Pulsed Active Steering:
The effects of non-linear tire relaxation behavior on
the effectiveness of PAS
DC2010.002
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January 11, 2010
Acknowledgements

I would like to thank Professor A. Khajepour for his patience, support and guidance and the opportunity to do this internship at the University of Waterloo.

I would also like to thank all the people I have met during my stay in Canada. They have made my stay much more enjoyable. A special thanks goes to Christoph Baum and Matthew Millard for their contribution to my work.

Finally, I would like to thank my family, girlfriend and all my friends for their support and encouragement to make this thesis possible.
Abstract

This report explores the effects of non-linear tire behavior on the effectiveness of Pulsed Active Steering (PAS). The goal of PAS is to reduce vehicle rollover by subtracting a pulse from the steering signal. Previous studies have been performed into the subject, showing that PAS has the potential to reduce vehicle rollover. In addition, changing the frequency and shape of the pulse has clear effects on the vehicle roll and yaw behavior. This report focuses primarily on the vehicle’s roll behavior, which is quantified using a roll coefficient.

The vehicle model used consists of a three degree-of-freedom model. The degrees of freedom are the yaw angle, lateral translation and roll angle. For the tire model, a Magic Formula tire model is used as described by Pacejka. The tire relaxation model consists of a tire contact patch attached by a spring and damper to the rim. The forces acting in the contact patch are again described by the Magic Formula tire model.

The simulations show that considering tire relaxation is of utmost importance, since large differences in the roll coefficient as well as the side slip angles are found between the model with and without tire relaxation. It was also found that PAS can reduce the roll coefficient when applying a sinusoidal shaped pulse. The mean value of the roll coefficient is lower than that of a constant subtractive steering angle. The peak values, however, are still higher than the roll coefficient achieved by a constant subtractive angle. A non-sinusoidal pulse does reduce the roll coefficient even further, but still surpasses the constant subtractive angle. One benefit of pulsing the steering angle has been found. The roll coefficient reduces faster. Simulations have been performed to find the required increase in average over a constant subtractive angle.
Glossary

SUV  Sports Utility Vehicle
AFS  Active Front Steering
NHTSA  National Highway Traffic Safety Administration
PAS  Pulsed Active Steering
\( r \)  Yaw velocity
\( v \)  Lateral velocity
\( \phi \)  Roll angle
\( u \)  Longitudinal velocity
\( V \)  Vehicle velocity
\( \delta_{lf} \)  Steering angle left front
\( \delta_{rf} \)  Steering angle right front
\( \alpha_{lf} \)  Side slip angle left front
\( \alpha_{lr} \)  Side slip angle left rear
\( \alpha_{rf} \)  Side slip angle right front
\( \alpha_{rr} \)  Side slip angle right rear
\( a \)  Distance from front axle to center of gravity
\( b \)  Distance from rear axle to center of gravity
\( T \)  Track width
\( R \)  Roll coefficient
\( COG \)  Center of gravity
\( m \)  Total vehicle mass
\( m_s \)  Sprung mass
\( m_u \)  Unsprung mass
\( I_{xx} \)  Inertia around x-axis
\( I_{zz} \)  Inertia around z-axis
\( F \)  Force
\( R_c \)  Corner radius
\( h_r \)  Distance between road surface and roll axis
\( h \)  Distance between roll axis and COG of the sprung mass
\( k_\phi \)  Roll stiffness
\( d_\phi \)  Roll damping
\( a_y \)  Lateral acceleration
\( S_V \)  Horizontal shift factor for the Pacejka tire model
\( S_H \)  Vertical shift factor for the Pacejka tire model
\( \gamma \)  Camber angle
\( V_x \)  Tire velocity in x-direction
$V_{s,y}$  Tire velocity in y-direction
$V_{x}'$  Velocity of tire contact patch in x-direction
$V_{y}'$  Velocity of tire contact patch in y-direction
$v_r$  Tire deformation
$k_r$  Tire lateral stiffness
$d_r$  Tire lateral damping
$\sigma$  Tire relaxation length
$C_{Fa}$  Cornering stiffness
$m_c$  Mass of tire contact patch
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Chapter 1

Introduction

Traveling by motorized vehicle is the most common mode of transport both in the United States and Canada. It provides an unprecedented possibility of going everywhere, whenever the user chooses to. However, for all its advantages, traveling by motorized vehicle is the number one cause of death for people between 3 and 34. A staggering 5,811,000 police reports concerning motor vehicle crashes were filed in the United States in 2008 alone. In these crashes 37,261 people were killed. The economic cost for these crashes was $230.6 billion [10].

From the 37,261 fatalities in 2008, 46% was caused by a single vehicle crash. Single vehicle crashes are further split up into rollover (47%) and non-rollover (53%) crashes. Rollover crashes thus accounted for 8386 fatal rollover accidents in the United States. SUVs contributed greatly to this number, 33% of these deaths were caused by SUVs, making them twice as likely to rollover compared to normal passenger cars. A lot of research has already been performed to reduce vehicle rollover, the next section will discuss this in more detail.

1.1 Previous research

Techniques for reducing vehicle rollover can generally be subdivided into three categories:

- Active Front Steering (AFS), in which the front wheel steering angle is actively corrected in order to change the vehicle stability behavior. This system is already implemented by BMW [5] in order to provide a variable steering ratio, but also to ensure vehicle yaw stability.

- Yaw moment control which functions by applying the brakes individually to create an extra yaw moment to ensure stability of the vehicle. It has been shown by NHTSA that fatalities could be reduced by up to 30% if all vehicles were fitted with this system [7].

- Active Rear Steering, in which the rear wheels are steered. This not only benefits the maneuverability of a vehicle, but also the stability by steering in the same direction as the front wheels, according to Renault. The system can also detect asymmetric braking situations. In these cases, the rear wheels are steered such that vehicle stability is ensured, without the driver noticing.

Obviously, multiple combinations of these systems are possible. All three systems have their own benefits. Ackermann [2] states that with active steering, the side force that has to be
developed by the tire is only one fourth of the longitudinal force that has to be generated by braking to ensure the same contribution to the yaw moment. Ackerman also suggests a control system to ensure rollover stability by actively controlling the steering angle and applying the brakes, in order to reduce vehicle rollover yet achieve the desired trajectory.

Kuo [6], however says that neither differential braking, nor active steering can reduce rollover enough to prevent the vehicle from flipping. Kuo states that the full non-linear vehicle model influences stability too much to use a PD controller in combination with AFS to reduce the rollover coefficient sufficiently. Using braking to reduce the rollover coefficient, he states, is too heavily influenced by the possible shift of vertical tire load. A combination of both active steering and differential braking is also heavily influenced by this. Kuo suggests using Pulsed Active Steering (PAS) to reduce vehicle rollover instead. This will be discussed in more detail in the next section.

1.2 Pulsed Active Steering

In pulsed active steering, the front wheel steering angle is controlled in a similar way as brake force is controlled in ABS braking: a pulse is added or subtracted from the drivers steering input to ensure vehicle stability. Kuo has investigated multiple pulse forms in order to find a correct pulse from. A symmetric pulse increases the original rollover coefficient. A non-symmetric pulse shows better results, however. He states that the control frequency as well as the pulse amplitude are important control parameters. A suitable control frequency is 2 Hz.

Vos [14] has continued this research. He has found that the pulse form suggested by Kuo is not the optimal form for PAS and suggests a new shape. He has also found that low frequency pulsed actually increases the rollover coefficient. Higher frequencies, however, show great potential in reducing the rollover. A pulse frequency of 8 Hz is optimal, since this does not influence the path deviation as much as a 4 Hz pulse does. He, however, never uses high speeds (maximum 40 km/h) in combination with large steering angles and thereby does not achieve high rollover coefficients. An important remark made during this report is that a constant subtractive steering angle would be more effective. Vos furthermore states that he does not use real SUV dimensional parameters.

A third research into this topic has been done by Abdel-Rahman [1]. His research is twofold, one part being the use of modern simulation programs to determine optimal parameters for PAS. The other part is building a Hardware In Loop (HIL) setup in order to do further simulations and research. In his simulations he has found that frequency modulation does not influence the yaw dynamics, but does have a clear effect on the roll dynamics. Using a pulse frequency close to the vehicle’s roll eigenfrequency improves the roll dynamics the most. Abdel-Rahman furthermore built an experimental setup. The results achieved with this setup were similar to the results obtained with the simulations.

1.3 Problem Statement

One problem does arise with the previous research: Kuo states that he does not consider tire relaxation length because tire relaxation only accounts for about 10% of the duty cycle of a sinusoidal pulse at 100 km/h. It has however been shown previously that tire relaxation
behavior plays a significant role in transient vehicle dynamics. Vos did consider a tire relaxation model, but only performed simulations at one speed and one steering angle. This report will therefore consider the full nonlinear tire relaxation characteristics as shown by Maurice [8]. This means that the relaxation length will not only be considered as a function of side slip angle but also as a function of vertical load. The main research question for this report is: what is the benefit of Pulsed Active Steering compared to conventional active steering when considering the full non-linear tire behavior including non-linear relaxation effects? A new pulse will also be proposed as was suggested by Vos and Abdel-Rahman in their reports.

1.4 Report overview

The overview of this report is as follows:

Chapter 2 first describes the yaw and roll model; After this the non-linear tire model is discussed. Finally, tire relaxation will be described.

In Chapter 3, the simulation results are shown and discussed. First a comparison is made between a tire model without relaxation and with relaxation. Secondly, a constant subtractive angle is discussed. This is then compared to the results of a sinusoidal pulse. Finally, a new, non-sinusoidal shaped pulse is formulated and simulated.

Chapter 4 presents the conclusions based upon the simulations performed. Some further research is also proposed in this chapter.
Chapter 2

Vehicle Models

This chapter will discuss the vehicle model as used in this report. First the yaw model will be discussed, after that the roll model will be dealt with. The third section of this chapter will discuss the non-linear tire model, finally the fourth section discusses the tire relaxation model. The vehicle model considered has seven degrees-of-freedom. These are the yaw velocity \( r \), the lateral velocity \( v \) and the roll angle \( \phi \). Furthermore each of the four tires has one degree of freedom.

2.1 Yaw model

The yaw model describes the vehicle’s yaw and lateral motion. Assumptions made for this model are:

- All angles are small, hence \( \sin(x) \approx x \) and \( \cos(x) \approx 1 \).
- Longitudinal velocity is kept constant. Since all angles are small it is assumed that \( u \approx V \).
- The steering motion of the front wheels will only be about the vertical axis.
- The front wheels are steered with an angle \( \delta_{rf}, \delta_{lf} \) for the right and left front wheel respectively.
- Longitudinal forces are ignored because it is assumed there is no rolling resistance in x-direction. Air drag is also neglected.
- Suspension compliance and free play are ignored.

Figure 2.1 shows the yaw model. In this figure, \( a_{ij} \) represents the side slip angle. The steering angle is represented by \( \delta_{ij} \). Here \( i \) represent the left (l) or right (r) side, \( j \) represents the front (f) or rear (r). The distances \( a, b, T \) represent the distance of the front axle to the center of gravity (COG), distance of the rear axle to the COG and the track with respectively. The force equilibrium in y-direction can be derived as being:

\[
m(\dot{v} + ur) = \sum F_y
\]
The moment equilibrium around the COG is:

$$I_{zz} \dot{r} = a \sum F_{yf} - b \sum F_{yr}$$  \hspace{1cm} (2.2)

From these equations, the lateral velocity and yaw velocity can be derived. The force $F_y$ in the above equations has to be generated by the tires (explained in more detail in chapter 2.3). The steering angles $\delta_{lf}$ and $\delta_{rf}$ are defined by the Ackerman principle:

$$R_c = \frac{a + b}{\tan \delta}$$  \hspace{1cm} (2.3)

$$\delta_{lf} = \arctan \frac{a + b}{R_c + T/2}$$  \hspace{1cm} (2.4)

$$\delta_{rf} = \arctan \frac{a + b}{R_c - T/2}$$  \hspace{1cm} (2.5)

The side slip angles $\alpha_{ij}$ are then defined as:

$$\alpha_{lf} = \delta_{lf} - \frac{1}{u} (v + ar)$$  \hspace{1cm} (2.6)

$$\alpha_{lr} = -\frac{1}{u} (v - br)$$  \hspace{1cm} (2.7)

All vehicle parameters can be found in Appendix B.
2.2 Roll model

The roll model describes the angular motion of the vehicle around the x-axis. The following assumptions have been made:

- The wheels stay on the ground. This is necessary for the model to be valid.
- Road unevenness is not considered.
- The roll axis is parallel to the ground plane and runs down the center of the vehicle; the distance from the wheel contact patch to the roll axis is $T/2$.
- The COG of the unsprung mass is situated in the ground plane.
- The springs and dampers are considered torsional around the roll axis.

Figure 2.2 shows the roll model. Here, $\phi$ represents the roll angle of the vehicle. The roll model consists of two masses; the sprung and unsprung mass. The center of gravity of the unsprung mass is considered to be in parallel with the ground plane and is made up by the weight of the tires and part of the suspension. The sprung mass has an inertia $I_{xx}$ around an axis parallel to the x-axis. Furthermore, $h_r$ represents the height of the COG above the ground plane and $h$ is the height of the COG of the sprung mass from the roll axis. The spring and damper forces are supposed to be torsional around the roll axis. The moment equilibrium about the roll axis is:

$$m_s g h \sin (\phi) - k \phi \dot{\phi} + m_s h \left( \dot{v} + u r \right) = \left( I_{xx} + m_s h^2 \right) \ddot{\phi} $$  \hspace{1cm} (2.8)

Figure 2.2 also shows vertical forces. Each tire has its own specific vertical force, depending on the state of the vehicle. From the force and moment equilibria the vertical forces can be derived:

$$F_{zf} = \frac{b}{2(a+b)} \left( mg + \frac{1}{2T} \left( (h \cos \phi + h_r) m_s a_y + m_s g \sin \phi \right) \right) $$  \hspace{1cm} (2.9)

$$F_{zd} = \frac{a}{2(a+b)} \left( mg + \frac{1}{2T} \left( (h \cos \phi + h_r) m_s a_y + m_s g \sin \phi \right) \right) $$  \hspace{1cm} (2.10)

$$F_{zr} = \frac{b}{2(a+b)} \left( mg - \frac{1}{2T} \left( (h \cos \phi + h_r) m_s a_y + m_s g \sin \phi \right) \right) $$  \hspace{1cm} (2.11)

$$F_{zr} = \frac{a}{2(a+b)} \left( mg - \frac{1}{2T} \left( (h \cos \phi + h_r) m_s a_y + m_s g \sin \phi \right) \right) $$  \hspace{1cm} (2.12)

In the above equations, the mass $m$ is the sum of the sprung mass ($m_s$) and the unsprung mass ($m_u$).

Since the main objective of the pulsed active steering system is to reduce rollover, a parameter to indicate the amount of rollover has been defined [3]:

$$R = \frac{F_{zr} - F_{zd}}{F_{zr} + F_{zd}} = \frac{2m_s}{mT} \left( (h \cos \phi + h_r) \frac{a_y}{g} + h \sin \phi \right) $$  \hspace{1cm} (2.13)

This equation indicates that the roll coefficient is defined as the difference between the vertical force on the right to that of the left, normalized by the sum of the two forces. If this equation goes to 1 or $-1$, the tires on one side of the vehicle loose contact with the road and rollover is likely. Again, all vehicle parameters can be found in Appendix B.
Figure 2.2: Vehicle roll model, seen from the rear.
2.3 Steady state tire model

The tire is the only contact between the road and vehicle, it is therefore important to understand the forces being generated by the tire at various conditions such as different lateral accelerations and vertical loads. Literature presents various models, ranging from simple, empirical, linear models up to highly non-linear physical models [13]. The model that will be used in this report is the Magic Formula (MF) tire model [12]. The MF tire model is a widely used, semi-empirical tire model that calculates the forces and moment characteristics for a tire. The general formula reads:

\[ y = D \sin \left( C \arctan \left( Bx - E (Bx - \arctan Bx) \right) \right) \]  

(2.14)

The force then becomes:

\[ F = y + S_V \]  

(2.15)

\[ x = X + S_H \]  

(2.16)

with \( S_V \) and \( S_H \) being shift factors. The parameters \( B, C, D, E \) are all based on a set of parameters unique for each type of tire. Their exact formulas are explained in Appendix A. Figure 2.3 shows the side force as a function of side slip angle. As can be seen, for small angles (\( \alpha < 2.5^\circ \)) the tire can be treated as a linear system. The stiffness experienced in this region is called the cornering stiffness of the tire. The maneuvers used in this report are outside the linear range. Therefore the full non-linear behavior will be considered.
2.4 Tire relaxation model

A tire has to deform before it can generate side force. This lag in side force and side slip angle is called relaxation length. For small side slip angles, this relaxation length can be taken constant. It has however been shown that for larger side slip angles, the relaxation length diminishes [8]. Figure 2.4 shows the relaxation model used in this report in top view. It is a simplified version of the model proposed by Pacejka et al [11] and is based on the separation of contact patch slip not through the use of relaxation lengths, but by the use of carcass stiffness. Some assumptions for this model are:

- The contact patch is only allowed to translate in y direction with respect to the lower part of the rim, no rotation is allowed.
- The frequency of the wheel motion has to be well below the first natural eigenfrequency. It is suggested by Pacejka to use the model up to a maximum frequency of 15Hz.
- The response to slip of the contact patch is instantaneous, since no deflection has to develop here.
- No tire inertia is considered
- The camber angle ($\gamma$) is zero.
- $V'_y$ is assumed to be equal to $V_y$ since no slip in longitudinal direction is considered.

Together with the non linear tire model as proposed in section 2.3 the model takes care of the diminishing relaxation length and of the vertical load dependent relaxation length. The equation of motion for this model is:

$$m_c V'_y + d_r \dot{v}_r + k_r v_r = F(\alpha', F_z)$$

The force acting from the ground on to the contact patch can be calculated using the steady state formula. The force acting on the wheel center now is:

$$F_y = k_r v_r + d_r \dot{v}_r$$

Here, the displacement $v_r$ can be found by solving:

$$\dot{v}_r = V'_y - V_{sy}$$

Where $V_{sy}$ equals $V_y$ because no camber angle is considered. The side slip angle $\alpha'$ can be found by:

$$\alpha' = -\frac{V'_y}{|V_x|}$$

The stiffness and damping as measured on a standing tire can now be used to calculate the relaxation length. If, however, the typical relaxation length is know, the lateral stiffness can be calculated using:

$$\sigma = \frac{C_{fr}}{k_r}$$
In this report a constant value for the mass and damping will be used as described in Pacejka [12]. The stiffness parameter, however, varies significantly, depending on the type of tire, as was shown by Besselink [4]. A low value of the lateral stiffness gives the longest relaxation length as can be seen in Figure 2.5. Therefore, a low value is chosen. The values of the mass, damping and stiffness can be found in Table 2.1.

Table 2.1: Parameters used for relaxation length model

<table>
<thead>
<tr>
<th>Variable</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m_c$</td>
<td>1</td>
<td>kg</td>
</tr>
<tr>
<td>$d_r$</td>
<td>2000</td>
<td>Ns/m</td>
</tr>
<tr>
<td>$k_r$</td>
<td>140</td>
<td>kN/m</td>
</tr>
</tbody>
</table>
Figure 2.5: Relaxation length as a function of step in side slip angle

(a) Different vertical loads, $k = 140000 \text{ N/m}$

(b) Different lateral stiffness, $F_z = 4500 \text{ N}$
Chapter 3

Simulation Results

In order to get a good insight in the effectiveness of pulsed active steering when considering the a non-linear relaxation model, a suitable input signal must be used. This means that only the dynamic effects resulting from the pulsed steering input should show. Therefore the following input signal is used (Figure 3.1):

- A sinusoidal increase of the steering angle up to $\delta$.
- A constant steering angle over time $t_{relax}$ which ensures all dynamic effects of increasing the steering angle have damped out.
- A subtractive pulsed section with amplitude $\delta_{pulse}$ and time $t_{pulse}$. The length of this pulsed section has to be chosen such that the transient behavior can be observed as well as the steady state behavior to a pulsed steering input.
- A sinusoidal decrease of the steering angle to zero.

This input results in a long J-turn. Its trajectory depends on the forward speed, the maximum steering angle, the relaxation time, the pulse amplitude and the pulse time. For the pulsed section a sinusoidal signal has been chosen, since Vos [14] has shown that this type of input closely matches the pulse form that reduces the roll coefficient the most. He suggests a pulse amplitude of 2/5 of the total steering amplitude.

Since the relaxation length is a function of the side slip angle and vertical load, different parameters, such as the vehicle speed and steering input, have to be varied to clearly see the influence of pulsed active steering on the vehicle roll behavior. When choosing these parameters, care must be taken not to surpass the maximum roll coefficient of 1, since the model is no longer valid above this value. Figure 3.2 shows the maximum allowable steering angle for various speeds for $|R|$ not to surpass 1.

3.1 Tire model comparison

Previous research ([1] and [6]) into PAS has mostly used tire models without a suitable relaxation model. This section will show the difference between the models and show the importance of the use of tire relaxation effects when assessing the effectiveness of PAS. Both tire models will use the Pacejka tire model as a relation between the side slip angle and the
Figure 3.1: Steering input.

Figure 3.2: Maximum steering angle allowed for the roll coefficient not to surpass 1.
side force generated. The simulations shown are made at 80 km/h with a maximum steering angle of 3.1°.

Figure 3.3 shows the roll coefficient of both tire models. It is clearly visible that there is a significant difference between the tire models, since the tire model that does not consider relaxation achieves a much higher roll coefficient. This difference is explained in Figure 3.4 where the side slip angles are shown. The side slip angles achieved with the non relaxing tire model are about 5% larger. The side slip angles also clearly show that the achieved side slip angle variations are smaller when considering the full tire model. This also appears in the roll coefficient. The maximum reduction is much smaller (0.17 for the non relaxation versus 0.11 for the tire model with relaxation). Using a tire model without relaxation may produce unrealistic results.

3.2 Constant subtractive input

As was discussed in the introduction, changing the steering angle is already used as a method of ensuring vehicle stability. This is usually implemented using by changing the steering angle to a smaller value as is visible in Figure 3.5. For ease of implementation this reduced angle will be considered constant. The subtracted angle will be the same as the average value of the sinusoidal shaped subtracted pulse and will from now on be called the constant subtractive input.

Figure 3.5 shows the roll coefficient for a 80 km/h J-turn maneuver with an unpulsed subtractive angle as a reference. The steering angle required to achieve this roll coefficient is 3.1° and the subtracted angle is 0.62°. This subtracted angle corresponds to the average
subtracted angle by a sinusoidal shaped pulsed steering with an amplitude of 2/5. As can be seen the roll coefficient is reduced from 0.80 to approximately 0.70. Taking a closer look at the roll coefficient reveals that it takes approximately 2.5 seconds for the roll coefficient to reduce to its lowest value.

Figure 3.5: Constant subtractive angle at 80 km/h and $\delta = 3.1^\circ$. 
### 3.3 Pulsed subtractive input

Figure 3.6 shows the effectiveness of pulsed active steering versus a constant subtractive input. This effectiveness is determined by:

\[
effectiveness = \frac{R(t_{\text{relax}}) - \text{mean}(R(t_{\text{pulse steady}}))}{R(t_{\text{relax}}) - R(t_{\text{steady}})}
\]  

(3.1)

Or in words, the ratio of the change in average rollover coefficients in the pulsed and unpulsed scenarios. This means that a pulsed system has a larger average reduction if the effectiveness is larger than 1. Obviously a constant subtractive steering angle is more beneficial if the effectiveness is smaller than 1.

As is visible, the effectiveness decreases significantly for higher speeds. At 40 km/h the pulsed active steering is 42% more effective than a constant subtractive steering angle. Its peak value is reached at 2.8 Hz, which is approximately two times the vehicle’s roll eigen frequency. The 80 km/h effectiveness also has its peak value at 2.8 Hz, this effectiveness indicates that the PAS is, on average, still 4.1% better than a constant subtractive angle. The decrease in effectiveness can be explained by the decrease in achieved side slip angle as can be seen in Figure 3.7. This figure also shows that at 120 km/h the side slip angle variation is even smaller. This is also visible in the effectiveness, which is now only 1% better than a constant subtractive angle.

When looking closer at the roll coefficients shown in Appendix C, it is clearly visible that, although the mean roll coefficient achieved by PAS is better, the peak values still surpass the roll coefficient achieved by a constant subtractive input. It is also clear that for 120 km/h the
constant subtractive input has not reached a steady level yet, indicating that the relaxation length is very long at that speed. The times to steady state for the 40 and 80 km/h are approximately 1.1, respectively 2.5 seconds. This time is slightly smaller for the pulsed input as is shown in Figure 3.7, where the side slip angle of a constant input and of a 2.8 Hz sinusoidal input are shown. This shorter time is explained by (2.18). The larger reduction of the extension of the spring \( v_1 \) makes for a larger force reduction. This time is not influenced by frequency.

It is further clear that the pulse frequency of 2.8 Hz does not produce the lowest roll coefficient. Depending on the forward velocity, the lowest roll coefficient is achieved between 1.1 Hz and 1.7 Hz, approximately the vehicle’s eigenfrequency. The fact that the pulse frequency of 2.8 Hz is the most effective frequency but not the frequency that reduces the roll frequency the most can be explained by the fact that a frequency of 2.8 Hz does reduce the roll coefficient the most when looking at the minimum roll coefficient.

A delay between the steering input and the roll coefficient also occurs due to the relaxation of the tire. Figure 3.8 shows that for low frequencies the delay is relatively large (about 1/5 of the frequency), and constant for higher frequencies. As can be expected, applying PAS at 40 km/h gives the largest delay, since tire relaxation is a distance related phenomenon rather than a time related phenomenon. Since the distance traveled per second at 40 km/h is the smallest, the time delay is the largest. When considering the time delay as a distance delay, all three speeds produce the same average delay.
3.4 Non sinusoidal pulse

The previous section showed that tire behavior plays an important role in the effectiveness of pulsed active steering: with increasing speed the effectiveness of PAS decreased. This is mainly caused by the tire relaxation effects. With higher speed, the steering angle required for the vehicle to reach a high roll coefficient becomes smaller. When the specific characteristics of tire relaxation are recalled (Figure 2.5), it becomes clear that the tire relaxation length is much greater at higher speed. To make the PAS more efficient, it is suggested to increase the depth of the pulse. This decreases the slip velocity of the wheel and therefore also decreases the deflection of the tire contact patch (equation 2.19).

Figure 3.9 shows the roll coefficient in detail for a pulse size of $\frac{3}{5}$ and $\frac{4}{5}$ of the steering angle. For ease of implementation a sinusoidal input is chosen. Also shown is the roll coefficient achieved by a constant subtractive angle. As is visible, a larger maximum pulsed angle does not let the roll coefficient drop below the constant subtractive angle (both constant subtractive angles are corrected for the average input). It does however give potential to reduce the roll coefficient further since the drop below the constant subtractive angle is larger. The larger pulse reduces the roll coefficient faster. It takes the $\frac{2}{5}$ steering input approximately 2.2 seconds to decrease it to its steady state value, whereas this is only 1.5 seconds for the $\frac{4}{5}$ input.

The previous section has shown that a large pulsed input does influence the vehicle behavior positively, it however, also comes with some drawbacks such as the larger difference between the minimum and maximum value. The following signal is therefore proposed:
- A large pulse amplitude.
- The same average as a sinusoidal input of 2/5 of the total steering input.
- A base frequency of 1.2 Hz.

Figure 3.10 shows this pulse. It consists of a sinusoidal shaped top part, with a quadratic lower section. Care is taken to ensure that the function is continuous. The average of this new function is kept the same as that of the sinusoidal pulse. Its depth therefore also determines the width. Combinations that give an average pulse of 2.48° are shown in Table 3.1. The first entry of the table is the sinusoid itself, indicating that its width is 0.417 seconds at the middle.

<table>
<thead>
<tr>
<th>Table 3.1: Alternate pulse parameters for a 1.2 Hz pulse</th>
</tr>
</thead>
<tbody>
<tr>
<td>Width [s]</td>
</tr>
<tr>
<td>0.417</td>
</tr>
<tr>
<td>0.4</td>
</tr>
<tr>
<td>0.39</td>
</tr>
<tr>
<td>0.38</td>
</tr>
<tr>
<td>0.37</td>
</tr>
<tr>
<td>0.36</td>
</tr>
<tr>
<td>0.35</td>
</tr>
<tr>
<td>0.34</td>
</tr>
</tbody>
</table>

The results of the non sinusoidal pulse can be found in Figure 3.11. The figure shows that the new pulse is capable of reducing the peak roll coefficient further than the sinusoidal pulse. It is, however, dependent of the width of the quadratic section. A large width, almost equal to the sinusoid reduces the roll coefficient the most. It is also visible that a deeper pulse gives a lower roll coefficient, but, as was shown before also has a larger difference between
the minimum and maximum value. The peak value therefore is higher than the peak value of a less deep peak. This phenomenon is explained by the time difference between the roll angle and lateral acceleration being the smallest as can be seen in Figure 3.12. Since the roll coefficient is a function of both, the closer the peak values are together, the higher the peak value of the roll coefficient is. If this did not happen, a pulse with a width of 0.35 seconds would reduce the roll coefficient more, because its lateral acceleration is the lowest, as is the roll angle.
Figure 3.11: Roll coefficient for the non sinusoidal pulse at 1.2 Hz

(a) Lateral acceleration

(b) Roll angle

Figure 3.12: Lateral acceleration and roll angle for the non-sinusoidal pulse
Although PAS is not capable of reducing the roll coefficient to a value below a constant subtractive pulse, it does provide one clear benefit: PAS can reduce the roll coefficient faster than a constant subtractive pulse. Simulations revealed that the average value of the pulsed input needs to be 8% larger compared to a constant subtractive input for this vehicle. Figure 3.13 shows the roll coefficient achieved by applying such an increase. It also shows that at $t = 8.65$ seconds the pulsed vehicle has reached a steady state, whereas the constant subtractive vehicle’s roll coefficient is still decreasing.
Chapter 4

Conclusions and Recommendations

4.1 Conclusions

The effects of non-linear tire relaxation on the effectiveness of Pulsed Active Steering have been investigated. A three degree of freedom vehicle model is developed: a model with the lateral motion, yaw angle and roll angle is used. The tire model consists of a Magic Formula tire model, describing the relation between the side slip angle and lateral force. It furthermore consists of a contact patch attached by a spring and damper to the rim, which is used to describe the relaxation behavior of the tire. The simulations are performed using a J-turn maneuver. A sinusoidal subtractive input is chosen, the range of frequencies considered is $1 - 12$ Hz. Three speeds are used: 40, 80 and 120 km/h. After the sinusoidal shaped pulse a non sinusoidal input is considered at 80 km/h.

The results from the simulations show that a pulsed input has potential in reducing the vehicle rollover. For every speed the average reduction is greater for PAS than for a constant subtractive input. The effectiveness varies with frequency. For all three velocities the frequency that reduces the average the most seems to be 2.8 Hz. The reduction of the average achieved with PAS is 42%, 4.1% and 1% for 40, 80 and 120 km/h respectively. This reduction of the average roll coefficient compared to the constant subtractive angle is due to the difference in side slip angle.

The simulations also show that the peak values still surpass the roll coefficient achieved by a constant subtractive input. The frequencies that produce the lowest roll coefficient are between 1.1 and 1.7 Hz, which is close to the vehicle’s eigenfrequencies.

Since the sinusoidal pulse is not capable of reducing the roll coefficient below the constant subtractive input, a new pulse is considered. This pulse is deeper than a normal sinusoid, but narrower, resulting in the same average value as a the sinusoid used before. A base frequency of 1.2 Hz is used for this pulse, as this frequency has the lowest roll coefficient at 80 km/h. The new pulse proves to reduce the roll coefficient further than the sinusoidal pulse can but still not below the roll coefficient achieved by a constant subtractive steering angle.

Simulations revealed that PAS can reduce the roll coefficient faster than a constant subtractive angle can. It is therefore investigated what average a pulsed steering angle should have to achieve a lower roll coefficient. This average proves to be 8% larger than a constant reduced steering angle.
4.2 Recommendations

This report has mainly focused on vehicle roll behavior and has not considered its yaw behavior in finding a good pulse. A study into the yaw dynamics of the vehicle could therefore be performed as it is also important for a vehicle to maintain its desired trajectory. It could also be combined with other vehicle stability systems such as yaw moment control to be able to combine trajectory tracking and prevention of vehicle rollover.

In this thesis PAS is used to prevent vehicle rollover. PAS can also be used to increase the steering angle in a case of understeer or decrease the steering angle in case of oversteer. Further research can be performed to see if PAS can be effective in any of these situations.

Although the purpose of PAS is to reduce vehicle rollover in critical situations, research can be performed into the passenger comfort when the PAS is active.

The pulse shape influences the vehicle roll behavior. It is possible that a pulse shape exists that is capable of reducing the roll coefficient more. Further research can be performed into this subject, to find a pulse that reduces the vehicle roll behavior even more than the non sinusoidal pulse that was defined in this report. A more analytical approach can be chosen for this.

Finally, a study into the practical implementation can be performed. A suggestion for this is using a cam based mechanism, making the construction cheaper to implement compared to conventional active steering. The frequency can then be simply adjusted by changing the rotational speed of the cam.
Bibliography


Appendix A

Magic Formula

The magic formula tire model contains various parameters that define the tire properties. For the lateral force these parameters will be further explained in this chapter. First of all some general parameters:

- Adapted nominal load
  \[ F'_{z0} = F_{z0} \lambda_{Fz0} \]  

- Nominal vertical load increment
  \[ df_z = \frac{F_z - F'_{z0}}{F'_{z0}} \]  

All the parameters \( p_i \) and \( \lambda_j \) in the following equations are parameters measured for each type of tire. Their values will be treated in section A.

Shape factor C

The shape factor C defines the limit range of the sine function and thereby the shape of the total function. Its quantity is calculated by:

\[ C_y = p_{Cyl} \lambda_{C_y} \]  

Peak value D

The peak value D determines the height of the peak as is visible in Figure 2.3. It is calculated by:

\[ D_y = \mu_y F_z \]  

here, the friction coefficient \( \mu_y \) is defined as:

\[ \mu_y = (p_{Dy1} + p_{Dy2} df_z) (1 - p_{Dy3} y^2_y) \lambda_{\mu y} \]
Curvature factor $E$

The curvature factor is:

$$E_y = \left( p_{Ey1} + p_{Ey2}d_f \right) \left( 1 - (p_{DE3} + p_{Ey4}) \, \text{sign} \alpha_y \right) \lambda_{Ey} \tag{A.6}$$

Stiffness factor $B$

The stiffness factor $B$ is defined as:

$$B_y = \frac{K_y}{C_y D_y} \tag{A.7}$$

with:

$$K_y = p_{Ky1} F_{z0} \sin \left( 2 \arctan \left( \frac{F_z}{p_{Ky2} F_{z0} \lambda_{Fz0}} \right) \right) \left( 1 - p_{Ky3} |\gamma_y| \right) \lambda_{Fz0} \lambda_{Ky} \tag{A.8}$$

Tire parameters

All MF tire parameters are based on the parameters used by Morency [9]. The parameters that are used in this report will be presented here.

| Table A.1: Parameters of the Magic Formula tire model |
|----------------|----------------|----------------|
| Variable        | Value          | Unit          |
| $\lambda_{Cy1}$ | 1              | -             |
| $\lambda_{\mu y}$ | 1              | -             |
| $\lambda_{\mu v}$ | 1              | -             |
| $\lambda_{Ey}$   | 1              | -             |
| $\lambda_{Ky a}$ | 1              | -             |
| $\gamma$          | 0              | $\text{rad}$ |
| $p_{Cy1}$         | 1.3507         | -             |
| $p_{Dy1}$         | 1.0489         | -             |
| $p_{Dy2}$         | -0.18033       | -             |
| $p_{Dy3}$         | -2.8821        | -             |
| $p_{Ey1}$         | -0.0074722     | -             |
| $p_{Ey2}$         | -0.0063208     | -             |
| $p_{Ey3}$         | -9.9935        | -             |
| $p_{Ey4}$         | -760.14        | -             |
| $p_{Ky1}$         | -21.92         | -             |
| $p_{Ky2}$         | 2.0012         | -             |
| $p_{Ky3}$         | -0.024778      | -             |
| $S_v$             | 0              | -             |
| $S_h$             | 0              | -             |
| $V_0$             | 16.6           | $\text{m/s}$ |
# Appendix B

## Vehicle parameters

<table>
<thead>
<tr>
<th>Variable</th>
<th>Value</th>
<th>Unit</th>
<th>Description</th>
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<td>kg</td>
<td>Sprung mass</td>
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<tr>
<td>( m_u )</td>
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<td>kg</td>
<td>Unsprung mass</td>
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<td>kgm(^2)</td>
<td>Inertia round z-axis</td>
</tr>
<tr>
<td>( I_{xx} )</td>
<td>743</td>
<td>kgm(^2)</td>
<td>Inertia round x-axis</td>
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<td>( a )</td>
<td>1.216</td>
<td>m</td>
<td>Distance of front axle to COG</td>
</tr>
<tr>
<td>( b )</td>
<td>1.506</td>
<td>m</td>
<td>Distance of rear axle to COG</td>
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<td>Roll center height from ground</td>
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<tr>
<td>( h )</td>
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<td>Distance from roll center to COG</td>
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<td>Nm/rad</td>
<td>Roll stiffness</td>
</tr>
<tr>
<td>( d_\phi )</td>
<td>6600</td>
<td>Nms/rad</td>
<td>Roll damping</td>
</tr>
</tbody>
</table>
Appendix C

Figures

Roll coefficients zoom

Figure C.1: Roll coefficient at 40 km/h.
Figure C.2: Roll coefficient at 80 km/h.
Figure C.3: Roll coefficient at 120 km/h.