Control oriented system analysis and feedback control of a numerical sawtooth instability model

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Sawtooth instabilities can trigger secondary instabilities (NTMs) and are associated with the mixing of fusion products in the plasma core, and premature losses of alpha-particles in a fusion reactor. To optimize between these effects, control of the sawtooth period is essential.

Experiments demonstrate the effect of the ECCD deposition radius on the sawtooth period, recently leading to some first closed loop results for sawtooth control on TCV and Tore Supra \cite{1,2}. In this paper we use a numerical sawtooth instability model to carry out control oriented system analysis, to enable structured feedback control design for the sawtooth period in plasmas with a circular cross section using ECCD.

The study assumes the availability of a line-of-sight ECE system \cite{3}. A combined Kadomtsev-Porcelli model is numerically implemented in Matlab / Simulink. The resulting infinite dimensional impulsive dynamical system is sufficient for control purposes, as it yields realistic steady-state input-output behavior. The inputs to this model are the total EC driven current and the poloidal angle of the EC-mirror, while the sawtooth period is the output. This model has been used for system identification purposes: simulations are carried out to identify the system response to stepwise changes in input variables. This way, linear approximations of the system dynamics around various operating points are obtained. The resulting linear time-invariant (LTI) models can be used for local controller design. These controllers (PI, i.e. proportional and integrating) are then feedback connected with the original impulsive dynamical system to show that we can indeed obtain desired sawtooth periods without steady-state errors with this strategy. These results suggest that feedback control of the sawtooth instability on actual tokamaks is indeed feasible.

References

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Introduction

A numerical model is used to carry out control oriented system analysis of the sawtooth instability, and design a feedback controller using ECCD for the sawtooth period.

Sawtooth model and steady state simulations

Model: \( \text{infinite dimensional impulsive dynamical system} \)

\[
\frac{\partial}{\partial t} B_\theta = \frac{\partial}{\partial r} \left( \eta \mu_0 r \left( B_\theta + r \frac{\partial}{\partial r} B_\theta \right) - \eta J_{\text{CD}} \right)
\]

\[
B_\theta(r, t^+) = \begin{cases} 
B_\theta(r, t^-) & \text{for } r \geq r_{\text{mix}} \\
\frac{1}{\eta r} B_\theta & \text{for } r < r_{\text{mix}} 
\end{cases}
\]

if \( s_q = 1 \leq s_{\text{crit}} \) (1a)

where (1a) denotes magnetic diffusion, (1b) full reconnection during a crash and \( s_{\text{crit}} \) the condition for a crash. Also

\[
\eta(r) \propto T_z(r)^{-3/2}, \quad T_z(r) = T_0 \left( 1 + q_a \left( \frac{r}{a} \right)^2 \right)^{-4/3}
\]

\[
J_{\text{CD}}(r) = J_0 \cdot \exp \left( -\left( \frac{r - r_{\text{CD}}}{a} \right)^2 \right), \quad r_{\text{CD}} = r_{\text{CD}}(\vartheta)
\]

Input: ECCD mirror angle \( \vartheta \). Output: period \( \tau_s \). Model implementation in Simulink® S-function yields results of Fig. 1.

System identification and controller design

Estimation dynamics around operating points \( \tau_s^i \in [9.25, 11.75] \) using feedback in Fig. 2 (\( k \) is the crash number index):

- estimate Sensitivity \( S_i \) and Process Sensitivity \( PS_i \), i.e. the transfer functions from step to \( \Delta \vartheta \) and \( \tau_{s,h}^i \)
- obtain discrete time approximations \( H_i = PS_i/S_i \), i.e. the dynamic response of the model from crash to crash

The controller \( C(z) \) was designed to guarantee stability and performance for all these \( H_i \), yielding

\[
C(z) = -0.721 \frac{z + 1}{2(z - 1)}
\]

or equivalently, using \( e_k = \tau_{s,k}^i - \tau_{s,\text{ref}}^i \)

\[
\Delta \vartheta_k = \Delta \vartheta_{k-1} - 0.721 \left( e_k + e_{k-1} \right)
\]

\( C(z) \) achieves bandwidths between 0.033 and 0.1 \( (\text{Fig. 3}) \), i.e. settling times between 10 and 30 periods (Fig. 4).

Conclusions

For a specific operating region linear discrete time approximations of the sawtooth model were obtained, a controller was designed and perfect tracking was achieved for all setpoints.

References