Predictive Control of Gyroscopic-Force Actuators for Mechanical Vibration Damping

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Executive summary

Gyroscopic forces of rotating wheels can be used for vibration damping in mechanical structures—this is known as gyro-stabilisation. In these applications, a gyroscope is attached to a mechanical structure in a particular configuration so the gyroscopic forces react and oppose the excitation forces that produce the vibration motion. Gyro-stabilisers are commonly used, for example, in marine structures to reduce the motion induced by the waves, in wave-energy extracting devices to extract energy from wave-induced motion, and in video cameras to stabilise image.

Due their construction, most gyro-stabilisers must operate subject to hard constraints on state and the control input. This together with the fact that the control objective can be posed as motion minimisation, makes constrained predictive control an attractive tool to address the control design problem.

In this report, we propose and study the application of a constrained model predictive control strategy for precession control of gyro-stabilisers for vibration damping due to a narrow-banded stationary stochastic forcing process with zero mean. The proposed control action is computed based on the optimisation of an objective function that captures deviations from the zero vibration condition and control effort. The optimisation takes also into account that the disturbance is narrow banded, and therefore, it can be predicted. From the numerical studies performed, it was found sufficient to consider prediction horizons of one-quarter of the mechanism natural period. Even perfect knowledge of the future disturbance beyond this does not improve performance. This is somehow expected since the control actions taken at a particular instant do not affect the behaviour of the system for all future times.

By using a novel combination of two observers, we provide a basis for adaptation to changes in the characteristics of the disturbance. A disturbance observer is used to estimate the disturbance at the current time instance. This is a robust observer that uses a simple random-walk model. The tuning of this observer is done such that the innovations are used to correct heavily the predictions using the model. Having the estimates of the disturbance process, then a second-order innovation model is identified. This identification is not performed recursively, so it is necessary to buffer the disturbance estimates. Once the refined model of the disturbance is obtained, a second observer is used with an augmented model that estimates the mechanism and gyro precession rates as well as the refined disturbance model states. The latter state estimates, together with the disturbance model provides a mechanism for prediction. Therefore, all the ingredients needed to implement the predicted controller are available.

We illustrate the performance of the proposed control strategy using a numerical simulation study with precession angle constraints.
1 Introduction

The dynamic response of mechanical structures subject to excitation forces often result in oscillatory motion or vibration. Gyroscopic forces of rotating wheels can be used for vibration damping of such motions in mechanical structures—this is known as gyro-stabilisation. In these applications, a gyroscope is attached to a mechanical structure in a particular configuration so the gyroscopic forces react and oppose the excitation forces that produce the vibration motion. These stabilisation or motion damping devices are very attractive due their fast response and high-actuation force (\(?)\). Gyro-stabilisers are commonly used in marine structures to reduce the motion induced by the waves (\(?)\), in wave-energy extracting devices to extract energy from wave-induced motion (\(?)\), and in video cameras to stabilise the image (\(?)\).

Consider the general mechanism depicted in Figure 1. This system is comprised of a mechanical system with one rotational degree of freedom and a characteristic inertia, damping and stiffness. This system is subjected to a random disturbance torque \(T_d\). Mounted on the mechanical system there is also a gyroscopic force actuator or gyro-stabiliser, which consists of a spinning wheel with two degrees of freedom; rotation about a vertical axis (spin) and a rotation about a horizontal axis (precession). As the mechanical system vibrates in the rotational degree of freedom due to the action of the external disturbance torque, it produces a torque on the precession axis of the gyro-stabiliser. This induces precession, and as a result there is gyroscopic reaction torque on the mechanical system that opposes the motion—this gyroscopic moment is used to attenuate the motion.

In order to limit the precession of the gyrostabiliser, a control torque, \(T_p\) is applied on the precession axis. Due their construction, most gyro-stabilisers must operate subject to hard constraints on maximum precession angle. For large gyro-stabilisers, like the ones used in marine vessels, the precession rate may also need to be limited to prevent excessive loads on spinning bearings. In addition to these constraints, the control torque may also be subject to magnitude and rate constraints. These features, together with the fact that the control objective can be posed as the minimisation of vibration, or alternatively, as a maximisation of the energy captured by the gyroscope (like in the wave energy converter application), makes the constrained predictive control an attractive tool to address the control design problem.

In this report, we propose and study a constrained model predictive control strategy for precession control of gyro-stabilisers for vibration damping. It is assumed that the disturbance force inducing the vibrations can be represented as a realisation of a narrow-banded stationary stochastic process with zero mean.

2 Dynamics of a Gyroscopic-Force Actuator System

A mathematical model for the coupled mechanical system shown in Figure 1 and gyrostabiliser can be obtain from Lagrangian mechanics and be expressed as follows (\(?)\),

\[ I_m \ddot{\phi} + B_m \dot{\phi} + C_m \phi = T_d - K_g \dot{\alpha} \cos \alpha \] (1)

\[ I_g \ddot{\alpha} + B_g \dot{\alpha} + C_g \sin \alpha = K_g \dot{\phi} \cos \alpha + T_p. \] (2)

Equation (1) represents the dynamics of the one-degree-of-freedom mechanism, while equation (2) represents the dynamics of the gyro-stabiliser about the precession axis.
We consider that the spin angular velocity $\omega_s$ is constant with a value established by the mechanical design of the gyro-stabiliser to achieve a desired gyroscopic stabilisation moment given by the last term on the right-hand side of (1). The parameter $K_g$ in the model is the spin angular momentum, that is,

$$K_g = I_s \omega_s,$$

where $I_s$ is the moment of inertia of the gyro-stabiliser wheel about the spin axis. The rest of the variables and parameters of the model (1) - (2) are given in Table 1.

For precession angles limited to about 1 rad, which is the usual constraint for gyro-stabilisers, the following approximations hold,

$$\dot{\alpha} \cos \alpha \approx \dot{\alpha},$$

and

$$\dot{\phi} \cos \alpha \approx \dot{\phi}.$$
Using the above, we obtain a linearised model

\[ I_m \ddot{\phi} + B_m \dot{\phi} + C_m \phi = T_d - K_g \dot{\alpha}, \quad (6) \]
\[ I_g \ddot{\alpha} + B_g \dot{\alpha} + C_g \sin \alpha = K_g \dot{\phi} + T_p, \quad (7) \]

The last term on the right-hand side of (6) represents the gyroscopic stabilisation torque which counteracts that of the disturbance. This term indicates that the larger the precession rate and the spin angular momentum, the larger the gyroscopic moment will be. The precession of the gyro, however, is normally limited to a maximum angle, that is a constraint \(|\alpha| \leq \alpha_{\text{max}}\). In order to satisfy this constraint, a precession torque is generated by a controller. The control objective is then to let the gyro-actuator to develop precession as much as possible within the given constraints. In some applications, specially where large torques are involved, it may be necessary also to limit the precession rate since the rate induces the loading on the spinning and precession bearings.

3 A Predictive Control Approach

The dynamics of the mechanism with the gyroscopic actuator given in (6)-(7) can be put into a state-space form and be discretised. This leads to the following model representation

\[ x_{t+1} = Ax_t + Bu_t + Bd_t, \quad (8) \]

where, \(x_t = [\phi_t, p_t, \alpha_t, a_t]^T\), \(u_t\) represents the precession control torque \(T_p\), and \(d_t\) is the random disturbance torque \(T_d\). Due to the operational restrictions of the gyro-actuator, we consider the following constraints.

\[ u_m \leq u_t \leq u_M, \quad (9) \]
\[ \Delta_m \leq u_t - u_{t-1} \leq \Delta_M, \quad (10) \]
\[ y_m \leq Cx_t \leq y_M. \quad (11) \]

The constraints (9) and (10) are related to the control mechanism that generates the precession torque. The constraint (11) is related to the precession angle and rate constraints. Throughout this work, we assume that the system is appropriately designed such that the constraints are feasible in nominal operation, and that there is a safe system that can act in cases where the precession control is not large enough to satisfy the state constraint.

In order to obtain a control law, we can pose a finite-horizon optimisation problem:

\[ U^* = \arg \min_{U \in \mathcal{U}} V_N(x_0, U, D), \quad (12) \]

where the scalar-valued function \(V_N\) is a measure of performance deviation, \(x_0\) is the initial condition, \(U = [u_0, u_1, \ldots, u_{N-1}]^T\) is the sequence of control actions over the horizon \(N\), and the constraint set \(\mathcal{U}\) arises from (9)-(11). The variable \(D\) represents the sequence of future disturbances, \(D \equiv D^N_t = [d_t, d_{t+1}, \ldots, d_{t+N}]^T\).
In the case where the disturbance $d_t$ in (8) is correlated and narrow banded, $D$ can be replaced by predictions,

\[ \hat{D}_t^N = [\hat{d}_{t|t}, \hat{d}_{t+1|t}, \ldots, \hat{d}_{t+N|t}]^T, \]  

where the notation $\hat{d}_{t+j|t}$ is the prediction at the time $t+j$ based on the information given up to the time $t$.

Ideally, we should like to solve the problem for an infinite horizon, that is $N \to \infty$. However, due to the constraints, the type of objective function $V_N$ chosen, and the fact that the disturbance predictions may not be accurate too far into the future, such a solution may be hard or even impossible to find, except for special cases (\textbf{?}). A practical solution to the infinite horizon problem consists of implementing receding-horizon control law based on the solution of the above finite-horizon constrained optimal control problem invoking the certainty equivalence principle. This control strategy is known as Model Predictive Control (MPC), and it provides a natural framework to address the control design problem subject to constraints (\textbf{??}).

In this formulation, the control takes the form

\[ u_t = K_N(\hat{x}_t, \hat{D}_t^N), \]  

where $\hat{x}_t$ is an estimate of the state and $\hat{D}_t^N$ is a sequence of estimates of the disturbance over a prediction horizon of $N$ samples. The control law (16), however, is not known explicitly. Its value is obtained, at each sampling instant, from the numerical solution of the optimal control problem (12), which is solved at each sampling instant with $x_0 = \hat{x}_t$ and $D = \hat{D}_t^N$. The control value of (16) corresponds to the first element of the sequence of optimal control moves $U^\star$.

The above strategy suggest the control architecture depicted in Figure 2. The main components of the controller are an observer that estimates the state and the disturbance, a modelling and prediction block that estimates a model of the disturbance and generates the predictions over the desired prediction horizon, and the MPC controller, which solves the optimisation problem (12) given the estimate of the state and the predictions of the disturbance. In the next sections, we further specify the different components of the control system.

### 4 Control Problem Specialisation

For the system under study, namely (8), we consider a quadratic objective function as a measure of performance deviation over the finite horizon,

\[ V_N(x_0, U, D) = x_N^T Q_f x_N + \sum_{k=0}^{N-1} x_k^T Q x_k + u_k^T R u_k, \]  

where $Q_f$ and $Q$ are symmetric non-negative definite matrices and $R$ is a symmetric and positive definite matrix.

Propagating forward the model (8), we can define the following vector equation,

\[ X = \Gamma_u U + \Lambda x(0) + \Gamma_d D, \]
Figure 2: Block Diagram of the Proposed Control System for the Gyro-stabiliser.

where

\[
X = \begin{bmatrix}
x_1 \\
x_2 \\
\vdots \\
x_N
\end{bmatrix}, \quad U = \begin{bmatrix}
u_0 \\
u_1 \\
\vdots \\
u_{N-1}
\end{bmatrix}, \quad D = \begin{bmatrix}
d_0 \\
d_1 \\
\vdots \\
d_{N-1}
\end{bmatrix},
\]

(19)

and

\[
\Lambda = \begin{bmatrix}
A \\
A^2 \\
\vdots \\
A^N
\end{bmatrix},
\]

(20)

\[
\Gamma_u = \begin{bmatrix}
B_u & 0 & \ldots & 0 & 0 \\
AB_u & B_u & \ldots & 0 & 0 \\
\vdots & \vdots & \ddots & \vdots & \vdots \\
A^{N-1}B_u & A^{N-2}B_u & \ldots & B_u
\end{bmatrix},
\]

(21)

\[
\Gamma_d = \begin{bmatrix}
B_d & 0 & \ldots & 0 & 0 \\
AB_d & B_d & \ldots & 0 & 0 \\
\vdots & \vdots & \ddots & \vdots & \vdots \\
A^{N-1}B_d & A^{N-2}B_d & \ldots & B_d
\end{bmatrix},
\]

(22)
Expanding the objective function (17), we further obtain
\[ V_N = \overline{V}_N + U^T W U + 2 U^T V, \]  
(23)
where \( \overline{V}_N \) is independent of \( U \), and
\[ W = \Gamma_u^T \overline{Q} \Gamma_u + \overline{R} \]  
(24)
\[ V = \Gamma_d^T \overline{Q} \Lambda x(0) + \Gamma_d^T \overline{Q} \Gamma_d D, \]  
(25)
\[ \overline{Q} = \text{diag}[Q, \ldots, Q, Q_f], \]  
(26)
\[ \overline{R} = \text{diag}[R, \ldots, R]. \]  
(27)

Magnitude and rate constraints on the input and the output can be expressed as follows:
\[ u_{\text{min}} \leq u_k \leq u_{\text{max}}; \quad k = 0, 1, \ldots, N - 1 \]  
(29)
\[ y_{\text{min}} \leq y_k \leq y_{\text{max}}; \quad k = 1, 2, \ldots, N - 1 \]  
(30)
\[ \Delta u_{\text{min}} \leq u_k - u_{k-1} \leq \Delta u_{\text{max}}; \quad k = 0, 1, \ldots, N - 1 \]  
(31)

This constraints (9)-(11) can be expressed as linear constraint on \( U \) of the form
\[ L U \leq K, \]  
(32)
where \( L \) is defined as:
\[ L = \begin{bmatrix} I_N \\ -I_N \\ E_u \\ -E_u \\ G \\ -G \end{bmatrix}, \]  
(33)
where \( I_N \) is the \( N \times N \) identity matrix and \( G \) is the following \( N \times N \) matrix
\[ G = \begin{bmatrix} 1 \\ -1 \\ \ddots \\ 0 \\ \ddots \\ \ddots \\ 0 \\ -1 \end{bmatrix}. \]  
(34)

\( E_u \) and \( E_d \) are the following \( (N - 1) \times N \) matrices
\[ E_u = \begin{bmatrix} C_{B_u} & 0 & \ldots & 0 & 0 \\ C_{A_{B_u}} & C_{B_u} & \ldots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ C_{A^{N-1}B_u} & C_{A^{N-2}B_u} & \ldots & C_{B_u} \end{bmatrix}, \]  
(35)
\[ E_d = \begin{bmatrix} C_{B_d} & 0 & \ldots & 0 & 0 \\ C_{A_{B_d}} & C_{B_d} & \ldots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ C_{A^{N-1}B_d} & C_{A^{N-2}B_d} & \ldots & C_{B_d} \end{bmatrix}. \]  
(36)
The matrix $K$ in (32) is given by

$$K = \begin{bmatrix} U_{\max} \\ U_{\min} \\ Y_{\max} \\ Y_{\min} \\ V_{\max} \\ V_{\min} \end{bmatrix}$$  \hspace{1cm} (37)$$

where

$$U_{\max} = \begin{bmatrix} u_{\max} \\ \vdots \\ u_{\max} \end{bmatrix}, \quad U_{\min} = \begin{bmatrix} -u_{\min} \\ \vdots \\ -u_{\min} \end{bmatrix}$$  \hspace{1cm} (38)$$

$$V_{\max} = \begin{bmatrix} u_0 + \Delta u_{\max} \\ \Delta u_{\max} \\ \vdots \\ \Delta u_{\max} \end{bmatrix}, \quad V_{\min} = \begin{bmatrix} -u_0 - \Delta u_{\min} \\ -\Delta u_{\min} \\ \vdots \\ -\Delta u_{\min} \end{bmatrix}$$  \hspace{1cm} (39)$$

$$Y_{\max} = \begin{bmatrix} y_{\max} - CAx_0 \\ \vdots \\ y_{\max} - CA^{N-1}x_0 \end{bmatrix} - Ed \begin{bmatrix} d_0 \\ d_1 \\ \vdots \\ d_{N-1} \end{bmatrix}$$  \hspace{1cm} (40)$$

$$Y_{\min} = \begin{bmatrix} -y_{\min} + CAx_0 \\ \vdots \\ -y_{\min} + CA^{N-1}x_0 \end{bmatrix} + Ed \begin{bmatrix} d_0 \\ d_1 \\ \vdots \\ d_{N-1} \end{bmatrix}$$  \hspace{1cm} (41)$$

Combining the above results, we have that the finite horizon optimal control sequence that minimises the quadratic objective function (17), subject to the dynamic model (8) and the constraints (9)-(11) can be obtained from the following standard Quadratic Programme:

$$U^* = \arg \min_{U: L \leq K} \mathbf{U}^T W \mathbf{U} + 2 \mathbf{U}^T \mathbf{V}.$$  \hspace{1cm} (42)$$

The matrix $W$ in (42) depends only on the parameters of the model (8) and the matrices that define the objective function (17). The matrix $L$ in the constraint depends only on the parameters of the model. Therefore, these matrices can be computed off line. The matrices $V$ and $K$, however, depend not only on the model parameters, but also on the disturbance predictions and the initial state of the system. Therefore, these matrices need to be computed on-line at each sampling instant.

With regards to the stability of the proposed strategy, let us first start with the problem without constraints. If there are no constraints, and we set $D$ to zero, then the proposed control law reverts to

$$u_t = -e_1 W^{-1} V = -K_u x_t, \quad e_1 = [1, 0, \ldots, 0].$$  \hspace{1cm} (43)$$

Since the system under study is controllable, if we chose the matrix $Q_f$ in (17) as the solution of the algebraic Riccati equation associated with unconstrained linear-quadratic regulator (LQR) problem (no disturbance), then the controller of the above control law is indeed the LQR stabilising state
feedback control (\(\zeta\)). If we now consider the disturbance a feed-forward term appears in (43), which
does not modify the stability in the unconstrained case. Actually, it follows that the system can be
stabilised using only negative feedback of the precession components of the state (\(\zeta\)). Indeed, under
this reduced feedback control strategy, both the mechanism and gyro-stabiliser models are passive
from force to velocities. Therefore, their interconnection is also passive; and thus stable (\(\zeta\)).

If we now turn to the constrained control problem. We assume that input and state constraints
are such that the constrained problem is feasible. One can, then use the traditional approach that
considers the function \(V_N\) as a Lyapunov function. The state equation (8) and the objective function
(17) are continuous and they take null values when their arguments are null. The final state set \(X_f\)
given in(11)—related to the precession constraints—is closed and the control set related to (9)-(10)
is compact and both these sets contain the origin. Furthermore, for each state in \(X_f\) there exists an
invariant control law, that is, if \(x_t \in X_f\), then \(x_{t+1} \in X_f\) under such control law. The asymptotically
stability of the origin follows from the above characteristics of the problem—see \(\zeta\) page 123.

4.1 State and Disturbance Observer

The predictive controller requires the state and also the prediction of the disturbance. These can
be estimated using an observer provided a model of the disturbance is postulated. In this work,
we formulate this estimation in two stages. In a first stage, we adopt a random-walk model for the
disturbance, that is,
\[d_{t+1} = d_t + w^d_t,\] (44)
where \(w^d_t\) is a zero mean Gaussian iid sequence. With this simple model, we can consider an augmented
state-space model
\[
\begin{align*}
z_{t+1} &= A_z z_t + B_z u_t + w^z_t \\
y_t &= C_z z_t + v^z_t,
\end{align*}
\] (45)
where \(z_t = [d_t, x^T_t]^T\), and \(C_z\) is such that the measurement is a noisy version of mechanism and gy-
rostabiliser precession angles. The vector \(v^z_t\) represents the measurement noise. The state noise \(w^z_t\)
represents model uncertainty.

With the augmented model (45), we can design a Kalman filter. The filter provides estimates based
on a prediction using the model and a correction using the measurements. How much the filter trusts
the predictions and therefore how much it corrects them depends on the noise covariance matrices.
If there were no model uncertainty and the covariances of the noises were known, the filter would be
optimal in the minimum mean-square sense. However, since the model of the disturbance that we are
using may not be accurate, we can abandon the idea of optimality and use the state-noise covariance
matrices as tuning parameters. To this purpose, we make partition of the state-noise and its associated
covariance matrix as follows
\[w^z_t = \begin{bmatrix} w^d_t \\ w^z_t \end{bmatrix}, \quad Q_z = \begin{bmatrix} Q_d & 0 \\ 0 & Q_x \end{bmatrix},\] (46)
Since the simple model proposed for the disturbance (44) may not be very accurate, we set the value
of the variance \(Q_d\) to a high value, which is much higher than the value of the entries of the diagonal
matrix \(Q_x\). This way, the Kalman filter will put a heavy weight in the corrections based on the
measurements since the model is not accurate. This is a robust way of estimating the disturbance.
However, since the model used in the observer is not accurate for predictions. Therefore, in order to
implement a predicted controller, the disturbance model needs to be refined.
4.2 Disturbance Identification and Prediction

Having an estimate of the disturbance $\hat{d}_t$, we can use it for system identification to refine its model and then use it in a prediction scheme as illustrated in the block diagram shown in Figure 2. This is a second step in the disturbance estimation. The advantage of using the proposed method is that the tuning of the disturbance observer considered in the first stage does not change if the characteristics of the disturbance change.

If the disturbance $d_t$ is stationary, and it has a narrow-banded spectrum, one can make an approximation based on spectral factorisation (47),

$$S_{dd}(\omega) \approx |H_d(j\omega)|^2 \sigma_e,$$

where $H_d(s)$ is the transfer function of a linear filter driven by uncorrelated noise $\epsilon_t$ with variance $\sigma_e$.

For most applications a second order filter presents a good approximation. If we use the innovations representation of the approximating filter, a disturbance model can be expressed as

$$z_{d,t+1} = A_d z_{d,t} + K_d \epsilon_t$$

$$d_t = C_d z_{d,t} + \epsilon_t,$$

where $z_{d,t} \in \mathbb{R}^2$. Given the innovations model (48), and the estimates of $d_t$ provided by the observer considered in the previous section, one can use maximum likelihood to estimate $A_d$, $K_d$ and $C_d$—see (47).

After estimating the parameters of the model we can build an observer to estimate the disturbance model state together with the state of the mechanism and gyro-stabiliser based on an augmented model

$$z_{a,t+1} = A_a z_{a,t} + B_a u_t + E_a w_t$$

$$y_t = C_a z_{a,t} + v_t,$$

where

$$z_{a,t} = \begin{bmatrix} z_{d,t} \\ x_t \end{bmatrix}.$$

Having the innovations model (48) and the estimate of the state (50), then we can obtain optimal predictions by propagating the model forward (47). That is,

$$\hat{d}_{t+j|t} = C_d (A_d)^j \hat{z}_{d,t}.$$

5 Numerical Study

In this section, we consider a numerical study of a proposed control strategy for the mechanism shown in Figure 1 with a maximum precession angle constraint of 1 rad. The main parameters of the system are summarised in Table 2.

The model and the controller were implemented in Matlab and Simulink. The associated quadratic programme was solved using the function `quadprog` of Matlab’s optimisation toolbox. The mechanism was excited by a random torque with realisations generated by filtering white noise.
Table 2: Model parameters for Simulation Study.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>$I_m$</td>
<td>$3.39 \times 10^2$</td>
<td>Kg m²</td>
</tr>
<tr>
<td>$B_m$</td>
<td>$8.0 \times 10^1$</td>
<td>Nms/rad</td>
</tr>
<tr>
<td>$C_m$</td>
<td>$3.57 \times 10^2$</td>
<td>Nm/rad</td>
</tr>
<tr>
<td>$I_g$</td>
<td>$1.11$</td>
<td>Nms/rad</td>
</tr>
<tr>
<td>$B_g$</td>
<td>$1.91$</td>
<td>Nm/rad</td>
</tr>
<tr>
<td>$C_g$</td>
<td>$1.66$</td>
<td>Nm/rad</td>
</tr>
<tr>
<td>$K_g$</td>
<td>$0.84 \times 10^2$</td>
<td>Kgm²rad/s</td>
</tr>
</tbody>
</table>

Figure 3 shows the estimation results of the disturbance observer, which uses the model (45). Note that none of the variables shown in this figure are actually measured, since we assume measurements of the mechanism and gyro-stabiliser precession angle only. Note also from the top plot that the filter is able to estimate the disturbance torque very well even though the model (44) is not accurate.

Figure 3 shows the 3- and 10-step-ahead predictions using (51) with the identified second order model for the disturbance estimates shown in Figure 3. Finally, Figure 5 shows the performance of the controlled mechanism. The top plot shows the angle of the mechanism with and without control. The middle plot shows the precession angle and its constraints. In this plot, we can observe that the control is able to enforce the constraints on the maximum precession of 1 rad. The bottom plot shows the precession torque generated by the controller.

Figure 3: Disturbance and angular rate estimates obtained from a Kalman filter that uses a random walk as a disturbance model.
6 Summary and Discussion

In this work, we propose and study the application of a constrained model predictive control strategy for precession control of gyro-stabilisers for vibration damping. It is assumed that the disturbance force inducing the vibrations can be represented as a realisation of a narrow-banded stationary stochastic process with zero mean.

The proposed control action is computed based on the optimisation of an objective function that captures deviations from the zero vibration condition and control effort. The optimisation takes also into account that the disturbance is narrow banded, and therefore, it can be predicted. From the numerical studies performed, it was found sufficient to consider prediction horizons of one-quarter of the mechanism natural period. Even perfect knowledge of the future disturbance beyond this does not improve performance. This is somehow expected since the control actions taken at a particular instant do not affect the behaviour of the system for all future times.

By using a novel combination of two observers, we provide a basis for adaptation to changes in the characteristics of the disturbance. A disturbance observer is used to estimate the current disturbance. This is a robust observer that uses a simple random-walk model. The tuning of this observer is done such that the innovations are used to correct heavily the predictions using the model. Having the estimates of the disturbance, then a second-order innovation model is identified. This identification is not performed recursively, so it is necessary to buffer the disturbance estimates. Once the refined model of the disturbance is obtained, a second observer is used with an augmented model that estimates the mechanism and gyro precession rates as well as the refined disturbance model states. The latter state
estimates, together with the disturbance model provides a mechanism for prediction. Therefore, all the ingredients needed to implement the predicted controller are available.

We considered a numerical study, which shows very high performance of the controller and its ability to handle the maximum precession constraints. Extensions that consider constraints on the precession rate as well as angle are trivial. From the numerical experiments it is interesting to note that it seems possible to lock the gyro at maximum precession. This characteristics can be advantageous for applications of wave energy conversion.

Future work should focus on the issue of feasibility of the joint input and state constraints. Throughout this work, we assumed that the precession actuator had enough authority to be able to generate precession torque necessary to satisfy the state constraints (precession angle). In cases where the size of disturbance is large, the associated optimisation problem can be unfeasible. In this case, there must be a safety mechanism that locks the gyro-stabiliser. This is beyond the scope of the this work.