FREQUENCY DOMAIN BASED FEED FORWARD TUNING FOR FRICTION COMPENSATION

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ABSTRACT
In high precision motion control, performance is often limited by the presence of nonlinearities. In this study, the presence of nonlinear influences in a high precision transmission electron microscope stage is investigated using broadband multisine signals. These measurements yield the nature and level of nonlinearities as well as the best linear approximation of the dynamics. By quantitatively measuring the level of nonlinear influences, this method indicates the relevance of improved modeling. Next, the nonlinear influences are modeled explicitly by measuring the higher order sinusoidal input describing functions (HOSIDF) of the system which describe the 'direct' response of the system at the input frequency as well as at harmonics of the input frequency. Application of this technique yields a structured way to design Coulomb friction feed forward in the presence of nonlinearities. This procedure linearizes the input-output dynamics by applying feed forward and measuring the HOSIDFs which indicate the remaining nonlinear effects. Application of this technique yields a structured way to design feed forward in the presence of nonlinearities.

INTRODUCTION
When identifying and controlling (mechanical) systems, a linear model structure is often assumed. If nonlinear influences are small, such assumptions may be justified. In order to draw conclusions about nonlinear influences (type and magnitude) the authors in \cite{1, 2, 3} present a multisine based, frequency domain identification approach. This method yields both the Best Linear Approximation (BLA) of the systems dynamics and the magnitude and type of nonlinearities present. To further quantify the nonlinear influences the authors in \cite{4, 5, 6} present a nonparametric modeling technique referred to as Higher Order Sinusoidal Input Describing Functions (HOSIDF). This technique describes the response of a system by relating the magnitude and phase of the harmonics of a sinusoidal input, in the output signal due to nonlinear influences. Finally, this study extends the concept behind the HOSIDFs to optimal feed forward design for a friction dominated motion system.

EXPERIMENTAL SET-UP
In this paper two experimental set-ups used in this study are introduced. Next, the multisine based approach is used to measure the BLA of the dynamics of an industrial high precision motion stage. This method emphasizes the relevance of improved modeling by detecting the magnitude and type of nonlinearities present. Moreover, the HOSIDFs of the same system are measured to further quantify the nonlinear behaviour. Finally, the concept behind the HOSIDFs is used to design optimal feed forward control for a 4\textsuperscript{th} order motion system with Coulomb friction, minimizing the amount of nonlinear influence and largely improving the systems performance.

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used to control the system. Second, a 4th order mass, spring, damper system with Coulomb friction is used to illustrate the usage of the measured nonlinear response in feed forward tuning. In the sequel the experimental set-ups are discussed in detail.

(I) Transmission electron microscope stage

Figure 1 depicts the TEM stage, removed from the TEM. The stage is used as a SISO system driven by a Maxon DC motor, with the engine voltage as input and the position of the stage as an output. The application of the excitation signal and measurement of the response is performed using a SigLab 20-42 dynamics analyzer providing 90 dB aliasing protection. The sample position is measured using a MicroE Mercury 3500 encoder with a sensor accuracy of 5 nm.

(II) 4th order mechanical system with friction

Figure 2 shows a 4th order mass, spring, damper system with Coulomb friction applied at the motor side. The system consists of two rotating masses connected by a torsional element. The friction is applied by applying a constant normal force acting through a vertical guidance system. The contact between the rotating mass and the friction element consists of two cylindrical elements resulting in an approximate single point contact. The constant normal force, combined with the single point of steel to steel contact results in a close approximation of Coulomb friction. The rotation of both masses is measured using an optical encoder with an accuracy of 500 increments per rotation and the system is driven by a Maxon DC motor. The engine voltage is the input of the system, while the rotation of the mass on the motor side (right side in Figure 2) is the output.

DETECTING NONLINEARITIES

Methodology

In this section a multisine based method is introduced that allows measurements of both the best linear approximation of a system under test and the magnitude and type of nonlinearities present [1, 2, 3]. In general, output spectra $Y$ are composed of the harmonic content generated from the input $U$ by the best linear approximation $Y_{BLA} = H_{BLA} U$, stochastic disturbances (noise) $Y_{noise}$ and disturbances generated by nonlinear influences $Y_S$ [1]:

$$Y = Y_{BLA} + Y_S + Y_{noise}$$

Using signals with specific spectra and phase distributions called random odd multisines, the amount of information about nonlinear influences obtained from measurements in maximized. The $m^{th}$ realization of a random odd multisine is defined as:

$$u^{[m]}(t) = \sum_{n=1}^{N} \alpha_n \sin \left( 2\pi f_n t + \varphi_n^{[m]} \right),$$

with $\alpha_n \in \mathbb{R}_{\geq 0}$ possibly different for various frequencies, but constant over different real-
First, consider the excited spectral lines in the input signal. Performing $M$ experiments with $M$ realizations of the excitation signal yields $M \times P$ input and output time series and their corresponding spectra $U(\omega)$ and $Y(\omega)$. Averaging over multiple periods of the same realization yields the average spectrum and the variance on this spectrum due to stochastic distortions $\sigma^2_{BLA,noise}$, but not that due to nonlinear effects. Next, calculating the average spectrum over different realizations yields the best linear approximation $Y_{BLA}$ of the true spectrum and the variance on this averaged spectrum due to nonlinearities $\sigma^2_{BLA,NL}$. Second, consider the response at odd ($o$) and even ($e$) non-excited lines $P(o/e)$. The spectrum has random phase at these lines, hence calculating the variance over the different measured spectra yields the average power that occurs at these frequencies. This yields a measure for nonlinear behaviour as well.

**Application**

The BLA and the level of nonlinearities in the industrial high precision stage are measured using the described procedure. Measurements are performed with measurement frequency of $f_s = 2560$ Hz and a block length of $N_{block} = 8192$ measurement points. This yields a base frequency of the random odd multisine of $f_0 = \frac{f_s}{2N_{block}} = 0.3125$ Hz. Sufficient waiting time is allowed to assure that transient phenomena have damped out, avoiding leakage phenomena and no windowing is applied.

$M = 10$ realizations of the odd random multisine have been generated and the response has been measured for $P = 10$ periods. Furthermore, this experiment is repeated for 20 different rms values
of the multisines, logarithmically scaled between 0.3 V and 5.0 V. A typical output spectrum is depicted in Figure 3. The best linear approximation of the systems dynamics is depicted in Figure 4 as a function of both frequency and input power.

**Results**

From Figure 3 it becomes clear that nonlinearities have an average level 10 dB lower than the power generated in the output spectrum by the BLA of the system. Both odd and even nonlinearities are detected, but odd nonlinearities dominate by almost 20 dB. Finally, the variation due to stochastic influences is almost 30 dB lower than that due to nonlinear effects.

**QUANTIFICATION OF NONLINEAR EFFECTS**

**Methodology**

In order to use the obtained information about nonlinearities from the previous section in controller design, the Higher Order Sinusoidal Input Describing Functions (HOSIDF) of this system are identified. HOSIDFs describe not only the ‘direct’ response (gain and phase) of a system at the excitation frequency, but describe the response at harmonics of the excitation frequency as well. Consider the following input signal used to identify the HOSIDFs:

$$w(t) = \beta \sin(2\pi f_0^s t).$$  \hspace{1cm} (3)

Next, the k\textsuperscript{th} order HOSIDF is defined as:

$$|H_k(\beta, f_0^s)| = \left|\frac{Y(kf_0^s)}{U(f_0^s)}\right|$$  \hspace{1cm} (4)

$$\angle H_k(\beta, f_0^s) = \angle Y(kf_0^s) - k \angle U(f_0^s),$$  \hspace{1cm} (5)
a PD-controller and a reference signal $\theta_r$ as depicted in Figure 6. The rotation of the motor $\theta_1(t)$ is the output and controlled variable. Apart from feedback, a nonlinear feed forward is applied to compensate for Coulomb friction. Using a stabilizing PD controller, the feed forward parameter $K_{fc}$ will be tuned to linearize the input-output behaviour of the closed loop system. In other words, $K_{fc}$ will be tuned such that the amount of nonlinear (harmonic) content relative to the linear content in the output is minimal, or in terms of the systems HOSIDFs:

$$K_{fc}^* = \arg\min_{K_{fc} \in \mathbb{R}_+} \left| \frac{H_1(K_{fc})}{H_i(K_{fc})} \right|$$

In the sequel, numerical and experimental results are provided that illustrate this tuning method. In both simulations and experiments $K_{fc}$ is varied from 0 (no feed forward) until the system is slightly overcompensated. The HOSIDFs are measured yielding the required minimum. Since the feed forward friction model is a static model, the tuning procedure is performed for one frequency only, assuming Coulomb friction in the plant.

**Numerical Results**

Figure 7 depicts simulation results of a two mass, spring, damper system similar to the system depicted in Figure 2. The system is subject to Coulomb friction at the motor side and operates in feedback as depicted in Figure 6. From Figure 7 it becomes clear that input-output behaviour of the system becomes more linear with increasing $K_{fc}$ until an optimum is reached when the feed forward equals the Coulomb friction force. Furthermore, the phases of the HOSIDFs turn $180^\circ$ at the optimal setting.

**Experimental Results**

Figure 8 shows the same behaviour in experiments using the experimental set-up depicted in Figure 2. An optimum is reached at $K_{fc}^* = 0.1157$ V where the relative level of nonlinearities has decreased from 15% to less than 1.5%. The remaining nonlinear influences are due to effects that are not captured by the feed forward model. Note that even nonlinearities (not depicted) have a relative level of only 4% and are not influenced by the purely odd feed forward.

**Discussion**

The method presented in this paper enables optimal tuning of (feed forward) parameters in the sense that the input-output behaviour of the system is linearized. The procedure has been demonstrated for Coulomb friction feed forward but may be used to tune arbitrary controllers in the presence of nonlinearities as long as the complete system has a periodic response to a periodic input.

The optimal value $K_{fc}^*$ not necessarily yields the smallest tracking error. Figure 9 shows the response and error observed in experiments: in ab-
The presented method allows tuning of arbitrary controllers linearizing input-output dynamics of systems with a periodic response to a periodic input. It may be used to assess and compare the quality of different controllers and future research aims at optimization algorithms for fast, automated controller tuning. Furthermore, tuning of multiple parameter (non)linear and adaptive controllers is investigated.

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REFERENCES