elements is most critical with regard to mesh designing in delamination models which contain brittle interfaces. The maximum element size applied in delamination models containing ductile interfaces is usually not limited by the size of the cohesive zone elements, yet for example by accurately representing the geometry or stress field in the (bulk) material. However, estimations can be made regarding to the process zone length of types of delamination that are not considered here, e.g. plasticity, rate dependent delamination and multi-layer structures. A highly conservative approach with respect to the determination of the maximum element size of multi-layer structures would be to use the properties of the most brittle interface, the stiffness of the most soft layer, and the height of the thinnest layer and use the corresponding cohesive element size. In this respect, the present study can be used as a starting point for mesh design rules for more complex interfacial delamination models.

Finally, applying a cohesive mesh size as presented in the current study does not ensure a converged solution, as there are multiple reasons possible which result in a diverging solution, insufficient cohesive zone elements is just one of them.
Bibliography


Appendix A

Cohesive zone models

In this appendix two other cohesive zone models are examined and compared with the CZM of Van den Bosch that is used in this project. It is recommended that one first reads chapter 2 completely before this appendix, as it describes the Van den Bosch CZM and explains some basics of a CZM.

In recent history a number of interfacial damage models have been implemented in FEM software packages. In the commercial FE code MSC.Marc® implementations of Van Hal et. al [28] and Van den Bosch [24] are available at Philips Applied Technologies by the use of user subroutines. MSC.Marc® itself has also developed a CZM in its software package. Firstly the models of Van Hal and MSC.Marc® are described briefly, after which they are compared with the Van den Bosch model (which is described in chapter 2) to examine which model is most suitable to determine the process zone and the number of elements within in relevant laminates.

A.1 Van Hal

Van Hal et. al developed a CZM [28] which was aimed to analyse the structural integrity of IC interconnects, whereby the focus is on the so called metal peel off failure mode. Metal peel off is caused by delamination of the brittle interface between several layers in the interconnect structure. The quasi-static solution of the mechanical problems under consideration may exhibit so-called limit points (i.e., snap-through and snap-back points), caused by the brittleness of the interfaces. The presence of these limit points requires the use of an arc-length control method. A so-called local arc-length control method is used which allows to use only the damage in the active cohesive zones to control the load in the solution procedure. This procedure is based on the weighted subplane method proposed by Geers [5] and is more robust than conventionally used global arc-length methods. The local arc-length method does not reduce the oscillations observed in the global response during delamination, but allows more severe oscillation before convergence problems arise. This is shown in figure A.1 where the global numerical response of a DCB is simulated with the local arc-length method of Van Hal and a typical conventional CZM in this respect of Van den Bosch.
Figure A.1: (a) Oscillating global force-displacement response of a double cantilever beam specimen with an insufficiently refined mesh with both a conventional CZM of Van den Bosch (green) and the local arc length method of Van Hal (red) compared to the analytical solution as provided by (3.30) and (3.31) for respectively the initial response (blue dashed curve) and the response for a propagating crack (black dashed curve). (b) Zoomed in on a part of figure A.1(a) to have a closer look on the oscillating response.

The Van den Bosch CZM fails to converge when $u$ is about 3 $mm$ due to an insufficiently refined mesh, while the Van Hal implementation is able to converge. The average force $P$ is overestimated by the Van den Bosch model due to its inability to simulate snap-back points and shows the added value of the local-arc length method in this respect.

The Van Hal model does not have a parameter $\beta_2$ to define a ratio between the work of separation for Mode I and II loading, but it does have a parameter $\beta_1$ to define a ratio between the maximum shear traction $T_{s_{\text{max}}}$ and the maximum normal traction $T_{n_{\text{max}}}$ as follows

$$\beta_1 = \sqrt{\frac{T_{s_{\text{max}}}}{T_{n_{\text{max}}}}}.$$  \hfill (A.1)

The traction is defined as [28]

$$T = \sqrt{T_n^2 + \beta_1^2 T_s^2}.$$  \hfill (A.2)

The normal traction $T_n$ and shear traction $T_s$ are given by

$$T_n = T \frac{\delta_n}{\lambda},$$  \hfill (A.3)

$$T_s = T \frac{\beta_1^2 \delta_s}{\lambda}.$$  \hfill (A.4)
where $\delta_n$ is the separation in normal direction and $\delta_s$ the separation in shear direction. To visualise the influence of the $\beta_1$ parameter on the tractions, a single element test is performed with the Van Hal CZE; a 2D four-node element with Gauss integration points and an exponential TSL is used for this purpose. In this test the top line (line 3-4 in figure A.2) of the CZE is loaded by a load angle $\alpha$ of 45°, as schematically depicted in figure A.2.

![Figure A.2: Schematic overview of a single loaded ($\alpha = 45^\circ$) 2D cohesive zone element. $\delta_n$ is the separation in normal direction and $\delta_s$ the separation in shear direction.](image)

The resulting traction - separation curves are shown in figure A.3. A $G_c$ of 1 $Nmm^{-1}$, a $T_{max}$ of 1 $Nmm^{-2}$ and a $\beta_1$ value of 1 and 2 are used. The results with a $\beta_1$ value of 1 are obviously the same as the results obtained with a formulation in which the $\beta_1$ parameter is not included. Using a $\beta_1$ parameter value of 2 results in a shear traction (blue line) which is four times as large as the normal traction (blue triangles). The increase of the $\beta_1$ value effectively shortens the characteristic separation length in shear direction.

![Figure A.3: Resulting traction - separation curve for a value of 1 and 2 of $\beta_1$. $T_{max} = 1 Nmm^{-2}$, $G_c = 1 Nmm^{-1}$ and $\alpha = 45^\circ$ are used.](image)

A.2 MSC.Marc©

The MSC.Marc© documentation only offers a brief description of the implementation of its cohesive zone elements. Therefore some of its characteristics have examined by running some simple tests. There are eight different elements to choose from: quadrilateral planar (2D), hexahedral (3D), pentahedral (3D) and axi-symmetric (2D) for which linear and quadratic interpolation functions are available. All elements use the mid-line/plane as their reference line/plane on which the local basis is constructed. Exponential, bi-linear and linear-exponential traction separation laws are available as shown in figure A.4.
Furthermore both coupling parameters, $\beta_1$ and $\beta_2$, are included. The $\beta_1$ parameter is included similar to the Van Hal model. The $\beta_2$ parameter is included different to the Van den Bosch model. A simple single element test is run in which an MSC.Marc\textsuperscript{®} 2D CZE is loaded at different load angles $\alpha$ and a $\beta_2$ value of 2. The analyses have been repeated with the Van den Bosch model CZE. In figure A.5) one can see the difference between the two cohesive zone models in terms of the amount of dissipated energy during opening of the cohesive zone element in mixed mode conditions. Depending on measurements of the interface that is to be modelled, one could favour one model description over the other.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{figureA4}
\caption{Dissipated energy as function of load angle $\alpha$ for both the MSC.Marc\textsuperscript{®} and Marco van den Bosch implementations. Mode I corresponds to $\alpha = 0^\circ$ and mode II to $\alpha = 90^\circ$. $G_{Ie} = 1 \text{Nmm}^{-1}$, $G_{IIe} = 2 \text{Nmm}^{-1}$.}
\end{figure}
A.3 Comparison

In this section two simple tests with the three cohesive zone models are performed in order to examine their behaviour. Most tests consist of three elements: a solid top and bottom and one cohesive zone element which connects the two, such that a basic layered structure is simulated. In figure A.6 this is schematically shown. The Van den Bosch is a 3D implementation, but has been tied such that it behaves as a 2D element.

![Figure A.6: Schematic overview of the single cohesive zone element test model. CZE node numbers included.](image)

In figure A.6 the cohesive zone element is opened in normal direction to visualise it, but in reality it has an initial zero height. The solid top and bottom are two linear elastic plates with a length \( l \) of 1 \( mm \), height \( h \) of 0.1 \( mm \) and width \( w \) of 1 \( mm \). The bulk material has a Young’s modulus of 200 \( GPa \) and a Poisson’s ratio of 0.3. A plane strain state is assumed.

In table A.1 the interface properties of the cohesive zone element are listed. In some cases several values are changed in order to examine their influence on the solution, but if this is not mentioned the interface properties of table A.1 are used.

<table>
<thead>
<tr>
<th>( T_{n,max} [Nmm^{-2}] )</th>
<th>( T_{s,max} [Nmm^{-2}] )</th>
<th>( G_{Ic} [Nmm^{-1}] )</th>
<th>( G_{IIc} [Nmm^{-1}] )</th>
<th>( \lambda_c [mm] )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0.368</td>
</tr>
</tbody>
</table>

Table A.1: Interface properties of the cohesive zone element.

The MSC.Marc\textsuperscript{©} model has three different traction separation laws. The exponential law is used to simplify the comparison with the models of Van Hal and Van den Bosch, which also contain the exponential traction separation law. In the MSC.Marc\textsuperscript{©} model also the Newton-Cotes/Lobatto scheme can be selected, but the Gauss integration scheme is used for comparison.

A.3.1 Contraction

This test shows the difference between the user subroutines and MSC.Marc\textsuperscript{©} regarding the calculation of the traction with respect to their initial element area. In the test a cohesive zone element is separated by a mode I loading, but after 0.5 \( mm \) displacement, the left edge is translated to the right edge by 0.9 \( mm \). After this decrease of the element surface (from 1 \( mm^2 \) to 0.1 \( mm^2 \)), the mode I loading is continued such that complete delamination occurs. No bulk elements are used. This is schematically shown in figure A.7.

In figure A.8 the resulting reaction force-separation curves are shown. It can be seen that the two user subroutines behave similar and that the MSC.Marc\textsuperscript{©} model shows a different result.
Figure A.7: a) Initial configuration with applied boundary conditions, b) Mode I separation of 0.5 mm completed, c) Reduction of element area to 0.1 mm$^2$ completed, d) Mode I loading continued until complete separation.

Figure A.8: Reaction force-separation curve for mode I loading with a contraction after a normal opening of 0.5 mm. $G_c = 1 \text{Nmm}^{-1}$, $T_{max} = 1 \text{Nmm}^{-2}$, $\lambda_c = 0.368 \text{mm}$.

The area reduction by a factor of 10, also reduces the calculated reaction force by a factor 10 when the MSC.Marc$^\text{©}$ model is used with large strain formulation. If small strain formulation is used this does not occur. The traction of the two user subroutines on the other hand increases with a decreasing element surface such that the reaction force observed remains equal during contraction. From the Van den Bosch model it is known that this behaviour is expected due to the use of Piola-Kirchhoff tractions which are defined on its original configuration, such that traction equilibrium at the interface is always maintained.

A.3.2 Rotation

In this test a rigid rotation is applied before any loading takes place to show the difference between the different types of bases that are used by the model. The test model is first rotated by 180 degrees such that the top element becomes the bottom element and vice versa. Subsequently a mode I loading is applied. The results should be exactly the same as when there is no rotation prior to the mode I loading. This is however not the case with the Van Hal model as shown in figure A.9.

The Van Hal model does not show the expected exponential curve, but a linear response...
Figure A.9: Reaction force observed at the top bulk element for different cohesive zone models when Mode I loading is applied after 180° rotation. $G_c = 1 \text{ Nmm}^{-1}$, $T_{max} = 1 \text{ Nmm}^{-2}$, $\lambda_c = 0.368 \text{ mm}$.

with the same slope as the initial slope of the Van den Bosch and MSC.Marc® curves. This is the same behaviour as when a Van Hal CZE is compressed instead of the usual opening of the cohesive zone element. The Van den Bosch and MSC.Marc® curves have the exact same curves when the rotation is not applied before the mode I loading occurs, which is the correct behaviour. Other amounts of rotation like 110 and 140 degrees have also been applied and give a similarly incorrect response by the Van Hal model and a correct response of the Van den Bosch and MSC.Marc® models. The incorrect behaviour of the Van Hal model is explained by the inability of the local basis to rotate. The Van Hal model and implementation were constructed for brittle interfaces and therefore have not been designed to deal with large displacements and rotations.

A.4 Conclusions

The Van Hal model offers an important advantage over the other two models. It can cope with larger oscillations due to its local-arc length method and therefore also larger element sizes are allowed compared to the other models. However, one may question what is the value of strongly oscillating results as seen for example in figure A.1. In addition the tests with preliminarily rotation applied to the model showed that specimens that sustain large rotations behave incorrect due to an inability of the local basis to rotate accordingly. The brief documentation of the MSC.Marc® model and the insecurity if force equilibrium is maintained if more advanced geometries are applied to the interface give reasons to prefer the Van den Bosch model over the MSC.Marc® model. In addition, the Van den Bosch implementation is constructed for large displacements and rotations, which occur in several electronic applications. Concluding, the Van den Bosch model is considered to be the most safe and logical choice in determining the required maximum converging element size in relevant laminates.
Appendix B

Numerical process zone length

In this appendix the choice of the lower and upper boundary of the numerical process zone ($l_{pz} = \text{lower boundary} \leq \lambda \leq \text{upper boundary}$) is numerically examined for the T-peel (section 3.5.1), DCB (section 3.5.2) and ELS reference specimens (section 3.5.3).

In table B.1 the length of the numerical process zones for the three specimens are given for different choices of upper and lower boundaries. The analytical lengths of the process zones as determined with equations (3.20), (3.35) and (3.46) are 0.4 mm, 6.2 mm and 3.3 mm for the T-peel, DCB and ELS specimen respectively.

From table B.1 it is immediately clear that a lower boundary value near 0 is undesirable to define the numerical process length, because it will be infinitely large. This is explained by the fact that only an infinitesimal stress is necessary in the crack plane to cause separation. This effect is most noticeable in the mode II ELS configuration, where the minimum lower boundary should at least be 0.01 to 0.1 $\lambda_c$ to prevent an infinitely large process zone. To have a safe lower boundary in this respect a $\lambda_c$ value of 1 is chosen, which coincides with $T_{max}$ and therefore the definition of the numerical process zone is situated in the softening part of the TSL only. Note, however, that up to $T_{max}$, already 26.4 % of the total possible work of separation is provided. The upper boundary is set at 6 $\lambda_c$, after which 98.3 % of the

<table>
<thead>
<tr>
<th>Lower boundary [$\lambda_c$]</th>
<th>Upper boundary [$\lambda_c$]</th>
<th>$l_{pz}$ T-peel [mm]</th>
<th>$l_{pz}$ DCB [mm]</th>
<th>$l_{pz}$ ELS [mm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.001</td>
<td>1</td>
<td>$\infty$</td>
<td>$\infty$</td>
<td>$\infty$</td>
</tr>
<tr>
<td>0.01</td>
<td>1</td>
<td>1.544</td>
<td>6.1</td>
<td>$\infty$</td>
</tr>
<tr>
<td>0.1</td>
<td>1</td>
<td>0.088</td>
<td>5.4</td>
<td>$\infty$</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
<td>0.072</td>
<td>3.8</td>
<td>2.7</td>
</tr>
<tr>
<td>1</td>
<td>3</td>
<td>0.088</td>
<td>1.9</td>
<td>0.9</td>
</tr>
<tr>
<td>1</td>
<td>4</td>
<td>0.160</td>
<td>3.4</td>
<td>1.7</td>
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<tr>
<td>1</td>
<td>5</td>
<td>0.224</td>
<td>4.5</td>
<td>2.4</td>
</tr>
<tr>
<td>1</td>
<td>6</td>
<td>0.296</td>
<td>5.6</td>
<td>3.1</td>
</tr>
</tbody>
</table>

Table B.1: Process zone lengths of the reference T-peel, DCB and ELS specimens for various choices of the lower and upper boundary of the numerical process zone.
total possible work of separation is provided and the cohesive zone is considered to be fully delaminated. In figure B.1 the numerical process zone is shown by the hatched surface which is bounded by $\lambda_c$ and $6 \lambda_c$.

Figure B.1: Traction - separation curve with the exponential law. Hatched surface defines the numerical process zone.
Appendix C

Process zone length during delamination

In this appendix the evolution of the length of the process zone during delamination of a DCB and ELS test is examined. A DCB loaded by a force does not have a steady state as the energy release rate depends on the crack length [21]. To examine the evolution of the process zone length during delamination a finite element simulation with the reference DCB specimen is performed. The initial crack length is adjusted to 0 mm and the length to 300 mm to cover a large range of crack lengths.

![Figure C.1: The process zone length during delamination of a the reference DCB specimen with $a_0 = 0$ mm and $l = 300$ mm.](image)

Figure C.1 shows that the process zone length decreases during delamination from a peak value of 8.2 mm to a value of 6 mm. The process zone length seems to reach a plateau at the value of 6 mm, which is consistent with the findings by Suo et al. [21]. Herein it is stated that the process zone size of a point loaded DCB approaches a plateau when $a/l$ is sufficiently large.
Figure C.2: Development of the length of the process zone during delamination of an ELS specimen.

A similar test is conducted for the reference ELS specimen, of which the results are shown in figure C.2. In contrast to the DCB specimen the length of the process zone remains a constant as soon as it has fully developed.
Appendix D

Detailed data of convergence study

In this appendix the results of the convergence tests of the T-peel, DCB and ELS specimens are shown in tables. The convergence test is conducted by increasing $T_{\text{max}}$ with various element sizes $l_e$ until the model is no longer convergent, because the number of elements $N_e$ in the process zone is too small. The ELS specimen needs most elements in the process zone to obtain a converged solution, about 2.5, the DCB test 2 elements, while in the T-Peel test 1 element seems to be a minimum.
### D.1 T-Peel test

<table>
<thead>
<tr>
<th>$T_{\text{max}} \ [N\text{mm}^{-2}]$</th>
<th>$l_{pz} \ [\text{mm}]$</th>
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<th>Comments</th>
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<td>12</td>
<td></td>
</tr>
<tr>
<td>30</td>
<td>0.032</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>60</td>
<td>0.016 − 0.02</td>
<td>2 − 2.5</td>
<td></td>
</tr>
<tr>
<td>120</td>
<td>0.008 − 0.012</td>
<td>1 − 1.5</td>
<td></td>
</tr>
<tr>
<td>240</td>
<td>0.004 − 0.008</td>
<td>0.5 − 1</td>
<td>no convergence</td>
</tr>
</tbody>
</table>

Table D.1: Overview T-peel test results with $l_e = 0.008 \text{ mm}$.

<table>
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<th>$T_{\text{max}} \ [N\text{mm}^{-2}]$</th>
<th>$l_{pz} \ [\text{mm}]$</th>
<th>$N_e$</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>30</td>
<td>0.03 − 0.032</td>
<td>15 − 16</td>
<td></td>
</tr>
<tr>
<td>60</td>
<td>0.016 − 0.018</td>
<td>8 − 9</td>
<td></td>
</tr>
<tr>
<td>120</td>
<td>0.008</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>240</td>
<td>0.004 − 0.006</td>
<td>2 − 3</td>
<td></td>
</tr>
<tr>
<td>480</td>
<td>0.002 − 0.004</td>
<td>1 − 2</td>
<td></td>
</tr>
<tr>
<td>960</td>
<td>0.002 − 0.003</td>
<td>1 − 1.5</td>
<td></td>
</tr>
<tr>
<td>1920</td>
<td>0.002</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>3840</td>
<td>0.001 − 0.002</td>
<td>0.5 − 1</td>
<td>no convergence</td>
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Table D.2: Overview T-peel test results with $l_e = 0.002 \text{ mm}$.
## D.2 DCB

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<th>$T_{\text{max}} [Nmm^{-2}]$</th>
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<th>$N_e$</th>
<th>comments</th>
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<td>16</td>
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</tr>
<tr>
<td>1.5</td>
<td>12</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>3</td>
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<td>2</td>
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<td>6</td>
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<td>no convergence</td>
</tr>
<tr>
<td>12</td>
<td>4 – 8</td>
<td>1 – 2</td>
<td>no convergence</td>
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Table D.3: Overview DCB results with $l_e = 4 \, mm$.

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<th>comments</th>
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<td>5</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>4</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>3</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>24</td>
<td>2</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>48</td>
<td>1 – 2</td>
<td>1 – 2</td>
<td></td>
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<tr>
<td>96</td>
<td>1 – 2</td>
<td>1 – 2</td>
<td>Higher average force during delamination</td>
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Table D.4: Overview DCB results with $l_e = 1 \, mm$.

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<td>24</td>
<td>2</td>
<td>8</td>
<td></td>
</tr>
<tr>
<td>48</td>
<td>1.25</td>
<td>5</td>
<td></td>
</tr>
<tr>
<td>96</td>
<td>1</td>
<td>4</td>
<td></td>
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<td>1 – 2</td>
<td>no convergence</td>
</tr>
<tr>
<td>768</td>
<td>0.25 – 0.5</td>
<td>1 – 2</td>
<td>no convergence</td>
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Table D.5: Overview DCB results with $l_e = 0.25 \, mm$. 

---

73
## D.3 ELS

<table>
<thead>
<tr>
<th>$T_{\text{max}} [Nmm^{-2}]$</th>
<th>$l_{pz} [mm]$</th>
<th>$N_e$</th>
<th>comments</th>
</tr>
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<td>12</td>
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<td>3–4</td>
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<tr>
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<td>8–12</td>
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Table D.6: Overview ELS results with $l_e = 4 \, mm$.

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</tr>
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<td>4</td>
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</tr>
<tr>
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<td>2</td>
<td></td>
</tr>
<tr>
<td>200</td>
<td>–</td>
<td>–</td>
<td>no convergence</td>
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Table D.7: Overview ELS results with $l_e = 1 \, mm$.

<table>
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<th>$l_{pz} [mm]$</th>
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Table D.8: Overview ELS results with $l_e = 0.25 \, mm$.