GPS SENSOR FUSION

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Summary

For future car applications higher accuracy of position and orientation data of a vehicle is needed. Nowadays a Global Positioning System (GPS) is used for position and orientation in, for instance, a car navigation system, but this GPS data is subject to a fairly low sample frequency of typically 1 Hz and a position accuracy of about 10 meter. In this report it is shown that there is potential of fusing the GPS- and vehicle motion sensors, like for instance used for Electronic Stability Control (ESC) systems, to improve the position and orientation data of a vehicle.

To be able to fuse the sensors data a concept is made using a roll axis vehicle model to simulate the sensor values. The longitudinal velocity, lateral velocity, yaw rate, position and heading of the vehicle are taken to be the states of the process model. A so-called hybrid extended Kalman (hybrid EKF) filter is used for obtaining an estimate for these states. The internal estimator model is a standard bicycle model representation of the vehicle.

Available measurement data is used in an early stage to get some feeling in converting real measurement data using the sensor fusion concept. New measurements are taken at the Ford Lommel Proving Ground, to give an idea about the accuracy of standard car navigation systems compared to a high precision Real Time Kinematic GPS system.
Contents

1. Introduction 1

2. Background information 3
   2.1 Global Positioning Systems 3
   2.2 Sensor fusion 6
   2.3 State estimation 7

3. Implementing experimental data in simulations 8
   3.1 Simulation with ESC data 8
   3.2 Simulation with GPS and ESC data 9

4. New measurement data 11
   4.1 Experimental setup 11
   4.2 Experimental results 12

5. Sensor fusion script 14
   5.1 System vectors 14
      5.1.1 State vector 14
      5.1.2 Sensor vector 14
      5.1.3 Input vector 15
   5.2 Reference model 15
   5.3 Internal model 16
   5.4 State estimator 20

6. Conclusions and recommendations 23
   6.1 Conclusions 23
   6.2 Recommendations 24

Appendix A: Measurements LPG 25

Appendix B: Augmented input vector 29

Appendix C: Influence of sample frequency and input vector 32

Appendix D: Hybrid Extended Kalman Filter 36

References 37
1. Introduction

Nowadays, a part of all cars is equipped with a Global Positioning System (GPS), like for instance a TomTom. The system provides information about the position- and orientation of the vehicle. The system can be built-in or portable. At the moment the GPS data is used to inform the users about their current location and it provides driving directions to get to a destination. However, this GPS information is subject to a fairly low sample frequency of typically 1 Hz. Also the accuracy of the position is in a range of about 10m [1]. The orientation of the vehicle is determined using the position information and thereby suffering from delay [2].

Electronic Stability Control (ESC) systems prevent vehicles from spin, skid and rollover. These are active safety systems which support the driver when he or she ends up in a critical situation. It is an addition to safety systems like ABS or traction control. The system is also commonly named as ESP (Electronic Stability Program); however in this report the name ESC will be used. The system consists of sensors that measure the steer angle, wheel speed, lateral acceleration and yaw rate. Also sensors for longitudinal acceleration and roll rate can be included [3].

The European Parliament has approved a law which obliges all newly registered cars to be equipped with ESC. According to this law new models of cars and industrial vehicles registered in the EU will have to be fitted with a system of electronic stability control starting November 2011. After that starting in 2014 this measure will be applied to all newly registered vehicles, without exception [4].

For future car applications increased position- and orientation information of a vehicle is required. For instance, one of these applications is called Connected Cruise Control (CCC), which provides speed, headway and lane use advices to drivers with the aim to mitigate congestion. The application enables the vehicle to look beyond the vehicles that are right in front of it. It also enables the driver to anticipate on the traffic flow situations far in front of him. This application requires accurate positioning information of a vehicle in relation to the road infrastructure and other vehicles for the algorithms [5].

As in the nearby future probably a lot of cars will be equipped with GPS- and ESC systems, the goal of this project is to investigate if there is potential of fusing the GPS- and ESC sensors data for position and orientation of a vehicle and to see if there are benefits of this fused data. To be able to evaluate the opportunities of fusing GPS- and ESC sensor data, some simulations are performed with GPS- and ESC sensor data already present at TNO. Also a sensor fusion script, simulating a real time environment, is set up. The results and set-ups are discussed in this report. The accuracy of a portable navigation system its signals is also reflected in this report.
1 Introduction

This report is organized as follows. In Chapter 2 a short introduction is given about GPS systems and sensor fusion. In Chapter 3 Simulink\(^{(1)}\) models are used to evaluate available GPS- and ESC data of TNO. In Chapter 4 experiments are performed to obtain real GPS- and ESC data. In Chapter 5 the sensor fusion script is reflected. Also the results of using this script can be found in this chapter. In Chapter 6 the conclusions and recommendations of this report are mentioned.

\(^{(1)}\) “Simulink® is an environment for multidomain simulation and Model-Based Design for dynamic and embedded systems.” [“Simulink”, www.mathworks.com, attended 13 April 2010]
2. Background information

In this report it is assumed that the reader has some knowledge about ESC systems and vehicle models, like for instance a bicycle model. However, GPS and sensor fusion might be new topics. That is why in this chapter a briefly introduction about GPS systems and sensor fusion is given.

2.1 Global Positioning Systems

The Global Positioning System (GPS) is a space-based global navigation satellite system. The system is designed to be functional with as few as 24 satellites. In its original design, the satellite constellation consisted of 24 satellites arranged in 6 orbital planes with 4 satellites per plane [6]. The satellites are positioned on a height of approximately 20,200 km, and travel with a speed of approximately 11,250 km/hour. They send signals with a low power of circa 20 to 50 Watt [7]. Currently, there are between 24 and 32 satellites in the GPS system. The exact number varies as old satellites fail or are retired, and new satellites are sent up to replace them. They are equipped with solar collectors to provide energy and with batteries to supply energy when they are on the shadow side of the Earth [8].

GPS can provide service to an unlimited number of users since the user receivers operate passively (i.e. receive only).

‘The time-of-arrival (TOA) ranging concept is used by GPS to determine user position. Measuring the time it takes for a signal transmitted by an emitter at a known location to reach a user receiver is entailed in this concept. This time interval, referred to as the signal propagation time, is then multiplied by the speed of the signal to obtain the emitter-to-receiver distance. By measuring the propagation time of signals broadcasted from multiple emitters at known locations, the receiver can determine its position’ [6]. To determine its position a method is used which is known as ‘GPS triangulation’, although trilateration would be a better word, because there are no angles involved.

The idea behind this trilateration is shown in figure 2-1. When the place \((s_1)\) and distance \((r_1)\) of satellite-emitter one is know, the receiver could be on the whole surface of sphere one. When the position \(s_2\) and distance \(r_2\) are also known, the receiver could be on a path, defined by the crossing of the two spheres. This path is indicated in blue in the figure. With data about position \(s_3\) and distance \(r_3\), the receiver can only be on two positions, \(p_1\) or \(p_2\). With use of a fourth sphere (determined by a satellite or the Earth’s geometry) the exact place can be determined, as the altitude is known.

\[ fig. 2-1: \text{GPS trilateration} \]
2. Background information

To determine the distance, the ‘messages’ of the satellites are used. Every satellite transmits every 30 seconds a message with its codename, its new position, new atomic time, the satellites condition (‘ephemeris’) and the new location of all the active GPS-satellites (‘almanac’). This message is sent with pseudo random codes, starting with the satellite’s name. At the same time as the satellite produces its code, the GPS-receiver produces the codes for the names of all the satellites, and so it knows which signals it is looking for. When the satellite’s message is received, the travelling time can be determined by looking at the phase shift between the satellites pseudo random code and the receiver’s one [8].

However the determined distances to the satellites contain errors. Therefore receivers use four or more satellites to solve for receiver’s location and time. By determining the distances to satellites at the same time, the clock in the GPS-receiver can be updated. This is necessary because the receiver’s clock isn’t as accurate as the atomic clocks placed in the satellites [6].

The ranging errors can be grouped into seven following classes [6][9]:

1) Ephemeris data: Errors in the transmitted location of the satellite.
2) Satellite clock: Errors in the transmitted clock.
3) Ionosphere: Errors in the corrections of pseudorange caused by ionospheric effects.
4) Troposphere: Errors in the corrections of pseudorange caused by tropospheric effects.
5) Multipath: Errors caused by reflected signals entering the receiver antenna.
6) Receiver: Errors in the receiver’s measurement of range caused by thermal noise, software accuracy, and inter-channel biases.
7) The Earth’s physical model: Errors caused by the physical model of the Earth that is used.

The magnitude of these errors is not discussed in this report.

The standard physical model of the Earth that is used for GPS applications is the DOD’s World Geodetic System 1984 (WGS-84). An ellipsoidal model (oblate spheroid) of the Earth’s shape is provided by WGS-84, as can be seen in figure 2-2. The model is used for estimating the latitude, longitude and height of a GPS receiver. WGS-84 uses \(a = 6.378,137\) km, which is the mean equatorial radius of the Earth, and \(b = 6,356.7523142\) km, which corresponds to the polar radius of the Earth [6]. Thus, the eccentricity of the Earth ellipsoid, \(e\), second eccentricity, \(e'\), and the flattening, \(f\), can be determined by:

\[
e = \sqrt{\frac{a^2 - b^2}{a}} = \sqrt{1 - \frac{b^2}{a^2}} \quad (2.1)
\]

\[
e' = \sqrt{\frac{a^2 - b^2}{b}} = \sqrt{\frac{a^2}{b^2} - 1} = \frac{a}{b} e \quad (2.2)
\]

\[
f = \frac{a-b}{a} = 1 - \frac{b}{a} \quad (2.3)
\]
2. Background information

WGS-84 is not fixed to an earth plate; it is fixed to the center of the Earth. The WGS-84 receives updates.

As can be expected, the Earth isn’t an exact ellipse as described in the WGS-84 model, and therefore the model’s accuracy depends on the place of the Earth for which the model is used. For instance, in the Netherlands, the Earth deviates from the original WGS-84 model. It is more accurate in the Netherlands, to use the EuropeanDatum50 (ED50), which uses $a = 6,378.388$ km and $b = 6,356.912$ km. [8].

The position accuracy of a standard GPS is about 10 meters [1]. A GPS receiver can provide velocity information with accuracies of 3 cm/s (1 $\sigma$, horizontal) and 6 cm/s (vertical), even without differential corrections [1][2]. This accuracy can be achieved because the velocity information is determined by measuring the Doppler shifts of GPS carrier waves. GPS measurements are stable but subject to a fairly low update rate (1-10 Hz) [1]. The position accuracy is said to become a lot smaller (within 45 cm. instead of 10 m.) if the European space-based global navigation satellite system Galileo is operative and used instead of the GPS system [10].

GPS-locations are normally indicated with degrees of longitude (0° to 180°) and degrees of latitude (0° to 90°).

For instance; in figure 2-3 the east longitude is the angle $\alpha$ and the northern latitude is the angle $\beta$. The plane indicated by the blue circle is the equatorial plane; the one indicated by the red circle is the prime meridian through Greenwich. One degree [°] is split in 60 minutes [''], one minute in 60 seconds [''].

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*fig. 2-2: Ellipsoidal model of the Earth’s shape*

*fig. 2-3: Longitude and latitude*
2. Background information

2.2 Sensor fusion

Sensor fusion is combining data from multiple sensors and related information, whereby more specific inferences are achieved than were possible by using a single, independent sensor. Sensor fusion is not a new concept, humans and animals have the ability to use multiple senses to help them survive. Humans are able to apply this complex and adaptive process naturally. Data (sight, sound, scent and touch) from the body’s sensors (eyes, ears, nose and fingers) is combined with prior knowledge in order to assess the world and the events that occur [11]. For example, an advanced warning of impending dangers can be provided by the sense of hearing, when visibility is limited by structures or vegetation.

Benefits of using a multiple sensors application instead of a single-sensor approach may be [12]:

- Robust operational performance: Any of the sensors has the potential to provide information during unavailability, denying or lacking coverage of the other sensors.
- Extended temporal coverage: One sensor is able to detect or measure an event at times the other sensors are unable.
- Extended spatial coverage: One sensor can cover what other sensors cannot.
- Increased confidence: When multiple independent measurements provide information on the same event.
- Improved detection performance: When multiple separate measurements of the same event are integrated effectively.
- Reduced vulnerability to denial: As a result of the increased dimensionality of the measurement space.

Benefits of sensor fused data over single sensor data are [11]:

- Improved estimating: If several identical sensors are used, combining the observations may result in an improved estimate. A statistical advantage is gained by adding the independent observations, assuming the data are combined in an optimal manner.
- Improved observation process: Using the relative placement or motion of multiple sensors may result in an improved observation process. For example, two sensors that measure angular directions to an object can be coordinated to determine the position of an object by triangulation.
- Improved observability: Broadening the baseline of physical observables can result in significant improvements. If the observations are correctly associated, the combination of the sensors provides a better determination than could be obtained by either of the independent sensors. This results in a reduced error region.
2. Background information

2.3 State estimation

State estimation is the process of mathematically determining an estimate for a system’s state $x$ (e.g., position, orientation, velocity), based on measurements $z$ (output of the system) and $u$ (input of the system), related to the state. Note that in this report the name $z$ is used instead of $y$ to comply to the terminology used at TNO. Each discrete sensor measurement is often referred to as an observation. State estimation is typically performed by computer-implemented mathematical models. In order to provide estimations of the internal states a model of the real system is used [11].

The main linear estimators often applicable to both static and dynamic applications for fusion of spatial information are ‘Least Squares’, ‘Weighted Least Squares’, ‘Maximum Likelihood’ and ‘Minimum Variance (minimum mean square error)’. This information is taken from [11], where in Chapter 6 each estimator is discussed more extensively. Linear estimators are used because they have well-understood properties and the computations are a linear function of the observations. A static application could for instance be an estimation of the position of a fixed object using measurements. A dynamic application could for instance be an estimation of the positions of a moving object using multiple measurements.

The discrete time, recursive solution for the linear, minimum variance estimation problem is provided by the Kalman filter, which is the statistical estimator most often applied to dynamic tracking [11]. The continuous time solution is provided by the Kalman-Bucy filter. The filters consider the observable, linear system with input $u$, outputs $z$, states $x$ and system matrices $A, B, C$ and $D$:

$$\dot{x} = Ax + Bu + w$$
$$z = Cx + Du + v$$

Here $w$ is the process noise and $v$ is the measurement noise. The noises $w$ and $v$ are assumed to be independent of each other, white noise and with normal probability distributions with zero mean [13]. An extended form of the Kalman filter is used for this project and explained in Chapter 5. This filter is stated in [appendix D].
3. Implementing experimental data in simulations

Before making new dedicated measurements, see Chapter 4, some simulations are done with data already present at TNO. This is done to get some feeling with the data which can be expected from the measurements and to get some feeling with sensor fusion. The simulations are also performed to set some scripts up which proved to be useful in the developed sensor fusion script of Chapter 5.

3.1 Simulation with ESC data

With this simulation the conversion from yaw rate \( r \), longitudinal velocity \( u \) and lateral velocity \( v \) from measurement data to a \( (x, y) \) position is achieved using the formulas [14]:

\[
\dot{x} = u \cos(\psi) - v \cos(\psi) \tag{3.1}
\]

\[
\dot{y} = u \sin(\psi) + v \cos(\psi) \tag{3.2}
\]

Note:

\[ \dot{\psi} = r \tag{3.3} \]

As GPS-data was not available, but data from multiple measurements from the same test track was available, one of the measurements can be taken to be the reference in the simulation script. This is done to update the other measurements their \( x, y \) position and heading \( \psi \) at a sample frequency \( f_s \) of 1 Hz. This results in the adapted data points of figure 3-2. The measurements contain the yaw rate \( r \), longitudinal velocity \( u \), lateral velocity \( v \) and their related sample times.

In Simulink this is achieved by using an external reset and external initial condition source in the integrators and look-up tables to match the covered distance \( s \) of the different measurements and the reference. The diagram of this Simulink model can be seen in figure 3-1 for the \( x \)-position.

\[
\begin{align*}
\text{Calculations} & \quad u, v, r \\
\text{Look-up table} & \quad s \\
\text{\textit{Reference}} & \quad \text{Reset} \\
\end{align*}
\]

\[ \begin{array}{c}
\text{Integrator} \\
\text{\textit{Reference}} \\
x_0
\end{array} \]

\[
\begin{align*}
\dot{x} & \quad \text{Reset} \\
x & \quad \text{\textit{Reference}} \\
\end{align*}
\]

\[
\begin{array}{c}
\text{fig. 3-1: Diagram of the Simulink model}
\end{array}
\]

In the zoomed box of figure 3-2 the update can be clearly seen in the adapted data at \( x = -245m \) and at \( x = -230m \). The non-adjoining end in this figure is because the test track was not fully used. The deviation between the ‘Reference’ and ‘Unadapted’ data may be partly caused by sensor drift of the ESC sensors.
3. Implementing experimental data in simulations

3.2 Simulation with GPS and ESC data

The script used in section 3.1 is then extended to cope with real GPS-data. The GPS-data is converted to fit in the system of coordinates of the other sensors and to a \((x, y)\) position using a flat-earth assumption.

\[
x_{gps} = R \cdot \cos(\beta) \cdot (\alpha - \alpha_0) \tag{3.4}
\]
\[
y_{gps} = R \cdot (\beta - \beta_0) \tag{3.5}
\]

\(\alpha = \text{longitude [rad]}
\beta = \text{latitude [rad]}
R = 6.3649e+006 [m], \text{earth radius}

To check the GPS-data the GPSVisualizer \([15]\) was used, resulting in figure 3-3. Here it can be seen that the test are taken at the ATP Automotive in Papenburg.

The GPS-data was then added as the reference in the simulation synchronizing with \(x = y = 0 \text{[m]}\) at \(t_o\). The unadapted positions are determined using yaw rate \((\psi)\), longitudinal velocity \((u)\) and lateral velocity \((v)\). As the GPS-data was taken started at the same time as the other sensors, the time was used to synchronize instead of the covered distance \((s)\). This results in figure 3-4.

As can be seen in the ‘correction’ graph of figure 3-4 there is a correction used every second (1 Hz) in the adapted data, which results in a zero error every second. This can be seen in the ‘Error between adapted and gps’ graph in this figure. The deviation between GPS and motion based position can be up to 20 meter. The position accuracy of the GPS system is 3 meter, 95% Circular Error Probable \([16]\). This indicates that 95% of the time the position readings will fall within a circle of 3 meter.
3. Implementing experimental data in simulations

fig. 3-4: Result of simulation with GPS data
4. New measurement data

New dedicated measurements are taken at the Lommel Proving Ground. The experimental setup can be seen in section 4.1, the experimental results in section 4.2.

4.1 Experimental setup

To be able to take some measurements TNO's BMW test vehicle was equipped with a Trimble Real Time Kinematic Global Positioning System (RTK-GPS). This system provides positioning data with a sample frequency of 10 Hz and decimeter accuracy depending on the augmentation used [17]. Also two TomToms GO LIVE 940 with Enhanced Positioning Technology (EPT) were mounted on the windscreen, see figure 4-1. Enhanced Positioning Technology uses motion and gravity sensors to calculate the position when GPS signals are unavailable. This may occur for example when driving in a city with tall buildings, underpasses or bridges or when driving in a tunnel. The TomToms have a GPS sample frequency of 1 Hz.

Also some, already in the vehicle present, sensors are used to determine the longitudinal ($a_x$) - and lateral ($a_y$) acceleration, the yaw rate ($\dot{\psi}$) and the velocity vector ($V$) of the vehicle. These sensors, but also the altitudinal ($a_z$) acceleration, pitch-rate ($q$) and air pressure ($P$) are covered by the EPT, except from the velocity vector ($V$), which is provided by the GPS-sensor.
4.2 Experimental results

The measurements are taken at the Ford Lommel Proving Ground. The results of accelerating a clockwise circle can be seen in figure 4-2. The ‘Position (Longitude – Latitude)’ plot is also showed in this figure, as no research was done on the error caused by the flat-earth assumption from section 3.2. The other test results can be found in appendix A.
As can be seen in the pictures of figure 4-2 made with GPSVisualizer [15] this test was taken at the 240m diameter circle at the Vehicle Dynamics Platform of the Lommel Proving Ground. Also it can be seen that the position determined by the TomToms is less accurate than a RTK-GPS, as was expected. As in section 2.1 was mentioned, the position accuracy of a standard GPS is about 10 meters.

Although the determined speed of the RTK-GPS and the Corrsys sensor mounted on the test vehicle more or less coincide in all tests, the determined speed of the TomToms does not coincide with them. The TomToms signals do not always coincide with each other too. This was not expected, as the accuracy of the speed of a GPS can be found in section 2.1 and both TomTom’s were mounted in the same position on the windscreen, on a short distance from each other. They both had a clear view with the satellites. Note that the speed of all GPS-systems will suffer from roll [18].

For more information about the results of the inertial sensor measurements of the TomToms, see [19].
5. Sensor fusion script

To achieve sensor fusion between the GPS- and the ESC-sensors, state estimation is used in this research.

5.1 System vectors

The diagram of the simulation model used for this report can be seen in figure 5-1. The input vector \((u_s)\) is discussed in section 5.1.3, the sensor vector \((z_s)\) in section 5.1.2, the state vector \((x_s)\) in section 5.1.1, the ‘Vehicle’ reference model \((A)\) in section 5.2 and the ‘Estimator’ \((C)\) and its estimated vector of the states \((\hat{x}_s)\) in section 5.4.

![fig. 5-1: Diagram of simulation model](image)

5.1.1 State vector

First the state vector \((x_s)\) is defined [20]. The state estimators already present at TNO have as states: longitudinal velocity \((u)\), lateral velocity \((v)\) and yaw rate \((r)\). These states are augmented with the x-position \((x)\), y-position \((y)\) and heading \((\psi)\), giving the vector \((x_s)\) and (5.1):

\[
x_s = [u, v, r, x, y, \psi]^T
\]

So:

\[
\dot{x}_s = [\dot{u}, \dot{v}, \dot{r}, \dot{x}, \dot{y}, \dot{\psi}]^T
\]

Note that the state vector \((x_s)\) is normally not measurable in a car. However this vector is available in the sensor fusion model, figure 5-1, or a suitable test vehicle.

5.1.2 Sensor vector

As given by the supervisor, for the ESC-system the longitudinal acceleration \((a_{x_{esc}})\), lateral acceleration \((a_{y_{esc}})\) and yaw rate \((r_{esc})\) sensors are used in the sensor vector \((z_s)\):

\[
z_{s_{esc}} = [a_{x_{esc}}, a_{y_{esc}}, r_{esc}]^T
\]

For the GPS-system the x-position \((x_{gps})\), y-position \((y_{gps})\) and velocity \((V_{gps})\) are used:

\[
z_{s_{gps}} = [x_{gps}, y_{gps}, V_{gps}]^T
\]
Note that not the degrees of latitude and longitude are used, but the x-position and y-position like in (3.4) and (3.5). Also for the sensor vector the name $z_s$ is used instead of $y_s$ to comply to the terminology used at TNO. The sample frequency of the GPS sensor is 1 Hz, the frequency of the ESC sensors is assumed to be 100 Hz.

The heading of the GPS-system isn’t used. This heading is suffering from delay [2] and two GPS-systems would be needed to get an accurate heading with a small delay [21]. The model is suited to cope with heading for future work.

### 5.1.3 Input vector

The input vector $(u_s)$, see (5.5), contains the steering angle $(\delta)$:

$$u_s = [\delta]$$

(5.5)

Although in the model preparations are made to extend the vector to (5.6):

$$u_s = [\delta, F_{s_{\beta}}, F_{s_{fr}}, F_{s_{rl}}, F_{s_{rr}}]$$

(5.6)

This is to be able to see what the effect of adding forces in x-direction might be. The forces in x-direction of the front left- $(F_{s_{\beta}})$, front right- $(F_{s_{fr}})$, rear left- $(F_{s_{rl}})$ and rear right $(F_{s_{rr}})$ tire are used in this case. Measuring these forces could for instance be possible with the use of force-bearings [22].

### 5.2 Reference model

To be able to simulate sensor values and to check the results of the state estimator, a simple roll axis vehicle model (used as demo for the TNO Delft-Tyre toolbox) is adapted. Between the ground and the vehicle body a custom joint is used providing the x-position $(x)$, y-position $(y)$ and heading $(\psi)$, their time derivatives $\dot{x}, \dot{y}$ and $\dot{\psi}$, and second time derivatives $\ddot{x}, \ddot{y}$ and $\ddot{\psi}$. With them the longitudinal velocity $(u)$, lateral velocity $(v)$ and time derivatives $\dot{u}$ and $\dot{v}$ can be calculated using (5.7), (5.8), (5.9) and (5.10) to determine $\dot{x}$ and $\dot{y}$.

$$u = \dot{x}\cos(\psi) + \dot{y}\sin(\psi)$$

(5.7)

$$v = -\dot{x}\sin(\psi) + \dot{y}\cos(\psi)$$

(5.8)

$$\dot{u} = \ddot{x}\cos(\psi) - \dot{x}\dot{\psi}\sin(\psi) + \ddot{y}\sin(\psi) + \dot{y}\dot{\psi}\cos(\psi)$$

(5.9)

$$\dot{v} = -\ddot{x}\sin(\psi) - \dot{x}\dot{\psi}\cos(\psi) + \ddot{y}\cos(\psi) - \dot{y}\dot{\psi}\sin(\psi)$$

(5.10)

With some body sensors attached to the vehicle body the values for $z_{s_{esc}}$ and $z_{s_{gps}}$ can be simulated. The input vector $(u_s)$ is used as input for the roll axis vehicle model, see figure 5-2 or the A part of figure 5-1.
5 Sensor fusion script

5.3 Internal model

The model implemented in the state estimator is a bicycle model representation of the vehicle, which is already present at TNO. In this model, see figure 5-3, $a$ and $b$ are the distances to the center of gravity of the frame and $C_{fa1}$ and $C_{fa2}$ represent the cornering stiffness of the front and rear tire. The mass of the frame is indicated with $m_{frame}$ and $I_{zz\_frame}$ is the moment of inertia around the $z$-axis. For more details about a bicycle model, see [14] or [20]. The model equations $f(x_s, u_s)$ present at TNO are augmented with the equations (3.1), (3.2) and (3.3) to match:

$$
\dot{x}_s = f(x_s, u_s) = \begin{bmatrix}
  f_1 \\
  \vdots \\
  f_6
\end{bmatrix}
$$

(5.11)

With:

$$
f_1 = rv - \frac{C_{fa1} \arctan \left( \frac{\cos(\delta)(v + ar) - u \sin(\delta)}{\sin(\delta)(v + ar) + u \cos(\delta)} \right) \sin(\delta)}{m_{frame}}
$$

(5.12)

$$
f_2 = \frac{C_{fa2} \arctan \left( \frac{v - br}{u} \right) + C_{fa1} \arctan \left( \frac{\cos(\delta)(v + ar) - u \sin(\delta)}{\sin(\delta)(v + ar) + u \cos(\delta)} \right) \cos(\delta)}{m_{frame}} - ru
$$

(5.13)

$$
f_3 = \frac{C_{fa1} a \arctan \left( \frac{\cos(\delta)(v + ar) - u \sin(\delta)}{\sin(\delta)(v + ar) + u \cos(\delta)} \right) \cos(\delta) - C_{fa2} b \arctan \left( \frac{v - br}{u} \right)}{I_{zz\_frame}}
$$

(5.14)
5 Sensor fusion script

\[ f_4 = u \cos(\psi) - v \cos(\psi) \]  \hspace{1cm} (5.15)
\[ f_5 = u \sin(\psi) + v \cos(\psi) \]  \hspace{1cm} (5.16)
\[ f_6 = r \]  \hspace{1cm} (5.17)

The longitudinal force of the front tire of the bicycle model is called \( F_{x1} \) and of the rear tire \( F_{x2} \). The later forces are called \( F_{y1} \) (front tire) and \( F_{y2} \) (rear tire). The self aligning moments are called \( M_{z1} \) (front tire) and \( M_{z2} \) (rear tire). This results in (5.18), (5.19) and (5.20). In (5.19) the slip angle of the front tire is called \( \alpha_1 \) and in (5.20) the slip angle of the rear tire is called \( \alpha_2 \).

\[ F_{x1} = F_{x2} = M_{z1} = M_{z2} = 0 \]  \hspace{1cm} (5.18)
\[ F_{y1} = C_{f_{a1}}\alpha_1 \]  \hspace{1cm} (5.19)
\[ F_{y2} = C_{f_{a2}}\alpha_2 \]  \hspace{1cm} (5.20)

For the output equations \( h(x_s, u_s) \) the sensor equations of the measurable output \( z_s \) are used:

\[ z_s = h(x_s, u_s) \]  \hspace{1cm} (5.21)

Because of the different sample frequencies of the GPS and ESC- sensor, two different output equations and two different sensor vectors are used in the model. One pair of output equations and a sensor vector is used when only ESC sensors are available, the other when ESC- and GPS signals are available simultaneous. So during GPS outages, (5.22) will hold:

\[ z_s = z_{s\_esc} \]  \hspace{1cm} (5.22)

At times ESC- and GPS signals are available simultaneous, (5.23) is used for the sensor vector:

\[ z_s = \begin{bmatrix} z_{s\_esc} \\ z_{s\_gps} \end{bmatrix} \]  \hspace{1cm} (5.23)

See section 5.4 for more details.

To check the model equations \( f(x_s, u_s) \) and output equations \( h(x_s, u_s) \), (5.2) and (5.23) are simulated in Simulink. The block diagram representation of the setup of this check can be seen in figure 5-4 and 5-5. The state vector \( (x_s) \), its derivative \( (\dot{x}_s) \) and sensor vector \( (z_s) \) are known from the reference model from section 5.2.
The results of the check for the model equations are presented in figure 5-6.

In this simulation $m_{\text{frame}} = 1600\,\text{kg}$, $I_{zz,\text{frame}} = 3200\,\text{kg}\,\text{m}^2$, $a = b = 1.5\,\text{m}$ and $C_{fa1} = C_{fa2} = 85500\,\text{N/\text{rad}}$. Also $u_0 = 20\,\text{m/s}$, $v_0 = 0\,\text{m/s}$, $r_0 = 0\,\text{rad/s}$, $x_0 = y_0 = 0\,\text{m}$ and $\psi_0 = 0\,\text{rad}$. The steering angle ($\delta$) input can be seen in [appendix B].
The results of the check for the output equations are presented in figure 5-7. From these figures it is determined that the internal model and output description are acceptable to use in the state estimator. In [appendix B] the effect of adding longitudinal forces $F_{x1}$ and $F_{x2}$ can be seen. With the effect of adding these forces it is examined that equating these forces to zero, see (5.18), causes the bad results for the time derivative of longitudinal velocity ($u$) and the lateral acceleration ($a_{x_{loc}}$).

fig. 5-7: Results for output equations
5.4 State estimator

The vehicle state estimator, see figure 5-1 and 5-8, used for this research is the hybrid extended Kalman filter (hybrid EKF). As mentioned with the term ‘hybrid’, this filter considers systems with continuous-time dynamics and discrete-time measurements [13].

The filter is taken to be the best filter to find out which phenomena play an important role.

To be able to handle the different sample frequencies of the ESC-system and the GPS-system two different scripts are used for the discrete part of the filter. One of them deals with the case when only the ESC-sensors are available (‘Discrete part ESC’ of figure 5-9) and the other deals with the case when the ESC-sensors and GPS-sensors are available together (‘Discrete part GPS&ESC’ of figure 5-9). In Simulink there is a switch between those scripts, depending on which sensors are available at that time.

For the continuous part of the filter, also a script is used (‘Continuous part’ of figure 5-9). Depending on the availability of measurements there is a switch between only the continuous part of the filter or the continuous and discrete part, see figure 5-9. Note that using a switch may result in an unreliable filter, that is to say a filter that not necessarily converges in a meaningful manner. More investigation should be done on this part, see Chapter 6.
Running the simulation results in figure 5-10 for the estimated states \((\hat{x})\) and in figure 5-11 for the estimated sensors measurements \((\hat{z}_s)\).

In this simulation the same initial conditions and parameters are used as in the simulation of section 5.3. The process noise covariance matrix \(Q = 10^{-2}I_6\) and measurement noise covariance matrix \(R = 10^{-2}I_6\). The process noise and measurement noise are assumed to be independent of each other, white noise and with normal probability distributions with zero mean, see section 2.3.

In figure 5-10 the 1 Hz update of the GPS-system can be seen in the x-position \((x)\), y-position \((y)\) and heading \((\psi)\) graphs. This can for instance be clearly seen every second, starting around six seconds, in the y-position \((y)\) and heading \((\psi)\) graph of figure 5-10. Note that these graphs are made with a filter that isn’t optimized and the results will probably get a lot better when the filter is correctly optimized.
To optimize the filter the process noise covariance matrix $Q$ and measurement noise covariance matrix $R$ have to be adapted in the script. Also the hard transitions on 1 Hz might be influenced by optimizing the filter.

Also the sample frequency of 1 Hz of the GPS-system can be clearly seen in the GPS-graphs of figure 5-11.

Also from the $y_{(gps)}$ and $V_{(gps)}$ graphs it can be seen that the filter probably needs to be optimized. The influence of a higher sample frequency of the GPS sensor can be found in [Appendix C], also the influence of adding $F_{x1}$ and $F_{x2}$ can be seen there. Note that no inaccuracies of the sensors are implemented in the model yet.
6. Conclusions and recommendations

In this chapter the conclusions and recommendations of this report are stated.

6.1 Conclusions

Concluding from literature, simulations and measurements it can be said that there are benefits for fusing GPS- and ESC sensors for positioning and orientation of the vehicle. Between the GPS signals or during GPS outages the ESC sensors can provide data to determine the position and orientation of a vehicle. This data will be available at a higher sample frequency (100 Hz) compared to GPS (1 Hz), because of the higher sample frequency of the ESC sensors.

Using only the ESC sensors for position and orientation would result in an error which is increasing in time due to the drift of the sensors. In one of the performed simulations the deviation between GPS and motion based position is up to 20 meter, with a GPS position accuracy of 3 meter, 95% Circular Error Probable. This accuracy can be obtained using a professional sensor system. Using only ESC sensors would also give the problem of the initial heading of the vehicle. But for instance the orientation determined by GPS is bad and suffering from delay. Therefore it will be more convenient using the ESC yaw rate sensor for orientation of the vehicle.

The quality of position and orientation is determined by the quality of the GPS- and ESC sensors data and the sensor fusion algorithm. In real life the ESC sensors will suffer from drift and the GPS sensor will have position and orientation inaccuracies, which is not yet implemented in the reference model.

For vehicle state estimation and control there will be an added value because of the augmented sensor vector and the augmented state vector.

Systems like TomTom’s Enhanced Position Technology (EPT) are already used to determine the position and orientation during GPS outages. However using a portable system (like TomTom with EPT) for vehicle state estimation would result in the problem at which position and orientation the TomTom is mounted in the car. Also using a system like EPT instead of the ESC sensors would ask for more research on the quality of these EPT sensors, as no reference was available during the measurements. Also the steer angle is an input for the Vehicle State Estimator, which is not a sensor available inside a TomTom. Using only a system like TomTom with EPT wouldn’t incorporate the vehicle dynamics at all.
6 Conclusions and recommendations

6.2 Recommendations

For future work on this topic some recommendations are given in this section.

First of all a solution for the switching problem of the filter should be investigated. This will have to result in filter that can be proven to be reliable.

In order to make the sensor fusion script more realistic it is recommended to add GPS inaccuracies, jamming and outages in the reference model and to let the script deal with this, see [23]. The GPS inaccuracies could also be replaced by Galileo its inaccuracies to investigate the difference in results. Note that more investigation about Galileo should be done first in order to achieve this. It is also useful to add sensor drift for ESC sensors in the reference model and bias factors in the script to deal with this drift. The sensor specifications of the ESC sensors should be investigated first. The difference between the sensor specifications of the ESC sensors and EPT sensors could also be investigated. The GPS signals might be useful to determine the yaw rate sensor bias, but more research has to be done for this [2]. It is recommended also to use measurement data instead of the reference model.

The results of the sensor fusion script might become a lot better if the filter is optimized. To optimize the filter the process noise covariance matrix $Q$ and measurement noise covariance matrix $R$ have to be adapted in the script. Also the results might become better if the cornering stiffness for the bicycle model is determined more accurately.

The added value of sensor fusion for vehicle state estimation and control, compared to the Vehicle State Estimator which is already present at TNO could also be investigated.
Appendix A: Measurements LPG

In section 4.2 the results of the measurements taken at the Lommel Proving Ground (LPG) can be seen. For the sake of surveyability of the report not all measurement results were placed there. The other measurement results can be found in this appendix.

Anti-clockwise circle, accelerating

![Graphs showing measurement results for anti-clockwise circle, accelerating](image)

**fig. A-1: Results anti-clockwise circle, accelerating**
Appendix A: Measurements LPG

**Anti-clockwise circle, power-off, oversteer**

---

![Position (Longitude - Latitude)](image1)

![Position (x - y)](image2)

![Speed](image3)

![Longitudinal acceleration](image4)

![Lateral acceleration](image5)

![Yaw rate](image6)

*fig. A-2: Results anti-clockwise circle, power-off, oversteer*
Long time measurement with all kind of moves

fig. A-3: Results long time measurement with all kind of moves
Appendix A: Measurements LPG

Sinus with ascending frequency

fig. A-4: Results sinus with ascending frequency
Appendix B: Augmented input vector

As mentioned in section 5.1.3 the input vector \((u_s)\) can be extended to (5.6). The results of extending this vector can be seen, with use of (5.11) and (5.21), on the time derivative of the state vector \((\dot{x}_s)\), see (5.2), and sensor vector \((z_x)\), see (5.23).

The forces in \(x\) direction for the bicycle model were changed in:
\[
F_{x1} = F_{x_{-\beta}} + F_{x_{-\psi}} \quad \text{(Of the roll axis vehicle model)} \tag{B.1}
\]
\[
F_{x2} = F_{x_{-\eta}} + F_{x_{-\psi}} \quad \text{(Of the roll axis vehicle model)} \tag{B.2}
\]

The results can be seen in figure B-2 and B-3 and can be compared with figure 5-6 and 5-7, see section 5.3.

In this simulation the same initial conditions and parameters are used as in the simulation of figure 5-6 and 5-7. The steering angle \((\delta)\) input can be seen in figure B-1.

![fig. B-1: Steering angle input](image)
Appendix B: Augmented input vector

fig. B-2: Results for model equations
Appendix B: Augmented input vector

fig. B-3: Results for output equations
Appendix C: Influence of sample frequency and input vector

In this appendix the influence of changing the sample frequency and the input vector on the estimated states and measurements can be seen. The results of the unadapted system can be seen in figure 5-10 and 5-11 of section 5.4.

In figure C-1 and C-2 the results can be seen using a sample frequency \( f_s \) of 10 Hz for the GPS signals. This sample frequency could be possible using a RTK-GPS system instead of a normal GPS system. In the simulations in this appendix the same initial conditions and parameters are used as in the simulation of section 5.4.

![Diagram showing results for estimated states](image)

*fig. C-1: Results for estimated states*
Appendix C: Influence of sample frequency and input vector

fig. C-2: Results for sensor vector
Appendix C: Influence of sample frequency and input vector

In figure C-3 and C-4 the results can be seen of extending the input vector \( (u_i) \) like in appendix B. The sample frequency \( (f_s) \) is 1 Hz for the GPS signals.

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**fig. C-3: Results for estimated states**
Appendix C: Influence of sample frequency and input vector

fig. C-4: Results for sensor vector
Appendix D: Hybrid Extended Kalman Filter

In this appendix the working principle of the filter mentioned in section 5.4 is explained. For more information about this filter, see [13] where this information is taken from.

The system equations with continuous-time dynamics and discrete-time measurements are given as follows:

\[
\dot{x} = f(x,u,w,t) \tag{D.1}
\]

\[
z_k = h_k(x_k,v_k) \tag{D.2}
\]

\[
w(t) \sim (0,Q) \tag{D.3}
\]

\[
v_k \sim (0,R_k) \tag{D.4}
\]

For \(k = 1,2,\ldots,n\):

- The state estimate and its covariance are integrated from time \((k-1)^+\) to time \(k^-\) as follows:

\[
\hat{x} = f(\hat{x},u,0,t) \tag{D.5}
\]

\[
\hat{P} = A\hat{P} + P\hat{A}^T + \tilde{Q} \tag{D.6}
\]

This integration process is started with \(\hat{x} = \hat{x}^{+}_{k-1}\) and \(P = P^{+}_{k-1}\) and at the end there is \(\hat{x} = \hat{x}^{-}_k\) and \(P = P^{-}_k\).

- At time \(k\), the measurement \(y_k\) is incorporated in to the state estimate and estimation covariance as follows:

\[
K_k = P^+_k H_k^T (H_k P^-_k H_k^T + \widetilde{R}_k)^{-1} \tag{D.7}
\]

\[
\hat{x}^+_k = \hat{x}^-_k + K_k (z_k - h_k(\hat{x}^-_k,0,t_k)) \tag{D.8}
\]

\[
P^+_k = (I - K_k H_k)P^-_k (I - K_k H_k)^T + K_k \widetilde{R}_k K_k^T \tag{D.9}
\]

With:

\[
\tilde{Q} =QLQ^T \tag{D.10}
\]

\[
\tilde{R}_k = M_k \tilde{R}_k M_k^T \tag{D.11}
\]

and

\[
A = \left. \frac{\partial f}{\partial x} \right|_{\hat{x}}, \quad L = \left. \frac{\partial f}{\partial w} \right|_{\hat{x}}, \quad H_k = \left. \frac{\partial h_k}{\partial x} \right|_{\hat{x}^-}, \quad M_k = \left. \frac{\partial h_k}{\partial v_k} \right|_{\hat{x}^-} \tag{D.12, D.13, D.14, D.15}
\]
References


[10] “New clocks to keep time for Galileo satellite navigation system,” www.esa.int, attended 19 September 2009


References
