Control design for disturbance rejection on a HCCI model

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Master's thesis

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Abstract

Today's demand on engine emissions and fuel efficiency are driving the search for other combustion concepts. HCCI (homogeneous charge compression ignition) is a concept with low NO\textsubscript{x} (nitrogen oxide) and PM (particle matter) emissions, while still maintaining high fuel efficiency. Closed-loop control is necessary to operate a HCCI engine.

A first objective of this study is to select the best sensor and actuator for controller design. The second objective is to design a combustion phasing controller for disturbance rejection. In this study, a linear controller is designed to control the combustion phasing of a numerical HCCI model.

First HCCI and the need for control is explained. Then a literature study is presented to investigate the current status on HCCI combustion control. Several sensors, actuators and control strategies are explained. After this, a schematic overview of the papers found on HCCI control is given. The most favorable actuator, based on this literature study, is chosen to be VVA (variable valve actuation).

Next, the principles and implementation of the nonlinear HCCI model is explained. This nonlinear model is recreated from [4]. The results of the implemented model and the model presented in literature are similar, but do not completely coincide. This is due to uncertainty which specific equations and parameters are used in the paper. Nevertheless, the results are useful and the implemented model offers a platform for controller design.

Finally, the model is analyzed and its stability and equilibria are investigated. The most favorable actuator available in the nonlinear model is chosen to be RBL (rebreathing lift), which is a form of VVA. The linear controller is designed using a linearization of the model, after which it is implemented on the nonlinear model. The controller is able to reduce the error induced by disturbances and keep the combustion phasing within the specified boundaries. Furthermore, the controller is robust for the mismatch between the linear design method and the nonlinear model. When operating close to a stability bound, the controller is successful in keeping the system in the stable operating region. The experiments show some actuator saturation, which can be avoided by changing the temperature in the inlet manifold.

Future research needs experimental data to fit the nonlinear model on. This will take away the uncertainty which exact equations and parameters should be used in the model. The model can then be expanded to include engine power, knock and emissions. Finally, other control strategies can be investigated.
Acknowledgements

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I would also like to thank my parents for providing me the means to study and their never-ending unconditional support.

Thanks to my friends, for giving me the opportunity to relax.

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Chapter 1

Introduction

Today there are environmental and energy issues where legislation orders to reduce combustion engine emissions, while still maintaining high fuel efficiency. A concept to accomplish this is HCCI which is short for homogeneous charge compression ignition. This combustion process uses an homogeneous mixture (like spark ignition (SI) engines) which is ignited by compression (like in compression ignition (CI) engines). To prevent the mixture from combusting too viciously, it is highly diluted. HCCI has the advantage of low soot emissions, compared to CI, due to the homogeneous mixture. It has a higher fuel efficiency, compared to SI, due to the high compression ratio and lack of throttle losses. The diluted mixture results in a lower combustion temperature resulting in low NOx emissions.

There are however several downsides to HCCI combustion. First, there is no direct combustion trigger, like in SI or CI engines. This results in the combustion being very sensitive for its initial conditions (pressure, temperature, composition and homogeneity of the mixture). These initial conditions need to be closely controlled to obtain the correct combustion phasing. Secondly, the process is unstable, especially when the engine load is increased. A small disturbance can cause the phasing to drift away, causing misfire, stall or even engine damage. Closed loop control is needed to compensate for disturbances. Thirdly, HCCI has a limited operating region compared to conventional operation. Control can be used to increase the operating range. To summarize, closed loop combustion control is needed to operate an HCCI engine. Using control, the initial conditions can be influenced, instabilities can be avoided and eventually, the operating range can be enlarged.

The goal of this study is to select the best sensor and actuator for controller design. A model will be chosen to simulate the combustion process for which a controller will be developed. The controller is designed for disturbance rejection to guarantee good performance of the HCCI engine, while avoiding actuator saturation and being robust for modeling errors.

First HCCI and the need for control is explained in Chapter 2. Then a literature study is presented to investigate the current status on HCCI combustion control in Chapter 3. Next, in Chapter 4, the principles and implementation of the nonlinear HCCI model is explained. After this, the model is analyzed and its stability and equilibria are investigated in Chapter 5. Finally, a controller is designed of which the simulation results are presented in Chapter 6.
Chapter 2

Homogeneous Charge Compression Ignition

This chapter will give further details on Homogeneous Charge Compression Ignition (HCCI) combustion. The principles of HCCI combustion and the difference with knock will be explained in 2.1 and 2.2 respectively. Limitations on the operating range will be discussed in 2.3 and finally, the need for control is explained in 2.4.

2.1 HCCI

HCCI uses a homogeneous mixture, like in a SI engine, which is compression ignited, like in a CI engine. The mixture will autoignite and burns simultaneously at multiple sites in the cylinder. The absence of fuel-rich zones will greatly reduce the particulate matter (PM) emissions, compared to that of CI engines. To ensure that the combustion does not take place too aggressively, the mixture is diluted. Dilution can be achieved by EGR and/or excess air. This will produce a cold (1500 - 1800 K) combustion, which will lower the NO\textsubscript{x} emissions. However, the CO (carbon monoxide) and HC (hydro carbons) emissions are higher than that of conventional spark ignition (SI) or combustion ignition (CI) engines, due to wall wetting and the lower combustion temperature. The emissions of HC will result in a lower thermal efficiency than that of a CI engine. The absence of throttle losses and the increased compression ratio, compared to that of SI engines, increase the thermal efficiency. The difference between conventional and HCCI combustion is shown in Figure 2.1 as a function of equivalence ratio (\(\phi\)) and combustion temperature (\(T\)). The equivalence ratio is the fuel to air ratio divided by the stoichiometric fuel to air ratio. HCCI assumes the mixture is completely homogeneous. Because the mixture in the cylinder will never be completely homogeneous due to nozzles, crevices, etc., the term PCCI (premixed charge compression ignition) is sometimes also used.

There are several ways to create a homogeneous mixture.

1. The first method is to premix the fuel outside of the combustion chamber using PFI (port fuel injection), like in a spark ignited (SI) engine [13]. The fuel can be injected or atomized [34] in the inlet manifold. The inlet manifold needs to be adapted with respect to the conventional CI engine.

2. The second method is early direct-injection, which is used by for example [48]. The fuel is able to mix during the compression stroke to form a homogeneous mixture. There is no need for alteration of the conventional hardware, however some alternations may be needed on the injectors. The general diesel injectors are designed to inject fuel in the highly compressed mixture. If early-injection is used, the mixture will have a low density and temperature. The fuel will not
evaporate or mix easily and travels a considerable way into the cylinder. This will cause problems with wall impingement and fuel penetration. A different HCCI injector with more holes and different spray profile could be used, but block the way to dual-mode operation. [47] has experimented with a adjustable injector, in order to use different spray profiles for HCCI and CI operation.

3. The third method is late direct-injection (after TDC). Swirl is applied to promote mixing and a homogeneous mixture can be formed just in time to ignite. This method does not have as much problems with wall impingement and fuel penetration as the early injection method. There is however very little time to form a homogeneous mixture. Therefore there is no guarantee that it is indeed homogeneous. Late direct-injection is for example used in the Nissan MK system [43].

2.2 Knock and HCCI

Knock in a conventional combustion engine is a different phenomena than the combustion in a HCCI engine. Knock in a spark ignition engine is spontaneous formation of combustion zones in the combustion mixture. This will produce flame fronts which travel through the mixture. These flame fronts will compress the rest of the mixture, which will also form spontaneous combustion. This will produce pressure waves which travel through the combustion chamber, form high frequency standing waves and produce the “knocking” noise.

During normal HCCI operation, the combustion takes at multiple sites in the combustion chamber simultaneously. The combustion is not actuated by the pressure waves of the flame fronts, but by the piston motion. The pressure difference in the mixture is small and there will be no pressure waves. However, when the mixture is inhomogeneous or when operating at high load or temperatures, an overly rapid combustion can occur. The corresponding high pressure rise rate \( \frac{dp}{d\alpha} \) still can create high frequency pressure pulsations or a combustion front which travels through the cylinder [52]. In order to prevent engine damage, this knock in HCCI combustion needs to be avoided.

2.3 Operating range

A major disadvantage of HCCI combustion is the limited operating range, which is smaller than that of SI and CI engines. There are several factors which limit the operating range of a HCCI engine of
which a schematic graph is shown in Figure 2.2. The limits of HCCI are plotted against the load on the engine and the combustion timing (\(CA_{50}\), the crank angle where 50 percent of the fuel is burnt).

![Figure 2.2: Operation limits for HCCI. [38]](image)

On the x-axis retarding (units TDC) Early and late.

At early timings, with increasing load, the peak pressure and peak pressure rate (\(dp/d\alpha\)) increase. The engine can endure only a maximum pressure and the mechanical limits will bind the maximum load. In order to reduce the mechanical load, the combustion timing can be retarded. This will slow the chemical reactions and reduce peak pressure and peak pressure rate.

Secondly peak temperature will increase with increasing load and will form too much NO\(_x\) emissions. The maximum acceptable NO\(_x\) emission will limit the load.

Thirdly, when the combustion is retarded, the variation in combustion phasing becomes an issue. After TDC, the cylinder moves down, lowering the cylinder pressure. The on-going combustion will suffer from the lower temperature before any heat is released, which leads to more variation in phasing. A large variation may lead to misfire and the engine may stall. A good measure for this variation is the standard deviation of the combustion phasing [38].

It must be noted that here the lower limit is missing. This limit will be discussed next with use of Figure 2.3. Here, the limits are plotted against engine speed in stead of the combustion timing. A lower limit for HCCI operation at low engine speed and load is visible in this figure. At low load and engine speed, there is not enough thermal energy to initiate the auto-ignition of the mixture. This results in increasing cycle-to-cycle variation, which will result in increasing HC and CO emissions [24]. At higher engine speed, the shorter time to combust the mixture may cause partial combustion or misfire.

### 2.3.1 Increasing the operating range

There have been several attempts to increase the operating range of HCCI, of which a selection of methods is listed below. The actuation of the HCCI combustion process will be discussed in Section 3.1, where a lot of these methods will be further explained.

- A controller can have a positive effect on the operating range of the engine (Section 2.4)
- Cooling of the EGR or inlet air [11] or using water injection [39]. Cooling the combustion mixture reduces the rise rate of the pressure (\(dp/d\alpha\)), peak pressure and NO\(_x\) emissions.
• Boosting will have a beneficial effect on $NO_x$ formation [41].

• Operating in conventional CI or SI mode at high loads. This is referred to as dual-mode operation [40].

• The fuel that is used can be altered using an adjustable blend of two fuels [7], so that its autoignition properties fit the operating conditions.

• Changing the compression ratio can reduce the $NO_x$ formation [24].

Most of these methods are not able to enlarge the operating range to the size of conventional SI or CI operation. Therefore dual-mode operation is still regarded a good alternative for the transition towards HCCI operation on the complete operating range [20].

2.4 Controlling HCCI

A driver’s first concern regarding the engine is a correct response on his input on the accelerator. As a result, the main loop to control is the engine output torque, directly related to IMEP or indicated mean effective pressure. By varying for instance the amount of injected fuel, the IMEP can be set.

HCCI combustion is very sensitive to its initial conditions, but at low torque operation ranges (but not too low, see Section 2.3), the system is stable. Apart from optional disturbance rejection on the combustion phasing there is no need for additional feedback control. At medium/high torque operating ranges however, the wall temperature acts as a positive feedback on the combustion timing [13]. When the controller is turned off during high load operation, the combustion phasing may drift away, see Figure 2.4. This will ultimately result in misfire and engine stall when drifting towards late combustion phasing. When drifting towards earlier combustion phasing, pressure limits may be reached (Section 2.3).

Controlling only the IMEP by varying the amount of fuel in higher load regions no longer suffices. Additional control of the combustion phasing is necessary. A HCCI engine is a non-linear system [8] and the two control objectives are coupled; controlling the IMEP also influences the combustion phasing and vice versa [9]. Moreover, heating and cooling of the engine has a large influence on the combustion process [14]. The system is time varying due to transient load and temperature changes introduced by for example starting the engine. By reducing cycle-to-cycle variations, transients can be made possible, which is a very important property of a road-going engine. Actuation and sensing of the combustion process and calculation time between control actions need to be very fast. This eliminates time delay, which otherwise would deteriorate the performance of the controller. Cycle-to-cycle control
also prevents the occurrence of bimodal modes [9]. A bimodal combustion occurs for example when the engine has poor ignition due to too high amounts of EGR. A misfire will be followed by cycle with good combustion, which again will be followed by a cycle with partial combustion or misfire.

Closed-loop control of the engine has an additional advantage on the operating range of HCCI. At first because part of the unstable region of HCCI can be used. Secondly, a smaller deviation of the combustion phasing allows the operating point to be put closer to the operation bounds. Even then, a more reliable operation can be guaranteed. Each cylinder will have a different operating condition due to, for example: wear, machining tolerances, inlet manifold geometry, intake manifold conditions or cylinder cooling. Individual cylinder control ensures the difference between the operation conditions can be accounted for. Part tolerances can be loosened and calibration of the engine in the factory can be reduced to a minimum, which will reduce costs [36]. The power of the cylinders will be better balanced, reducing wear, noise and optimizing efficiency.

The second concern of the driver is the fuel economy of the engine. Controlling the combustion phasing in combination with IMEP can optimize the combustion process. This will lead to improved thermal efficiency, resulting in improved fuel economy.

There are some constraints which need to be taken into account: The government puts constraints on the engine in the form of noise, gas and particle emission regulations. Emissions of $NO_x$, CO, HC and PM depend on the combustion process. Noise emissions result from pressure rise and peak pressure in the engine. Other constraints originate from mechanical limits and saturation of the actuators. The previously mentioned peak pressure and pressure rise also need to be bound to reduce the stress on the engine. These constraints can be included in the controller design.

\[2.5\] Research objective

The performance of an HCCI engine largely depends on the combustion phasing. If the combustion is phased correctly, the engine efficiency and emissions can be optimized, while knock can be avoided. This study focusses on developing a disturbance rejection controller, ensuring the combustion phasing stays closely to its optimal value.

A next step is the stabilization of the combustion at high engine loads and facilitating transient operation. These subjects are outside the scope of this project.
First, a literature study is performed in Chapter 3 to investigate the current work in combustion control research. The different actuators, sensors and controllers are presented and after this, the most favorable actuator is selected.

The controller will be designed for a HCCI model, which is reconstructed from literature. This model is presented and explained in Chapter 4. The model will be analyzed and its most favorable actuator will be selected.

Finally a controller is designed in Chapter 5, using loop shaping. The results are presented in Chapter 6.
Chapter 3

Literature Study on HCCI control

This literature study will provide more detail on the current work done in HCCI control research. First the different actuators and sensors are discussed. After this, the controllers and modelling of the combustion process is explained and at last a brief conclusion is presented.

3.1 Actuators

In C.I. engines the fuel injector starts injecting fuel, which starts combustion. In S.I. engines the spark starts the combustion. There is no method to directly control the ignition of the mixture for HCCI. The mixture is too lean to use spark ignition and, when the initial conditions are set, can only auto-ignite. The initial conditions are therefore of great importance to the HCCI combustion process. The initial conditions can be actuated in several different ways. Overall, the actuators can be grouped by their influence on the combustion mixture: temperature or chemical composition (Table 3.1). It can be seen that some actuators have an influence on both temperature and composition. An attempt to directly actuate the combustion process are for example pulsed flame jet [25] or stratified injection [56] in order to volumetrically ignite the mixture or using glow plugs during operation. Methods like pulsed flame jet or the used of glow plugs are outside the scope of this project and thus will not be treated.

<table>
<thead>
<tr>
<th>Temperature</th>
<th>Chemical composition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fast thermal management</td>
<td>Boosting</td>
</tr>
<tr>
<td>Boosting</td>
<td>Variable valve actuation</td>
</tr>
<tr>
<td>Variable valve actuation</td>
<td>Fuel injection</td>
</tr>
<tr>
<td>Variable compression</td>
<td>Dual fuel</td>
</tr>
</tbody>
</table>

Table 3.1: Different actuators, sorted by influence on the combustion mixture

At first, this Section will deal with the actuators which mainly act on the mixture temperature. After this, actuators which alter the chemical composition of the mixture will be explained.

3.1.1 Temperature

Fast thermal management Modifying the inlet air temperature has a direct influence on the temperature of the combustion mixture, changing the combustion phasing. The inlet air can be heated by the exhaust gas indirectly through a heat exchanger or directly with the use of EGR. An intercooler can be used to cool the inlet air. When cool and hot air are combined in different
inlet channels, a so called “fast thermal management” (FTM) system can be created [22]. The system will switch between the different air flows with a valve. This will reduce the effect of the thermal inertia of the heat exchanger and result in a time constant of 8 engine cycles [22]. This is fast, but still not enough for cycle-to-cycle actuation. Another disadvantage is that, at the startup of the engine, the exhaust gas is cold and the heat exchanger will not produce hot air, needed for the HCCI combustion.

Boosting The inlet air can be boosted using a turbo or supercharger. Boosting will increase the inlet air temperature. The higher pressure in the cylinder at intake valve closing will also add to the temperature. The turbo of a HCCI engine will have different specifications than that of a conventional diesel engine, due to the lower exhaust gas temperatures [7]. Variable vane geometry of the turbo can be used as an actuator for the boost pressure and guarantees a good operation of the turbo at different operating conditions [9]. Boosting is used to manipulate the operating conditions, while combustion phasing control is covered by another actuator.

Variable valve actuation The intake and exhaust valves can also be actuated to control the combustion phasing. Closing the intake valve early in the induction stroke reduces the amount of intake air and therefore reduces the pressure and temperature at the end of the compression stroke. Late closing (after BDC) will have a same effect. Lowering the temperature and pressure at the end of compression will delay the combustion. During normal operation, the exhaust valve will not be closed before the intake valve opens. This is called a valve overlap and can be used as an actuator. Valve overlap can be changed by either changing IVO (intake valve opening) or EVC (exhaust valve closing). Both having the same effect to the ratio of inducted fresh reactants and exhaust gas. The temperature of the combustion mixture depends on this ratio. The reinducted exhaust gas is generally referred to as i-EGR, internal EGR or RGF (residual gas fraction). Another technique used to apply i-EGR is exhaust rebreathing, where the exhaust valve is opened again during the intake phase [53]. A downside of fully cylinder individual variable valve actuation is that these systems are expensive. A variety of less complicated flexible valve trains have been developed. Here the camshaft is still used, but the valves can be actuated (electro-mechanical, electro-hydraulic or electro-magnetic) whether or not to skip certain parts of the cam profile. This allows for cylinder independent actuation, but valve opening timings are not fully flexible and these systems are still costly.

Variable compression Another way to change the compression ratio is to change the swept volume of the pistons [24]. This can be accomplished by tilting the cylinder block relative to the crankshaft. By changing the distance between the crankshaft and the cylinder head, the swept volume is changed. In this case, the compression ratio of all the cylinders are changed simultaneously. Not all the cylinders can be actuated independently, so there is still another actuator needed to control individual cylinder combustion phasing. These systems are expensive and complicated as well. Response time to a step change of 1 on the compression ratio is reported to be 3 cycles (180 ms) [42].

3.1.2 Composition

Variable valve actuation Next to changing the temperature in the cylinder variable valve actuation influence the chemical composition as well. The moment of IVO determines the moment the fresh reactants and exhaust gas begin to mix, which will influence the chemical kinetics of the combustion [6].

Fuel injection When using direct injection, fuel injection timing (early, late, split injection) has an impact on the composition and homogeneity of the mixture [20]. The fuel injector itself also has an influence on the mixture [47].

Dual fuel A dual fuel solution mixes easily self-ignited n-heptane and more self-ignition resistant isoctane [7]. This method can be used to alter the octane number of the fuel, which is of great
importance to the ignition delay. The fuel is sprayed into the intake air stream very close to the valves. Fuel additives like DME (dimethyl ether) or a fuel reformer can be used to change the auto-ignition properties of the fuel as well [15].

3.2 Sensors

A sensor is needed to feedback information about the combustion. An option is to put this sensor directly in the combustion chamber. That sensor has to withstand harsh conditions including: heat shock, pressure shock and the influence of chemicals and radicals. Another option is to place the sensor outside the combustion chamber. These sensors have a far less stringent demands on durability, but the larger distance between the sensor and the combustion process deteriorates the signal to noise ratio.

There are a variety of sensors that can be used to detect different parameters of the combustion process. These can be grouped into in-cylinder and outside the cylinder placement (Table 3.2).

<table>
<thead>
<tr>
<th>In Cylinder</th>
<th>Outside the cylinder</th>
</tr>
</thead>
<tbody>
<tr>
<td>Piezo-electric pressure transducer</td>
<td>Sound sensor</td>
</tr>
<tr>
<td>Optical pressure transducer</td>
<td>Piezo-electric element</td>
</tr>
<tr>
<td>Ion-current sensor</td>
<td>Knock sensors</td>
</tr>
<tr>
<td>Optical combustion sensor</td>
<td>Engine speed</td>
</tr>
</tbody>
</table>

Table 3.2: Sensors sorted by in- and outside cylinder position

**Piezo-electric pressure transducer** is the most widely used pressure sensor. It uses a diaphragm in direct contact with the combustion chamber. The deflection of this diaphragm is measured using a piezo-electric element. This sensor type is only able to measure relative changes in pressure. Nevertheless an offset can be estimated to measure absolute pressure. They however are the most expensive sensors and have a limited life span, resulting in the search for a cheaper solution [9].

**Optical pressure transducer** uses a diaphragm like the piezo-electric sensors, but the deflection is measured using an optical interferometer [49]. This type of sensor is less accurate than the best piezo-electric transducers [35]. An advantage is the lower magnetic interference on the sensor signal.

**Ion-current sensor** consists of two electrodes in the combustion chamber over which a DC voltage is applied. The radicals in the cylinder during combustion will produce a small current. This current is used to observe the combustion process. These sensors give only local information and the signal is dependent on fuel properties and operating conditions [18]. Nevertheless, they are very cheap and when a SI engine is used, no engine modifications are needed.

**Optical combustion sensor** makes use of a phototransistor to observe the optical intensity of the chemical reaction in the engine [45]. They can withstand high temperatures and just like the optical pressure transducers, are less susceptible to magnetic interference. Depending on their design, they have a limited view angle.

**Sound sensor** are used in knock detection on conventional engines and could be used to detect the combustion phasing. At this moment however these sensors have problem with combustion detection in the low load region.

**Piezo-electric elements** can be used to attach an engine component like an injector, spark plug or cylinder head. The washer is then able to measure the force on the engine component, which
is related to the cylinder pressure [35]. The sensor is placed outside the combustion chamber, making the durability demands less stringent. The distance from the combustion chamber deteriorates the signal to noise ratio, which means they are less accurate than the pressure transducer sensors.

**Knock sensors** will detect vibrations in the engine. The filtered signal is normally used to detect knock or misfire in a conventional engine. It can also be used to estimate the pressure trace of the combustion [50]. The sensor is placed a distance away from the combustion chamber, making this cheap solution less accurate than the transducer sensors.

**Engine speed** is measured to determine the combustion phasing. It can be used for misfire detection, but the signal to noise ratio is low due to flexibility of the drive train and influences from bumpy roads [17].

### 3.3 Control parameter

The signal from the sensor (pressure, ion current, etc.) needs to be interpreted to determine the combustion phasing. An objective measure is needed, so there is a parameter which we can control. The parameter will be expressed in degrees crank angle, making it independent of the engine speed. Various different parameters have been developed, the ones most used are presented below. They are based on two different principles; heat release analysis and pressure analysis.

#### 3.3.1 Heat release analysis

A heat release analysis uses the pressure data to calculate the amount of fuel burnt (Appendix E). The heat release can be determined in several degrees of accuracy.

**Full/Gross heat release, \( \alpha_{50} \)** The point where 50 percent of the energy is released is calculated using a full heat release analysis. The different parameters in the heat release calculation need to be calibrated and tuned on the engine. The heat release can only be performed when all the pressure data is available, after the expansion stroke finishes. It is the most accurate method and has good noise suppression, but it is the most computational intensive.

**Net heat release, \( \alpha_{50}^{net} \)** The pressure samples are integrated to find the point where 50 percent of the energy is released. Not all of the heat transfer is taken into account and it has a fixed average polytropic coefficient. Since the slope of the heat release is steep, the error is small compared to the full heat release. The heat release data is available during the combustion stroke and \( \alpha \) is available immediately after the expansion stroke finishes. This method is faster and computational less intensive than a full heat release, though less accurate.

**Maximum Heat Release, \( \alpha_{max}^{dQ} \)** This method uses a heat release analysis, \( \alpha \) is chosen to be the point where the maximum heat is released. This makes it more susceptible to noise, because the slope of the heat release is flat at the top. A small error in the calculation of the heat release will amount to a greater error in the combustion phasing. \( \alpha \) is available as soon as the maximum heat release has taken place, which makes it even faster than the net heat release.

#### 3.3.2 Pressure analysis

Then there are methods based on pressure measurement. They do not calculate a heat release signal, but use the raw pressure data to determine the combustion phasing. As a result, the absence of averaging makes these methods very susceptible to noise. On the other hand, only relative pressure is needed and due to the absence of a heat release calculation, computational effort is greatly reduced [32].
Maximum pressure, $\alpha_{p_{\text{max}}}$: Maximum pressure can be used as a measure for combustion phasing, but does not precisely coincide with 50 percent energy released. When there is early or late combustion, the motored pressure peak is the global maximum. With combustion around TDC, the estimation of the maximum pressure is hard. Therefore, maximum pressure is not a reliable indicator of the combustion phasing.

Maximum pressure increase by combustion $\alpha_{p,c_{\text{max}}}$: This method deducts the motored pressure from the pressure signal. This makes it easier to find the maximum pressure and makes this method more reliable. Still, a disadvantage is that the maximum pressure does not precisely coincide with the point where 50 percent heat is released.

Pressure increase due to combustion, $\alpha_{p,c_{50}}^v$: Next to deducting the motored pressure from the pressure signal, the volume change of the cylinder is also accounted for. It is called $\alpha_{p,c_{50}}$ when it does not. In stead of using the maximum pressure, this method uses the point where the pressure increase has reached half its maximum. As a result, the point where 50 percent heat is released is found more accurately than when using the maximum pressure method.

Rassweiler and Withrow method for 50 percent mass fraction burned $\alpha_{MFB_{50}}$: This parameter uses the pressure change due to combustion, so the pressure signal with the deduction of a polytropic compression pressure. This pressure change is proportional to the mass fraction burned. This method is computational less intensive than a heat release analysis, but still needs pressure measurement data from a complete combustion cycle.

When an ion-current sensor is used, half of the rising flank of the ion current signal is defined as $\alpha_{50}^{\text{ion}}$.

### 3.4 Controllers

The actuator and sensor will be linked with a controller. This paragraph will explain the controllers which are commonly used in research. An overview of the research on HCCI combustion control is given in the next Section. A lot of research is concentrated on the "simple" SISO PID controller [9]. The combustion process is nonlinear, therefore different controllers are needed for the different operating points. This controller can be tuned manually using gain scheduling [37] or an extremum seeking algorithm [29]. Both methods work better than one regular PID, but the nonlinearity of the system still results in the controller having sub-optimal gains. PID has reasonable results regarding stability of the combustion phasing in a constant operating point. A feedforward controller can be applied to lighten the load on the PID controller during for example transients.

An linear quadratic Gaussian (LQG) or linear quadratic regulator (LQR) algorithm is an alternative. These controllers need a model of the system to calculate a control input which will minimize a cost-function. This is a problem, since a good model of a HCCI engine is hard to create, see Chapter 4 for more information. The better the model, the better the performance of the controller will be. The states of the system are sometimes hard to obtain, since not every state can be observed by sensors. Kalmann filters are used to reconstruct the states, making it possible to implement this controller [21].

The second alternative is a model predictive controller (MPC). This controller needs a model of the system as well. With this, the controller predicts the future output of the system and determines an optimal control input over several control steps using a optimization routine. Usually only the first control input is fed into the system. After this, the process starts again and another optimal set of inputs is determined. This controller can deal with constraints in the system, being very useful for example when meeting environment emission restrictions or actuator saturation. The optimization routine is a computational costly action. This is why these controllers are used widely in the chemical industry, where the time constants are large and computation time can be long. With the coming of faster computers, MPC nowadays is used more and more in mechanical control. The upper level of control is then performed by the MPC controller. Low level control is done by PID controllers. MPC can be used in a MIMO system, this will however increase the computation time. If the model of
the system is kept allowably small, the computation time can be reduced, so cycle-to-cycle control is possible.

### 3.5 Current work on HCCI control

HCCI is undergoing a lot of research in the last couple of years. This Section investigates the work done in the area of control. From different papers the controller, control parameters, sensors and actuators have been extracted and put together in table 3.3. This table will give a quick overview of were today’s research is headed. In closed-loop control, two different sensors are used, piezo electric pressure transducers and ion current sensors. The columns under these sensors are divided in the different control strategies used in combination with these sensors. The rows consist of the different actuators or actuator combinations used.

The abbreviations used for the actuators:

- FTM Fast thermal management
- DF Dual Fuel
- VVT Variable valve technology
- i-EGR Internal exhaust gas recirculation
- IVC Intake valve closing
- FFR Fuel flow rate
- VCR Variable compression ratio

Most of the projects use a heat release analysis to determine the point where 50 percent of the heat is released. A heat release analysis will be indicated with the superscript $^h$. A superscripted $^p$ indicates peak pressure is used as a combustion parameter. A Rassweiler and Withrow method will be indicated with the superscript $^r$.

<table>
<thead>
<tr>
<th>Actuators</th>
<th>In-Cylinder Pressure Transducer</th>
<th>Ion Current Sensor</th>
</tr>
</thead>
<tbody>
<tr>
<td>FTM</td>
<td>PI(D)</td>
<td>LQG</td>
</tr>
<tr>
<td>DF &amp; VVT (IVC)</td>
<td>[33]$^h$</td>
<td>[33]$^h$</td>
</tr>
<tr>
<td>VVT(i-EGR &amp; IVC)</td>
<td>[26]$^h$</td>
<td>[8]$^p$</td>
</tr>
<tr>
<td>VCR &amp; FTM</td>
<td>[22]$^h$</td>
<td>[28]$^h$</td>
</tr>
</tbody>
</table>

Table 3.3: Overview of HCCI control research. Combustion parameters are indicated using superscripts. Heat release analysis is indicated with the superscript $^h$. A superscripted $^p$ indicates peak pressure. A Rassweiler and Withrow method is indicated with the superscript $^r$.

What can immediately be noticed is that a lot of papers are more or less heading in different directions. There is no clear main point of interest, apart from the use of only two different sensors. Various different actuators and actuator combinations are being used. There is more interest in research using a pressure transducer as a sensor, but a lot of research (from Lund university) also takes the ion-current sensor into consideration. Different control strategies are being used, although Model Predictive control and PID control are most common.
3.6 Selection of actuators and sensors

Of all the actuators presented in literature, dual fuel and valve timing seem to be the subject most studied. Both actuators are fast enough to actuate the combustion on a cycle-to-cycle basis. A VVA system is more complicated and expensive than a dual fuel solution, but VVA has the advantage that only an adaptation of the engine is required. No change in infrastructure or use of a reformer is needed and a single fuel can be used. Therefore a VVA system is chosen to be the best actuator.

Variable compression ratio and fast thermal management systems are not fast enough to actuate on a cycle-to-cycle basis. Furthermore, when using VCR, there is still need of another actuator for cylinder individual control. Fast thermal management has the disadvantage that no thermal energy is available when the engine is started, so dual mode operation or an additional heater is needed.

The sensor most widely used is the piezoelectric pressure transducer. These sensors are expensive and have a limited life span. At the moment this sensor is the best option. These sensors however need to be made cheaper before they can be implemented on a road going engine. Ion current has a good potential as a combustion sensor and control results look promising. Other sensors are found in literature ([35] [49] [45] [50] [17]), but none of these is reported to be used in closed-loop control studies.
Chapter 4

Modeling of the HCCI combustion process

There are two types of models describing the HCCI combustion process. The first one being models based on system identification, basically a black box model. These models are used for control purposes and are kept fast and simple for online control. With an additional noise signal added to the input signal, the behavior of the system can be obtained. The system identification can be done in several ways: for example multi-variable output-error state space (MOESP) [9], [12], [21], [30] and auto regressive moving average with exogenous input (ARMAX) [16]. These models are mainly linear and are operating point dependant, which implies averaging or model switching among different operating points. In [9] input-output linearization is used in order to use a linear model with a better agreement with the real system.

The second type of models are based on a combustion model. These models can be computational intensive models regarding all chemical species in the mixture during combustion which can also account for fluid motion, multizone combustion models [23]. These numeric models are slow and not suitable for control purposes. Reducing these models will reduce computation time, sometimes without losing the required accuracy. Zero dimensional models assume the mixture burns homogeneous and simultaneous [13]. The cylinder flow may be omitted. Even the intake flow can be averaged over the combustion cycle in order to create a mean value model [2]. Some models even leave all the flow out of the model [31]. This created models using only the most relevant chemical species, which are much faster. Then there are models which use knowledge of the processes in the engine and use physically motivated system identification, so called “grey” box methods [10].

In order to understand and investigate the HCCI combustion process, a model is needed. It will also be used to design the combustion phasing controller. A black box model, which is often presented in literature, cannot be used. A lot of measurement data is needed to fit these models, something which was not available. A finite element combustion model is complex and would require much computational effort. Therefore a trade off grey box model is used. From all the models used in literature, the model of [2] seems to be the best documented. Therefore, this model is used for controller design in the remainder of this report. In Section 4.1 the model will be further introduced. In Sections 4.2, 4.3 and 4.5 a detailed description of the model is given. Section (4.6) will validate the implementation of this model.

4.1 The Mean Value Model

The model from [2] is fitted and validated on a single-cylinder Otto engine, with gasoline as fuel. It is a light-duty engine with 0.55L displacement and a compression ratio of 14. The experiments where
conducted at 1000 RPM and different fueling levels. The engine uses port fuel injection for premixed HCCI combustion. For more information on the experimental setup, fitting and validation, see [2]. A schematic overview of the engine is depicted in Figure 4.1. The air in the intake manifold is kept at 363 K, by a plenum heater. The engine uses internal EGR, i.e. the exhaust gases are fed back into the cylinder through the exhaust valve. This is done by opening the exhaust valve again during the intake phase. The exhaust runner is cooled, to limit the temperature of the rebreathed gas. The model does not account for engine knock or emissions like for example NOx, CO, HC or PM.

Figure 4.1: Manifold overview

The model is first introduced in papers [2] and [3]. It is further analyzed and applied in [4] and [5]. The models in these papers have a different number of states and equations, but are all derived from the model in [2]. This report tries to recreate the model discussed in [4]. This paper was chosen because of the stability study presented. This study would be helpful for system analysis in Section 5.1. Because the paper does not contain all parameters and equations needed to recreate the model, information is also obtained from [2] and [5]. Sometimes it is not clear which exact equation the author used; mail correspondence with the author clarified some of the questions. If a different equation than expected is used, it will be explained during the detailed description of the model. An overview of the differences between the models can be found in Appendices A and B. Another important thing is that, according to a mail from the author, the test engines used in [4] and [5] are different. The exact differences where however not mentioned.

A physically based parameterization of HCCI behavior is used to create a "grey" box model. It uses equations derived from conservation laws (mass, energy) and equations which are fitted and validated on experimental data. The cycle-averaged cylinder flows are combined with an Arrhenius rate integral to describe the combustion process. The model is separated into a continuous and discrete part (Figure 4.2). Three continuous differential equations are used to describe the manifold filling dynamics ($\dot{p}_1$, $\dot{m}_2$, $\dot{T}_2$). Three equations are used to describe the blowdown temperature and the flow from the cylinder ($W_{c2}(t+\tau), T_{bd}(t+\tau), T_{ad}(t+\tau)$). These are discrete difference equations and will be simulated using sample and zero order hold to account for the cycle delay of the engine. Engine speed determines the cycle delay, but is not depicted in the scheme, for clarity.

The conditions of the gas (pressure, temperature, mass) in the several volumes of the model are addressed by a subscript of a number or character (Figure 4.1). Ambient, intake manifold and outlet manifold conditions will have the subscript 0, 1 and 2 respectively. The in-cylinder conditions are subscripted by the character c. Flows are addressed by a subscript of two volumes: the first denotes the volume from which the flow is originated, followed by the volume where the flow is heading. For example $W_{c2}$ addresses the flow from the cylinder to the outlet manifold. In Appendix A the values of the different parameters are presented. Other subscripts will be discussed later on in this Chapter.
4.2 Intake manifold

The ideal gas law combined with the conservation of mass results in a differential equation which describes the pressure in the inlet manifold.

\[ \dot{p_1} = \frac{RT_1}{V_1} (W_{01} - W_{1c}) \]  

The pressure in the inlet manifold is the only state which is needed to describe the dynamics in the manifold. This is because the temperature in the intake manifold, \(T_1\), is kept at 363K by a plenum heater. And thus all dynamics are captured by using one continuous state. \(R\) is the gas constant and \(V_1\) is the volume of the intake manifold. The flow \(W_{01}\) is determined using the orifice flow equation in Appendix C from [1]:

\[ W_{01} = C_d A_{01} \sqrt{2(p_0 - p_1) \rho_0} \]  

Where \(C_d\) is the discharge coefficient and \(A_{01}\) the area of the opening between ambient conditions and the intake volume. These parameters will be lumped to form the orifice effective area of the inlet manifold \(C_d A_{01}\). \(\rho_0\) is the density of air in the inlet manifold. Due to the large manifold opening, the flow can be described using an incompressible flow function.

4.3 Cylinder filling

\(W_{1c}\) is the forced flow from intake manifold to cylinder due to cylinder pumping. Mail correspondence with the author of [4] pointed out that the equation from [5] and not [2] is used. \(T_{er}\) (exhaust runner temperature) is replaced by \(T_{rbl}\) (rebreathed gas temperature), because of the different heat transfer in the exhaust used in [4].

\[ W_{1c} \approx \frac{p_1 V_{BDC}}{RT_1} - \frac{T_{rbl}}{T_1} W_{2c} \]
Where the part before the minus sign is the total flow needed to fill the volume of the cylinder at bottom dead center, \(V_{BDC}\) with gas of temperature \(T_1\). \(\tau\) is the time it takes the cylinder to complete two revolutions or one cycle (\(\tau = \frac{360}{N}\)). Here \(N\) is the engine speed in rpm. The part after the minus sign is the flow from the exhaust manifold to the cylinder, corrected with the difference in temperature between the inlet \(T_1\) and rebreathing \(T_{rbl}\) flow. There is no correction for gas remaining in the cylinder, every cycle is assumed to have the maximum theoretical gas exchange.

\[
W_{2c} = \frac{1}{\tau T_{rbl}}(\kappa_0 + \kappa_1 \frac{p_1}{p_2}) RBL^\alpha
\]  

This equation again originates from [5] in stead of [2], where \(T_{er}\) is replaced by \(T_{rbl}\). This flow is fitted on the experimental data, where \(\kappa_1\), \(\kappa_2\) and \(\alpha\) are the fitted parameters. RBL is the amount of rebreathing lift in mm. The timing of the rebreathing is always the same [2], therefore there are no timing effects in this model. The temperature of the rebreathing flow is approximated by the temperature in the exhaust manifold plus a term, \(\Delta T_w\), accounting for the heat transfer between the exhaust gas and the exhaust runner when the gas moves back towards the cylinder.

\[
T_{rbl} = T_2 + \Delta T_w
\]  

The pressure in the cylinder after intake valve closing is approximated using a linear function. \(\beta_0\) and \(\beta_1\) are fitted on experimental data in [2].

\[
p_{ivc} = \beta_0 + \beta_1 p_1
\]  

The temperature in the cylinder just after intake valve closing is \(T_{ivc}\):

\[
T_{ivc} = (1 - x_r) T_1 + x_r T_{rbl}
\]  

\[
x_r = \frac{W_{2c} \tau}{m_c}
\]  

\[
m_c = \frac{p_{ivc} V_{ivc}}{RT_{ivc}}
\]  

Here \(T_{ivc}\) is taken from [2] \((T_{RBL} = T_{er})\), since the article [4] assumes steady state. \(T_{ivc}\) depends on the residual gas fraction, \(x_r\), and the temperature of the rebreathed gas, \(T_{rbl}\). The flow \(W_{2c}\) prescribes the residual gas fraction. When the temperature and pressure at intake valve closing are known, the mass in the cylinder \(m_c\) can be determined using the ideal gas law. Using these equations, \(T_{ivc}\) can be rewritten as:

\[
T_{ivc} = \frac{T_1}{1 + \frac{W_{2c} \tau R T_{ivc}}{p_{ivc} V_{ivc}} - \frac{W_{2c} \tau R T_{rbl}}{p_{ivc} V_{ivc}}}
\]

### 4.4 Combustion

The combustion process is described in [2] by 5 phases:

1. **Polytropic compression** The combustion mixture is compressed polytropically until the start of combustion (SOC) by autoignition. An Arrhenius integral is used to determine the start of combustion (SOC angle (1 % heat released) \(\theta_{SOC}\). The Arrhenius equation \(RR\) is the pressure and temperature dependance of the chemical reaction rate.

\[
RR = A p_0^n \exp\left(\frac{-E_a}{RT_c}\right)
\]
A is an Arrhenius scaling constant. \( E_a \) is the Arrhenius activation energy. \( n \) indicates the reactions sensitivity to pressure. \( T_c \) and \( p_c \) are the current temperature and pressure in the cylinder. When this equation is integrated a value corresponding to the activation energy is obtained. The compression factor of crank angle \( x \) is described as a function of crank angle \( y \) (4.12). The volume at crank angle \( x \) is divided by the current volume at crank angle \( y \). Summarizing, \( v_{ivc} \) is a scaled compression factor of intake valve closing at crank angle \( \theta \).

\[ v_x(\theta_y) = \frac{V_c(\theta_x)}{V_c(\theta_y)} \quad (4.12) \]

This factor helps to rewrite the temperature and pressure in Equation 4.11 as a function of crank angle (\( \theta \)).

\[ T_c = T_{ivc}v_{ivc}^{n_c - 1}(\theta) \quad (4.13) \]
\[ p_c = p_{ivc}v_{ivc}^{n_c}(\theta) \quad (4.14) \]

Here \( n_c \) is the polytropic coefficient during compression. Now the Arrhenius integral is calculated for each crank angle degree step \( d\theta \) from \( \theta_{ivc} \). When this function equals one, the activation energy is reached, indicating SOC.

\[ \int_{\theta_{ivc}}^{\theta_{SOC}} A p_{ivc}^{n_c} v_{ivc}^{n_c}(\theta) \exp(-\frac{E_a v_{ivc}^{1-n_c}(\theta)}{RT_{ivc}}) d\theta = 1 \quad (4.15) \]

The constants \( A \), \( E_a \), \( n \) and \( n_c \) are fitted and dependant on the engine and fuel type used.

2. **Combustion duration**

   The combustion process will take a certain amount of time. The crank angle degree where 90 percent of the heat is released follows from the SOC angle plus the combustion duration \( \Delta \theta \). \( CA_{50} \) is the crank angle where 50 percent of the heat is released.

\[ \theta_c = \theta_{SOC} + \Delta \theta CA_{50} = \theta_{SOC} + 0.55 \Delta \theta \quad (4.16) \]

The combustion duration is a function of the mean temperature \( T_m \) during combustion and the temperature at SOC, \( T_{SOC} \). \( \Delta T \) represents the instantaneous temperature rise at the end of combustion.

\[ \Delta \theta = k(T_{SOC})^{-2/3}(T_m)^{1/3} \exp\left(\frac{E_c}{3R_u T_m}\right) \quad (4.17) \]
\[ T_m = T_{SOC} + e \Delta T \quad (4.18) \]
\[ \Delta T = \frac{Q_{LHV} m_f}{c_v m_c} \quad (4.19) \]
\[ e = a_0 + a_1 k \quad (4.20) \]
\[ k = b_{k0} + b_{k1} \theta_{SOC} + b_{k2} \theta_{SOC}^2 \quad (4.21) \]

\( E_c \) is the activation energy and \( R_u \) the universal gas constant. \( k \) is a correction factor on \( \Delta \theta \). \( e \) represents an averaging of the released thermal energy during combustion. \( k \) and \( e \) are fitted parameters [2] and are used to include the effects of combustion efficiency and heat loss. \( Q_{LHV} \) is the lower heating value of the fuel, where \( m_f \) is the mass of the fuel included in each cycle.

3. **Instantaneous Heat release**

   The heat release is assumed to be instantaneous after the combustion process is finished. The temperature and pressure before combustion (bc) can be calculated using:

\[ T_{bc} = T_{ivc}v_{ivc}^{n_c - 1}(\theta_c) \quad (4.22) \]
\[ p_{bc} = p_{ivc}v_{ivc}^{n_c} \quad (4.23) \]
The temperature rise due to heat release is $\Delta T$ and can be added to find the temperature after combustion ($T_{ac}$).

$$T_{ac} = T_{bc} + \Delta T$$

$$p_{ac} = p_{bc}T_{ac}/T_{bc}$$

4. Polytropic expansion There is polytropic expansion (polytropic coefficient $n_e$) of the combusted mixture until the exhaust valve is opened.

$$T_{evo} = T_{ac}v_c^{(n_e - 1)}(\theta_{evo})$$

$$p_{evo} = p_{ac}v_c^{n_e}(\theta_{evo})$$

$n_e$ is the polytropic coefficient during expansion.

5. Adiabatic blowdown Adiabatic blowdown of the combusted mixture when the exhaust valve is opened.

$$T_{bd} = T_{evo} \left( \frac{p_{evo}}{p_{bd}} \right)^{(n_e - 1)/n_e}$$

The blowdown temperature is delayed to account for the cycle delay of the engine.

$$T_{bd}^d (t + \tau) = T_{bd}(t)$$

The flow from the cylinder to the exhaust manifold is the delayed combined flow from the intake manifold, the fuel and the rebreathed flow. It is assumed there are no residuals left in the cylinder.

$$W_{e2}(t + \tau) = W_{ic}(t) + W_f(t) + W_{2c}(t)$$

$W_f$ is the fuel flow rate and is determined by the mass of fuel injected per cycle divided by the engine cycle period $W_f = \frac{m_f}{\tau}$.

4.5 Exhaust manifold

The dynamics of gases in the exhaust manifold is captured by two differential equations:

$$\dot{m}_2 = W_{e2} - W_{20} - W_{2c}$$

$$\dot{T}_2 = \frac{1}{c_vm_2} [c_pW_{e2}T_{bd}^d - (c_vW_{e2} + R(W_{2c} + W_{20}))T_2 - Ah_{gw}(T_2 - T_w)]$$

Where $\dot{m}_2$ follows from the conservation of mass law. $T_2$ is derived from the conservation of energy and using an ideal gas [57].

$$\dot{E}_i - \dot{E}_{out} = \Delta \dot{E}_{system}$$

No work is performed in the outlet manifold and the kinetic energy change is considered negligible. The internal energy change $\dot{U}$ is equal to the heat transfer $\dot{Q}$ plus the energy transfer by mass $\Sigma \dot{m}h$.

$$\dot{Q} + \Sigma \dot{m}h = \dot{U}Ah_{gw}(T_2 - T_w) + W_{e2}c_pT_{bd}^d - W_{2c}c_pT_2 - W_{20}c_pT_2 = m_2c_v\dot{T}_2$$

The pressure in the manifold is calculated using the ideal gas law:
\[ p_2 = \frac{Rn_2 T_2}{V_2} \]  \hspace{1cm} (4.35)

The flow \( W_{20} \) is calculated using the orifice flow equations (Appendix C in [1]).

\[ W_{20} = C_d A_{20} \sqrt{2(p_2 - p_0)\rho_2} \]  \hspace{1cm} (4.36)

Where \( C_d \) is the discharge coefficient and \( A_{20} \) the area of the opening between the exhaust manifold and the ambient conditions. These parameters will be lumped to form the orifice effective area of the exhaust manifold \( C_d A_{20} \). \( \rho_2 \) is the density of air in the exhaust manifold.

### 4.6 Validating the implementation

To verify the implemented model, its results are compared to that of the original model. Figure 4.3 shows the results of the original model, presented in [4], and the results of the implemented model. The results for mass \( m_c \) are smaller in the implemented model. It is unclear how this difference arises. It could be that the parameters used for \( V_{ivc} \) and the determination of \( p_{ivc} \) are still different from the ones used in [4]. \( \theta_{SOC} \) is much earlier in the implemented model. If the results are shifted along the temperature \( T_{ivc} \) axis, they probably would coincide. \( \theta_{SOC} \) is calculated using the Arrhenius rate integral. Deviations may result from differences between the parameters \( \theta_{ivc}, A, E_a, n_c \), or \( n \).

The combustion duration \( \Delta \theta \) shows even more deviation, although the general trend seems to match. Most of the deviation is due to the incorrect \( \theta_{SOC} \), but again unequal parameters could be blamed. The curve of \( \Delta \theta \) is nonmonotonic, a thing which is unexpected and still cannot be explained [4]. Results for the blowdown temperature \( T_{bd} \) differ as well, but again the general trend fits.

It is impossible to check which exact model was used in [4]. Next to this, the results are of the same order and predict the same trends. Therefore it is decided to continue with the implemented model in the remainder of this report.
Figure 4.3: Results of implementation nonlinear model. solid: [4] dashed: implemented model.

$m_f = 9 \text{ mg/cycle}, N = 1000 \text{ rpm}, p_1 = 1.0048e5 \text{ Pa}.$
Chapter 5

Controller development

This Chapter will deal with the development of the controller. The controller’s objective is to keep the system’s combustion phasing ($CA_{50}$) at a desired setpoint to ensure stability and optimal efficiency.

\[ CA_{50} = \theta_{SOC}(T_{ivec}, p_{ivec}) + 0.55\Delta\theta(T_{ivec}, p_{ivec}, m_f) \]  

(5.1)

The system is stable in its nominal operating point, but when the operating point is shifted due to disturbances the system can become unstable (Section 5.1). Another effect of the drifting operating point is a deviation in combustion efficiency. Therefore a controller is needed to reject disturbances originating from, for example, ambient conditions or load changes.

A linear controller, designed using loopshaping, is chosen. This is done because this type of controller is easy to implement and tune. Furthermore, there are many quantitative ways to investigate the performance and stability.

Before we design a controller, we need more information on the model. To get a better insight into the nonlinear model which is presented in Chapter 4, the behavior of the model is analyzed in Section 5.1. Here the steady state behavior and the stability of the model is studied. The gathered information will be used to create a setpoint. The controller will be designed using a linearization of the nonlinear model (Section 5.2). The inputs of the model are divided into possible actuators ($T_1$, $RBL$, $m_f$) and disturbances ($p_0$, $T_w$). The actuators can be compared to a FTM, VVA with overlap and fuel flow rate system respectively. It is not possible to simulate a dual fuel solution in the current model. In Section 5.3, the most favorable actuator is selected. The actual controller design is presented in Section 5.5. In Chapter 6, the results of this controller are presented.

This Chapter will consider the nonlinear model in a nominal operating point unless stated otherwise. The most important values are given in Table 5.1. They are based on the nominal values used in [4]. Only $RBL$ is different, which is needed to reach the chosen setpoint (Section 5.1.4). The remaining parameters can be found in Appendix A.
### 5.1 Stability analysis

We will investigate the nonlinear model by means of a stability analysis used in [4] to get an idea of its steady state behavior, of the stability and the influence of the different actuators on the temperature equilibria. It uses a return map of the steady state breathing and combustion characteristics of the model. The part of the model which describes the process from exhaust gas blowdown to a new cylinder charge \((T_{bd} \rightarrow T_{ivc})\) is called the breathing characteristic. The process from intake valve closing temperature to the new blowdown temperature \((T_{ivc} \rightarrow T_{bd})\) is called the combustion characteristic.

![Combustion and breathing characteristics](image)

#### 5.1.1 Breathing characteristic

\(T_{ivc}\) is a function of \(W_{2c}\) and \(T_{rbd}\). \(T_{ivc}\) will be rewritten such that it is only a function of \(T_{bd}\).

\[
T_{ivc} = \frac{T_1}{1 + \frac{W_{2c} \tau R T_1}{p_{ivc} V_{ivc}} - \frac{W_{2c} \tau R T_{rbd}}{p_{ivc} V_{ivc}}}
\]  

(4.10)

\(T_{rbd}\) is a function of \(T_2\) (4.5). \(T_2\) in turn is a function of \(T_{bd}\) and \(W_{c2}\) (4.32). The breathing characteristic is determined under steady state conditions. Therefore, \(T_2 = 0\) and \(W_{c2}(t+\tau) = W_{c2}(t)\). This implies:

\[
T_2 = \frac{c_p W_{c2} T_{bd} + A h_{gw} T_{w}}{c_p W_{c2} + A h_{gw}}
\]

(5.2)

\[
W_{c2} = W_{01} + W_f + W_{2c} = W_{01} + \frac{m_f}{\tau} + W_{2c}
\]

(5.3)

The pressure in the inlet and exhaust manifold are both almost equal to atmospheric pressure, due to the large openings in the manifolds. Therefore, they are assumed to be constant and almost equal \((p_1 \approx p_2)\) for steady state conditions. This assumption simplifies the calculation of \(W_{2c}\) at steady state.

---

### Table 5.1: Nominal values of inputs, disturbances and parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>(T_1)</td>
<td>363 K.</td>
</tr>
<tr>
<td>(RBL)</td>
<td>3.65 mm.</td>
</tr>
<tr>
<td>(m_f)</td>
<td>9 mg/cycle.</td>
</tr>
<tr>
<td>(p_0)</td>
<td>1.01e5 Pa.</td>
</tr>
<tr>
<td>(T_w)</td>
<td>400 K.</td>
</tr>
<tr>
<td>(N)</td>
<td>1000 RPM.</td>
</tr>
<tr>
<td>(A h_{gw})</td>
<td>2.5 W/°K.</td>
</tr>
</tbody>
</table>
\[ W_{2c} = \frac{1}{\tau_{rbl}}(\kappa_0 + \kappa_1 \frac{p_1}{p_2})RBL^\alpha \approx \frac{1}{\tau_{rbl}}(\kappa_0 + \kappa_1)RBL^\alpha \] (5.4)

The set of equations (4.2, 4.5, 4.6, 4.10, 5.2, 5.3 and 5.4) are solved in Matlab using the symbolic toolbox. An algebraic equation for \( T_{ivc} \) as a function of \( T_{bd} \) is obtained:

\[ T_{ivc} = f(T_{bd}) \] (5.5)

The results are plotted in Figure 5.3 for two different values of heat transfer \( Ah_{gw} \), namely the nominal value, \( Ah_{gw} = 2.5 \text{ W/}^\circ\text{K} \), and \( Ah_{gw} = 0.75 \text{ W/}^\circ\text{K} \). In [4] perfect isolation of the exhaust manifold is used \( (Ah_{gw} = 0 \text{ W/}^\circ\text{K}) \). Due to differences between the models we use \( Ah_{gw} = 0.75 \text{ W/}^\circ\text{K} \) to get a similar result.

![Figure 5.3: Breathing characteristics.](image)

### 5.1.2 Combustion characteristic

\( T_{bd} \) is calculated using the formulae in Section 4.4 together with the equation for \( p_{ivc} \) (4.6). The only assumption here is that the steady state pressures \( p_1 \) and \( p_2 \) do not vary for different operating temperatures \( T_{ivc} \). \( T_{bd} \) is then only a function of \( T_{ivc} \), since \( p_{ivc}, p_2 \) and \( m_f \) will be known constants.

\[ T_{bd} = f(T_{ivc}, p_{ivc}, p_2, m_f) = f(T_{ivc}) \] (5.6)

The result is plotted in Figure 5.4. The temperature \( T_{bd} \) corresponds with the amount of waste heat in the exhaust gas. Therefore the point with minimal blowdown temperature (at \( T_{ivc}^{ref} \)) indicates a fuel-optimum where the most chemical energy is converted into work and not waste heat [4]. This is quite a simplification, but the model gives no other way to determine the efficiency.
5.1.3 Analysis

The axes of the breathing characteristic are switched and it is plotted together with the combustion characteristics to form a return map (Figure 5.5). The point where the breathing and combustion curve intersect is a temperature equilibrium point \((T_{ivc}^{eq}, T_{bd}^{eq})\). The intersection is a stable equilibrium when \(|\frac{\partial T_{bd}}{\partial T_{ivc}} \frac{\partial T_{ivc}}{\partial T_{bd}}| < 1\) [4]. This will be explained in the next paragraph.

Stability of the equilibrium

The stability of the equilibrium point can be determined via linearization of the system at that point [54].

The equilibrium temperatures are given by the following equations:

\[
T_{ivc}(k + 1) = T_{ivc}(k) = T_{ivc}^{eq} \tag{5.7}
\]
\[
T_{bd}(k + 1) = T_{bd}(k) = T_{bd}^{eq} \tag{5.8}
\]

We will investigate small perturbations \((\delta)\) around this equilibrium point:

\[
T_{ivc} = T_{ivc}^{eq} + \delta T_{ivc} \tag{5.9}
\]
\[
T_{bd} = T_{bd}^{eq} + \delta T_{bd} \tag{5.10}
\]

Around the equilibrium point, the breathing \((T_{ivc})\) and combustion \((T_{bd})\) temperatures can be approximated using a Taylor series approximation. Note that the next breathing temperature will only depend on the current blowdown temperature and visa versa.

\[
T_{ivc}(k + 1) + \delta T_{ivc}(k + 1) = T_{ivc}^{eq} + \frac{\partial T_{ivc}}{\partial T_{bd}} \delta T_{bd}(k) + \text{hot} \tag{5.11}
\]
\[
T_{bd}(k + 1) + \delta T_{bd}(k + 1) = T_{bd}^{eq} + \frac{\partial T_{bd}}{\partial T_{ivc}} \delta T_{ivc}(k) + \text{hot} \tag{5.12}
\]
The perturbations around the equilibrium will be small, therefore the higher order terms (hot) are neglected. Subtracting the equilibrium solution (Equations 5.7 and 5.8) from (Equations 5.11 and 5.12) yields:

\[
\begin{align*}
\delta T_{ivc}(k+1) &= \frac{\partial T_{ivc}}{\partial T_{bd}} T_{bd}(k) \\
\delta T_{bd}(k+1) &= \frac{\partial T_{bd}}{\partial T_{ivc}} T_{ivc}(k) = \left[ \begin{array}{cc} 0 & \frac{\partial T_{ivc}}{\partial T_{bd}} \\
\frac{\partial T_{bd}}{\partial T_{ivc}} & 0 \end{array} \right] \left[ \begin{array}{c} \delta T_{ivc}(k) \\
\delta T_{bd}(k) \end{array} \right] = [A_{stab}] \left[ \begin{array}{c} \delta T_{ivc}(k) \\
\delta T_{bd}(k) \end{array} \right]
\end{align*}
\]

(5.13)

The eigenvalues of the matrix $A_{stab}$ determine the stability of this equilibrium point. For discrete systems, this means that the eigenvalues need to lie within the unit circle ($|\lambda| < 1$). The eigenvalues of $A$ are $\sqrt{\frac{\partial T_{ivc}}{\partial T_{bd}} \frac{\partial T_{bd}}{\partial T_{ivc}}}$ and $-\sqrt{\frac{\partial T_{ivc}}{\partial T_{bd}} \frac{\partial T_{bd}}{\partial T_{ivc}}}$. The absolute value of the eigenvalues need to be smaller than 1, so they can be rewritten as $|\frac{\partial T_{ivc}}{\partial T_{bd}} \frac{\partial T_{bd}}{\partial T_{ivc}}| < 1$, which is the stability criterium found in [4].

Nonlinear analysis

Different equilibria are investigated with the nonlinear model (Chapter 4, $Ah_{gw} = 2.5 \text{ W/}^\circ\text{K}$). The simulations are plotted in the return map. The first equilibrium is the nominal operating point around $T_{ivc} = 444K$ and $T_{bd} = 612K$. The simulation is started at a distance from the equilibrium in the return map. It can be seen in Figure 5.6 that the nonlinear model converges towards the equilibrium point. The stability criterium at the equilibrium point, $|\frac{\partial T_{ivc}}{\partial T_{bd}} \frac{\partial T_{bd}}{\partial T_{ivc}}| = 0.02$, is smaller than 1 and is stable as the simulation converges. The nonlinear simulation (Figure 5.6) shows the expected staircase behavior, following the return map. At the end of the simulation, near the equilibrium, it does not follow the return map. This is probably due to the fact that the model is not in steady state while converging towards the equilibrium.

Next, the heat transfer in the exhaust manifold $Ah_{gw}$ is set to $0.75 \text{ W/}^\circ\text{K}$. There are two equilibrium points of which one is stable and the other is unstable (stability criteria 0.55 and 3.3 respectively). The unstable equilibrium will be investigated here ($T_{ivc} = 476K$, $T_{bd} = 638K$). The simulations show the model will stay in the equilibrium when there are no disturbances. If a disturbance is added, the
model will either converge to the stable equilibrium around $T_{ivc} = 463K$ and $T_{bd} = 634K$, or diverge towards higher temperatures (Figure 5.7).

The stability of the investigated equilibria also can be validated using the linearized model presented in Section 5.2. The eigenvalues of the system matrix $K$ of the state space system need to be in the unit circle for the system to be stable around that equilibrium point. The results of the linearized model agree with the analysis used here.

**Stability bound**

The actuator $T_1$ will be used to explain the stability bound. The influence of actuator $RBL$ and $m_f$ on the operating point are shown in Appendix C. In Figure 5.8 the return map is plotted for different temperatures of $T_1$. When temperature $T_1$ is increased to $388.5K$, the breathing and combustion characteristics at the equilibrium point ($T_{ivc} = 471K$, $T_{bd} = 657K$) are tangential. If the temperature $T_1$ is increased further, there is no intersection between the breathing and combustion curve. This means there are no equilibrium points possible for higher $T_1$. Next to this, the equilibrium at ($T_{ivc} = 471K$, $T_{bd} = 657K$) indicates the stability turning point. When $T_1$ is $388.5K$, there is only one equilibrium point. If $T_1$ is lower, there are two equilibrium points. The equilibria with a temperature $T_{ivc} < 471K$ are stable, equilibria with temperatures $T_{ivc} > 471K$ are instable.

### 5.1.4 Setpoint selection

The desired $CA_{50}^{ref}$ will be the point with the highest fuel efficiency and emissions within government regulations. The model does not account for emissions, therefore they can not be taken into account. First the point with minimal blowdown temperature and the corresponding temperature $T_{ivc}^{ref}$ are determined Section 5.1.2, see also Figure 5.4. The minimal blowdown temperature indicates the least amount of waste heat leaving the system, which is a course indication of the efficiency. The $CA_{50}$ at this temperature $T_{ivc}^{ref}$ is the desired setpoint. For $Ah_{gw} = 2.5 \text{ W/}^{\circ}K$, the setpoint is: $CA_{50}^{ref} = 6.6^{\circ}CA\ aTDC$. 

30
Figure 5.6: Nonlinear simulation plotted in the return map for the nominal, stable equilibrium point ($Ah_{gw} = 2.5 \text{ W/}^\circ\text{K}$).

Figure 5.7: Nonlinear simulation plotted in the return map, ($Ah_{gw} = 0.75 \text{ W/}^\circ\text{K}$) unstable equilibrium point with disturbances.
Figure 5.8: Influence T1 on the combustion process
5.2 Linearization

Linearizing the nonlinear model will enable us to evaluate the model using linear evaluation methods, which will be useful during actuator selection and during validation of the stability of the model. The frequency response of the state space model will be used for controller design in Section 5.5. Note however, that the linearized model will only be valid for a small region around its equilibrium conditions. The goal of this Section is to build a linear state-space model of the model presented in Chapter 4. Since this model is only accurate for dynamics slower than the cycle period $\tau$ a discrete model is chosen.

The continuous (Section 5.2.1) part needs to be linearized and discretized, therefore it will be linearized separate from the discrete part (Section 5.2.2). The states are partitioned into continuous $(x_c = [p_1 \ m_2 \ T_2])$ and discrete states $(x_d = [W_{c2} \ T_{bd}])$. Their variation around the nominal operating point is given by: $\bar{p}_1, \bar{m}_2, \bar{T}_2, \bar{W}_{c2}$ and $\bar{T}_{bd}$. The inputs are partitioned into actuator candidates $(w_a = [T_1 \ RBL \ m_f])$ and disturbances $(w_w = [p_0 \ T_w])$. The variation of these inputs around their nominal value is given by $\bar{T}_1, \bar{RBL}, \bar{m}_f, \bar{p}_0$ and $\bar{T}_w$. The continuous and discrete part of the state space model will be combined in Section 5.2.3. Finally, the order of the model will be reduced in Section 5.2.4. Note here that the equilibrium conditions were determined analytically in [4]. This however not be accomplished using the equations used in this report. The different interactions in the system, the Arrhenius integral, together with the nonlinear flow functions make it impossible to determine the equilibrium analytically using the symbolic toolbox in Matlab. Instead a nonlinear optimization routine is used.

5.2.1 Continuous model part

The continuous part of the model consists of the three continuous differential equations (4.1, 4.31, 4.32):

\[
\dot{p}_1 = \frac{RT_1}{V_1} (W_{01} - W_{1c}) \tag{4.1}
\]

\[
\dot{m}_2 = W_{c2} - W_{20} - W_{2c} \tag{4.31}
\]

\[
\dot{T}_2 = \frac{1}{c_v m_2} [c_v W_{c2} T_{bd} - (c_v W_{c2} + R (W_{2c} + W_{20})) T_2 - Ah_{gw}(T_2 - T_w)] \tag{4.32}
\]

First, the equations are linearized around the initial conditions at the nominal operating point.

\[
\dot{x}_c = f(x_c, x_d, w_a, w_w) = \begin{bmatrix} \dot{p}_1 \\ \dot{m}_2 \\ \dot{T}_2 \end{bmatrix} = \begin{bmatrix} f_1([p_1 \ m_2 \ T_2], [W_{c2} \ T_{bd}], [T_1 \ RBL \ m_f], [p_0 \ T_w]) \\ f_2([p_1 \ m_2 \ T_2], [W_{c2} \ T_{bd}], [T_1 \ RBL \ m_f], [p_0 \ T_w]) \\ f_3([p_1 \ m_2 \ T_2], [W_{c2} \ T_{bd}], [T_1 \ RBL \ m_f], [p_0 \ T_w]) \end{bmatrix} \tag{5.14}
\]

The equations are linearized using Matlab. First all the equations ($\dot{p}_1, \dot{m}_2, \dot{T}_2$) are programmed using the symbolic toolbox. They are partially differentiated using the jacobian function to obtain the following result:
period \[2\]. This continuous state space model can be discretized using a sample and zero order hold approximation under the assumption that the model is only accurate for dynamics slower than the cycle period \[2\]. The discrete versions of matrix \(A\), \(B\), \(C\) and \(D\) are \(A_d\), \(B_d\), \(C_d\) and \(D_d\) respectively and can be found using Equations 5.16-5.19. The discretization includes \(\tau\) and is therefore engine speed dependant. This means the model is only valid for this specific engine speed.

\[
\begin{bmatrix}
\dot{p}_1 \\
\dot{m}_2 \\
\dot{T}_2
\end{bmatrix} =
\begin{bmatrix}
\frac{\partial f_1}{\partial p_1} & \frac{\partial f_2}{\partial p_2} & \frac{\partial f_3}{\partial p_3} \\
\frac{\partial f_1}{\partial W_1} & \frac{\partial f_2}{\partial W_2} & \frac{\partial f_3}{\partial W_3} \\
\frac{\partial f_1}{\partial RBL} & \frac{\partial f_2}{\partial RBL} & \frac{\partial f_3}{\partial RBL}
\end{bmatrix}
\begin{bmatrix}
p_1(t) \\
m_2(t) \\
T_2(t)
\end{bmatrix} +
\begin{bmatrix}
\frac{\partial f_1}{\partial W_1} & \frac{\partial f_2}{\partial W_2} & \frac{\partial f_3}{\partial W_3} \\
\frac{\partial f_1}{\partial \sigma} & \frac{\partial f_2}{\partial \sigma} & \frac{\partial f_3}{\partial \sigma}
\end{bmatrix}
\begin{bmatrix}
W_{\text{evo}}(t) \\
T_{\text{evo}}(t)
\end{bmatrix} +
\begin{bmatrix}
0 \\
0 \\
0
\end{bmatrix}
\begin{bmatrix}
T_1(t) \\
m_f(t)
\end{bmatrix} +
\begin{bmatrix}
0 \\
0
\end{bmatrix}
\begin{bmatrix}
p_0(t) \\
T_{\text{vol}}(t)
\end{bmatrix}
\tag{5.15}
\]

The equilibrium conditions are substituted into the resulting jacobian, the result are the system matrices of the linearized model.

This continuous state space model can be discretized using a sample and zero order hold approximation under the assumption that the model is only accurate for dynamics slower than the cycle period \[2\]. The discrete versions of matrix \(A\), \(B\), \(C\) and \(D\) are \(A_d\), \(B_d\), \(C_d\) and \(D_d\) respectively and can be found using Equations 5.16-5.19. The discretization includes \(\tau\) and is therefore engine speed dependant. This means the model is only valid for this specific engine speed.

\[
A_d = e^{A\tau} \tag{5.16}
\]

\[
B_d = A^{-1}(A_d - I)B \tag{5.17}
\]

\[
C_d = A^{-1}(A_d - I)C \tag{5.18}
\]

\[
D_d = A^{-1}(A_d - I)D \tag{5.19}
\]

Where

\[
x_c(k + 1) = A_d x_c(k) + B_d x_d(k) + C_d w_a(k) + D_d w_w(k) \tag{5.20}
\]

### 5.2.2 Discrete model part

The equations 4.30, 4.28 and 4.29 describe the discrete dynamics.

\[
T_{bd} = T_{\text{evo}} \left( \frac{p_2}{p_{\text{evo}}} \right)^{(n_e - 1)/n_e} + \Delta T_{bd} \tag{4.28}
\]

\[
T_{bd}^d(t + \tau) = T_{bd}(t) \tag{4.29}
\]

\[
W_{evo}^d(t + \tau) = W_{evo}(t) + W_{f}(t) + W_{2e}(t) \tag{4.30}
\]

The same partitioning of states and inputs is used. The discrete part does not have a direct input of the disturbances \(w\), therefore they are omitted here.

Because \(T_{bd}\) depends on the Arrhenius integral (4.15), it is not easily linearizable. Therefore it is approximated using the equations in appendix D.

\[
T_{bd}^d(k + 1) = T_{bd}(k) = f(T_{\text{evo}}, p_{\text{evo}}, p_2, m_f) \tag{5.21}
\]
\[ x_c(k+1) = h(x_c, x_d, w_a) = \begin{cases} W_{c2}(k+1) = h_1([p_1, m_2, T_2], [W_{c2}^{d}], [T_1 \text{ RBL} m_f]) \\ T_{bd}^d(k+1) = h_2([p_1, m_2, T_2], [W_{c2}^{d}], [T_1 \text{ RBL} m_f]) \end{cases} \] (5.22)

This results in:

\[
\begin{bmatrix}
W_{c2}(k+1) \\
T_{bd}^d(k+1)
\end{bmatrix}
= \begin{bmatrix}
\frac{\partial h_1}{\partial p_1} & \frac{\partial h_1}{\partial m_2} & \frac{\partial h_1}{\partial T_2} \\
\frac{\partial h_2}{\partial p_1} & \frac{\partial h_2}{\partial m_2} & \frac{\partial h_2}{\partial T_2}
\end{bmatrix}
\begin{bmatrix}
p_1(k) \\
m_2(k) \\
T_2(k)
\end{bmatrix}
+ \begin{bmatrix}
\frac{\partial h_1}{\partial W_{c2}} & \frac{\partial h_1}{\partial T_{bd}^d} \\
\frac{\partial h_2}{\partial W_{c2}} & \frac{\partial h_2}{\partial T_{bd}^d}
\end{bmatrix}
\begin{bmatrix}
W_{c2}^c(k) \\
T_{bd}^d(k)
\end{bmatrix}
+ \begin{bmatrix}
\frac{\partial j_1}{\partial p_1} & \frac{\partial j_1}{\partial m_2} & \frac{\partial j_1}{\partial T_2} \\
\frac{\partial j_1}{\partial W_{c2}} & \frac{\partial j_1}{\partial T_{bd}^d}
\end{bmatrix}
\begin{bmatrix}
j_1([p_1, m_2, T_2], [W_{c2}^{d}], [T_1 \text{ RBL} m_f])
\end{bmatrix} \quad (5.23)
\]

The output of the discrete part of the model is the combustion timing. CA\(_{50}\) is calculated using the approximation in appendix D.

\[ y = CA_{50} = j(x_c, x_d, w_a) = j_1([p_1, m_2, T_2], [W_{c2}^{d}], [T_1 \text{ RBL} m_f]) \] (5.24)

\[ CA_{50}^- = \begin{bmatrix}
j_1 \\
j_1 \text{ RBL} \\
j_1 \text{ Wf}
\end{bmatrix}
\]

\[ (5.25) \]

### 5.2.3 Assembling

The continuous and discrete model parts will be combined to build the complete model:

\[
\begin{bmatrix}
\dot{p}_1(k+1) \\
\dot{m}_2(k+1) \\
\dot{W}_{c2}(k+1) \\
\dot{T}_{bd}^d(k+1)
\end{bmatrix}
= \begin{bmatrix}
A_d & B_d \\
C_d & D_d
\end{bmatrix}
\begin{bmatrix}
p_1(k) \\
m_2(k) \\
W_{c2}(k) \\
T_{bd}^d(k)
\end{bmatrix}
+ \begin{bmatrix}
0 & 0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
T_1(k) \\
RBL(k) \\
m_f(k) \\
p_0(k)
\end{bmatrix} \quad (5.26)
\]

The output CA\(_{50}\) can be written as:

\[ CA_{50} = \begin{bmatrix}
H & I
\end{bmatrix}
\begin{bmatrix}
p_1(k) \\
m_2(k) \\
W_{c2}(k) \\
T_{bd}^d(k)
\end{bmatrix}
+ \begin{bmatrix}
J & 0
\end{bmatrix}
\begin{bmatrix}
T_1(k) \\
RBL(k) \\
m_f(k) \\
p_0(k)
\end{bmatrix} \quad (5.27) \]
5.2.4 Model reduction

The condition number of the system matrix $A$ of the state space system is too large to perform calculation without loss of accuracy (order $10^{20}$). This is mainly due to the fast dynamics introduced by the pressure and the relative slow temperature dynamics. In order to reduce this condition number and to simplify calculations, model reduction using the Hankel singular values is performed. The Hankel Singular Values are obtained using the controllability and observability gramians, $P$ and $Q$, respectively.

$$P = \int_0^\infty e^{A\tau}BB^Te^{A^T\tau}d\tau$$  \hspace{1cm} (5.28)

$$Q = \int_0^\infty e^{A^T\tau}C^TCe^{A\tau}d\tau$$  \hspace{1cm} (5.29)

If these gramians are of full rank, the system is state controllable and observable. The multiplication of gramians $P$ and $Q$ will result in a matrix. The positive square roots of the eigenvalues of this matrix are the Hankel singular values $\sigma_i$.

$$\sigma_i = \sqrt{\lambda_1(PQ)}$$  \hspace{1cm} (5.30)

These values are an indication of how much influence the input has on the states (controllability) and the states on the output $CA_{50}$(observability). Small Hankel values indicate a small effect on the input-output behavior of the system, large Hankel values indicate a large effect.

Consider the example system $\dot{x} = Ax + Bv$. This system is brought in balanced form where the observability and controllability grammians are equal and diagonal. The state vector $x$ is split into $x_1$ with large Hankel values and $x_2$ with small Hankel values. Hence, the system can then be written as:

$$\begin{align*}
\dot{x}_1 &= A_{11}x_1 + A_{12}x_2 + B_1v \\
\dot{x}_2 &= A_{21}x_1 + A_{22}x_2 + B_2v \\
y &= C_1x_1 + C_2x_2 + Dv
\end{align*}$$

The derivative of the states with small Hankel values are set to zero. A residualization is then performed by writing $x_2$ as a function of $x_1$ and $v$. The steady state gain is preserved in this approach

$$x_2 = -A_{22}^{-1}A_{21}x_1 - A_{22}^{-1}B_2v$$  \hspace{1cm} (5.31)

This will be substituted into $x_1$ and $y$, which results in the following balanced realization:

$$\begin{align*}
A_{\text{new}} &= A_{11} - A_{12}A_{22}^{-1}A_{21} \\
B_{\text{new}} &= B_1 - A_{12}A_{22}^{-1}B_2 \\
C_{\text{new}} &= C_1 - C_2A_{22}^{-1}A_{21} \\
D_{\text{new}} &= D - C_2A_{22}^{-1}B_2
\end{align*}$$

The error bound on this model reduction is [55].

$$\|G(s) - G^k_a(s)\|_{\infty} \leq 2(\sigma_{k+1} + \sigma_{k+2} + ... + \sigma_n)$$  \hspace{1cm} (5.32)
Hankel values

<table>
<thead>
<tr>
<th>x_1</th>
<th>3.2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1.6</td>
</tr>
<tr>
<td></td>
<td>0.45</td>
</tr>
<tr>
<td>x_2</td>
<td>4.0e-5</td>
</tr>
<tr>
<td></td>
<td>3.3e-9</td>
</tr>
</tbody>
</table>

Table 5.2: Hankel Singular Values of the full order model

Where $G(s)$ denotes the original system and $G_k(s)$ is the reduced model containing $k$ states. Here it can be seen that the error is bound by the Hankel values of the omitted states.

In our linearized HCCI model, the balanced realization is obtained by using the Matlab function `balreal`. The Hankel singular values of the state space system are given in Table 5.2. The last two Hankel values are very small, therefore they are omitted using `modred`. The error is bound by the omitted Hankel values and will therefore also be small (Figure 5.9). The condition number of the reduced system is of order 2, which is good for calculation purposes.

Figure 5.9: Absolute error $\|G(s) - G_k(s)\|_{\infty}$ originating from model reduction

### 5.3 Actuator selection

The goal of this Section is to select the most favorable actuator for the system. This will again be done using the Hankel Singular Values. Larger singular values indicate a bigger influence on the input-output behavior. This can be used as a quantitative measure to select the actuator which has the biggest influence on the system. The Hankel Singular Values are calculated for the inputs $T_1$, $RBL$ and $m_f$ separately. The engine in this model is port fuel injected. This means the fueling $m_f$ determines the output power of the engine and can not be used to control the combustion phasing. Fueling is included for illustrative purposes only. The Hankel values are calculated for each individual actuator as only input on the system. Each input is divided by its maximum allowed variation in order
to compare the results. The maximum allowed variation is chosen to be the distance from zero to its nominal value. The nominal value of $T_1$ is 363 K. The distance from ambient temperature (293 K.) is 70 K. The nominal value of $m_f$ and $RBL$ are 9 mg/cycle and 3.65 mm. respectively. An overview of the scaling factors is given in Table 5.3. More detail on how the scaling is applied can be found in Section 5.4.

<table>
<thead>
<tr>
<th>$T_1$</th>
<th>RBL</th>
<th>$m_f$</th>
</tr>
</thead>
<tbody>
<tr>
<td>70 K</td>
<td>3.65 mm.</td>
<td>9 mg/cycle</td>
</tr>
</tbody>
</table>

Table 5.3: Maximum variation inputs

The Hankel singular values of each input are given in Table 5.4.

<table>
<thead>
<tr>
<th>$T_1$</th>
<th>RBL</th>
<th>$m_f$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.59</td>
<td>0.54</td>
<td>3.4</td>
</tr>
<tr>
<td>0.26</td>
<td>0.24</td>
<td>1.5</td>
</tr>
<tr>
<td>1.8e-7</td>
<td>4.3e-7</td>
<td>1.0e-4</td>
</tr>
</tbody>
</table>

Table 5.4: Maximum Hankel Singular Values of the different inputs

The conclusion of these results is that, of the two actuator candidates, the temperature of the inlet manifold has the most influence on the input-output behavior of the system. It is closely followed by the rebreathing lift. The model however does not account for the dynamics or delay of the heater or heat exchanger in the inlet manifold. In reality, due to thermal inertia, step changes of intake temperature are not possible. It is possible to get round these slow dynamics by using a fast thermal management system, however delay is still present. The inlet runner has a certain length and thus the air will take a certain time before it enters the cylinder. This will delay temperature changes. The amount of valve lift can be determined independently from cycle to cycle. This means $RBL$ has a faster influence on the input-output behavior than $T_1$. Therefore $RBL$ is chosen as the most favorable actuator.

The Hankel Singular Values of the fueling $m_f$ are large, in comparison with the other two actuators. If the load demand and therefore the fueling rate is changed, a considerable change in $CA_{50}$ is expected. Information about the load demand and fueling rate can be used in an additional feedforward controller [4]. This in order to minimize the control effort needed to maintain the correct $CA_{50}$. This, however, is outside the scope of this project.

### 5.4 Scaling

For control purposes, it is convenient to scale the linearized model. The inputs, disturbances and outputs are divided by the expected or maximum allowed change from their nominal value. For example, the output $y$ will be scaled using the maximum allowed error, $e_{\text{max}}$.

$$y = \frac{\hat{y}}{e_{\text{max}}}$$  \hspace{1cm} (5.33)

Here $\hat{y}$ indicates the unscaled situation. This results in a value with a magnitude of order one. This way, the performance of the system can be more easily interpreted. Note that the value of the scaling parameters are different from the scaling used in Section 5.3. In this Section the maximum allowed variation of for instance input $u$ is smaller, because of the chosen setpoint. The scaling factors are combined in matrices $D_e$, $D_u$ and $D_d$. $D_e$ and $D_u$ contain the maximum error ($D_e = e_{\text{max}}$) and
maximum input \( (D_u = a_{\text{max}}) \) respectively. \( D_d \) is a diagonal matrix \( D_d = d_{\text{max}} \) containing the values of the maximum disturbances. This way the scaled variables can be obtained [35]:

\[
d = D_d^{-1} \hat{d}, \quad u = D_u^{-1} \hat{u}, \quad y = D_y^{-1} \hat{y}, \quad e = D_e^{-1} \hat{e}
\]  

(5.34)

The scaled systems \( G \) and \( G_d \) can also be obtained using these matrices.

\[
G = D_c^{-1} \hat{G} D_u, \quad G_d = D_d^{-1} \hat{G}_d D_d
\]  

(5.35)

The scaling values which are used are given in Table 5.5.

**CA\(_{50}\)** The combustion phasing has a global minimum at \( T_{\text{ivc}}^\nabla \) (Figure 5.10). From its reference value \( CA_{50}^{\text{ref}} \), the combustion phasing of the model can only be advanced up to this minimum. Therefore the difference between the \( CA_{50}^{\text{ref}} \) and the minimum \( CA_{50} \) is the maximum allowed variation for the combustion phasing. Additionally, when the model operates at \( T_{\text{ivc}} \) lower than \( T_{\text{ivc}}^\nabla \), the model will never reach the temperature \( T_{\text{ivc}} \) from which the equilibria are unstable (Section 5.1.3). This will guarantee the stability of the model.

**RBL** The maximum possible rebreathing lift is 4 mm. [2]. Since the nominal value is 3.65 mm., the maximum allowed variation from its nominal value is 0.35 mm.

The most favorable actuator (u) is chosen in Section 5.1 to be **RBL**. This leaves the system with four inputs which from now on will be regarded as disturbances (d). Deviations from their nominal value act as a disturbance on the system. These arise due to changes in ambient conditions or load changes. The origin, magnitude and the active frequency range of the disturbance will be discussed below. The active frequency range is not needed for scaling, but will be used in Section 5.5.1 for controller design.

**\( \bar{T}_1 \)** The temperature \( T_1 \) in the inlet manifold is controlled by a plenum heater to keep the temperature constant at 363 K. Possible temperature changes in the manifold originate from changes in ambient temperature or engine speed changes. Ambient temperature change due to changing time is a slow process. A faster change in temperature is expected when driving at varying altitude. Even then, the time scale of the variation is in the range of tens of minutes. It is assumed the controller in the inlet manifold will be able to keep the temperature constant for deviations with such large time scales. When the engine speed is changed, the amount of air traveling through the plenum heater changes. This can result in a sudden temperature rise or drop. The magnitude and time constant of this change is assumed to be five degrees Kelvin and five seconds respectively. This leaves a relevant frequency range up to 0.2 Hz.

**\( \bar{m}_f \)** The fuel is delivered every cycle by a fuel injector. The fuel delivered will vary due to injector inaccuracies. This inaccuracy will be around three percent (0.27 mg/cycle). An injector will produce a slightly different amount of fuel every cycle. Therefore, this disturbance is active up to the sampling (cycle) frequency. Fueling changes due to driver demand are not a disturbance, but a change of operating point. It is a known change in input and this information can be used in a feedforward controller [4]. Only the nominal operating point will be considered in this report therefore the subject of feedforward is not treated.

**\( \bar{p}_0 \)** Ambient pressure changes are small. Major changes in ambient pressure occur when driving at different altitude. The time scale of this change is in the range of tens of minutes. The maximum amplitude of the pressure change is assumed to be \( 10^{3} \) Pa and relevant frequencies up to 0.01 Hz. Pressure changes due to overtaking, whirlwinds, tunnel driving have energy at a much higher frequency. Due to the small magnitude of these changes however, they will be neglected.
Table 5.5: Maximum variations used for scaling

<table>
<thead>
<tr>
<th>Variable</th>
<th>Maximum value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\epsilon_{\text{max}}$ (CA$_{50}$)</td>
<td>3.5 CA</td>
</tr>
<tr>
<td>$u_{\text{max}}$ (RBL)</td>
<td>0.35 mm.</td>
</tr>
<tr>
<td>$d_{\text{max}}$ ($T_1$)</td>
<td>5 K.</td>
</tr>
<tr>
<td>$d_{\text{max}}$ ($m_f$)</td>
<td>0.27 mg/cycle</td>
</tr>
<tr>
<td>$d_{\text{max}}$ ($p_0$)</td>
<td>0.01e5 Pa.</td>
</tr>
<tr>
<td>$d_{\text{max}}$ ($T_w$)</td>
<td>50 K.</td>
</tr>
</tbody>
</table>

The exhaust manifold is, like the inlet manifold, temperature controlled. Temperature in the exhaust manifold deviates due to changes in exhaust gas temperature. The exhaust gas temperature can change due to for example load changes or misfires of the other cylinders. The magnitude of these disturbances is estimated to be around 50 degrees K. The time scale of these changes is estimated to be five seconds. This leaves a relevant frequency range for these disturbances up to 0.2 Hz.

![Figure 5.10: CA$_{50}$ reference, minimal CA$_{50}$ and the maximum allowed change](image-url)
5.5 Controller design

In this Section the controller will be designed. A linear controller for the linearized model (Section 5.2) is designed using loop shaping. This controller will be used to control the nonlinear model. The scaling, explained in Section 5.4, is applied to easily interpret the performance criteria and results.

The controller uses a static setpoint (Section 5.1.4) and is designed for disturbance rejection. This in order to stay within the stability bound explained in Section 5.1.3. Additionally, this way the operating condition will be kept close to the optimal efficiency as explained in Section 5.1.2. The disturbances that act on the system are quantified in Section 5.4. The disturbance rejection will be examined in Section 5.5.1.

RBL is chosen as the most favorable actuator to control the system in Section 5.3. The valve lift is limited to 4 mm. Actuator saturation needs to be avoided, which is studied in Section 5.5.2.

The linear controller will be designed for the linearized model. This controller is then implemented on the nonlinear model. It is possible that the linearized model neglects some nonlinear dynamics. To ensure the controller performs well on the nonlinear model, its performance is checked on linearizations around three different operating conditions (Section 5.5.3). The performance criteria obtained in Sections 5.5.1, 5.5.2 and 5.5.3 will be used in Section 5.5.4 to design the controller.

The block diagram of the control strategy is depicted in Figure 5.11. The reference \( r \) will be 0, because the setpoint \( (CA_{50}^{ref}) \) will be constant. The error \( e \) is then equal to \(-y (e = r - y = -y)\). \( C \) is the linear feedback controller with \( u (RBL) \) as output. The linearized model \( P \) has \( CA_{50} \) as output \( y \). \( G_d \) is the linearized disturbance model, with \( T_1, m_f, \bar{p}_0 \) and \( T_w \) as inputs \( (d) \) and \( CA_{50} \) as output. To obtain \( P \) and \( G_d \) a laplace transformation of the state space model (Section 5.2.3) is used. This results in the complete system \( P_{\text{complete}} = M(sI - K)^{-1}L + N \) (Equation 5.36) with 5 inputs (4 disturbances and 1 actuator) and 1 output \( y \). Plant \( P \) is the transfer function from input \( RBL \) to the output \( y (CA_{50}) \). The disturbance model \( G_d \) is formed by the transfer functions from the rest of the inputs to the output \( y \).

Finally, it must be noted that no measurement noise is considered in this setup.

\[
P_{\text{complete}}(s) = M(sI - K)^{-1}L + N
\]

(5.36)

\[
C \bullet \quad u \quad \rightarrow \quad P \quad \rightarrow \quad y
\]

\[
\begin{align*}
\text{d} & \rightarrow \quad G_d \rightarrow \\
\text{r} & \rightarrow \quad e \rightarrow \quad C \rightarrow \quad u \rightarrow \quad P \rightarrow \quad y
\end{align*}
\]

Figure 5.11: Closed-loop control system with disturbances. \( r = CA_{50}^{ref}, \ u = RBL, \ y = CA_{50}, \ d = [T_1, m_f, \bar{p}_0, T_w] \).
5.5.1 Disturbance rejection

The goal of the controller is to reject disturbances acting on the output $y = CA_{50}$. Figure 5.12 shows the amplitude of the disturbance model $G_d$ for different disturbances. The frequency ranges where the disturbances will be active are indicated with arrows. $m_f$ is active over the complete frequency range as discussed in Section 5.4. The amplitude of $G_d$ with disturbance $T_w$ is approximately one in the relevant frequency range. This means this disturbance has a large influence on the output of the system and closed-loop disturbance rejection is needed.

![Figure 5.12: Disturbance models](image)

The closed-loop response from disturbance $d$ to output $y$ is equal to the sensitivity $S$ times the disturbance model $G_d$ [55]:

$$
y(j\omega) = \frac{1}{1 + C(j\omega)P(j\omega)} G_d(j\omega)d(j\omega) = \frac{1}{1 + L(j\omega)} S G_d(j\omega)d(j\omega) \quad (5.38)$$

Therefore the sensitivity function $S$ needs to be small ($<1$) to reduce the influence of the disturbances on the output. $T_w$ is assumed to have a frequency content up to 0.2 Hz. Therefore the sensitivity needs to be smaller than 1 up to the maximum frequency of $T_w$ (0.2 Hz.) to successfully reject disturbances. This consequently means the loop gain $L$ needs to be large ($>1$) for this frequency area.

5.5.2 Input saturation

The input $u$ of the system is limited in its magnitude. The plant should be able to counteract unwanted disturbances on $y$ without the input $u$ saturating. This because saturation of the input may introduce unwanted and unexpected behavior. The closed-loop response from disturbance $d$ to input $u$ is equal to the input sensitivity function $M$ times the disturbance model $G_d$ [55]:

$$
u(j\omega) = \frac{C}{1 + C(j\omega)P(j\omega)} M G_d(j\omega)d(j\omega) \quad (5.39)$$
To avoid actuator saturation, the input sensitivity $M$ should be small, preferably not larger than 1.

### 5.5.3 Robustness

The engine model used is nonlinear, while a linearized model and linear controller are used. This mismatch introduces a model fault which may introduce unwanted dynamics, destabilizing the system. Therefore the controller needs to be robust for these nonlinearities.

In order to check the performance and stability when the model varies, a linearization around two other operating points is made. The heat transfer coefficient in the exhaust manifold is varied 50\% from $Ah_{gw} = 1.25$ to $Ah_{gw} = 3.75$. It is assumed that, when the controller works satisfactory for these operating points, the controller is robust for the neglected dynamics. Figure 5.13 the return map for the three different operating points. Note here that the combustion characteristic does not change when the heat transfer in the exhaust manifold is changed. Figure 5.14 shows a Bode plot of the three linearized plant models which are used.

![Figure 5.13: Different operating points](image)

### 5.5.4 Controller design

Next, the controller will be designed, based on the following criteria:

- $S$ should be smaller than one for all frequencies lower than 0.2 Hz. (Section 5.5.1).
- $M$ should preferably be smaller than one. (Section 5.5.2).
- The performance of the controller should be guaranteed for linearizations around three operating points (Section 5.5.3).

Next to this, the controller needs to be stable. Peaking of the sensitivity above 2 should be avoided [55]. The minimum bandwidth of the system is 0.2 Hz, but a bandwidth of 1 Hz is chosen to improve performance.
The frequency response of the linearized state space model (Section 5.2) is given in Figure 5.14. The model is approximately a -1 gain. A gain of minus one is added, this way the loop gain is positive and has a phase close to zero.

Two integrators and a lead filter are used. The integrators are used to increase the low frequency gain. This will result in a small error for the low frequencies where most of the disturbances are active. The integrators, \( \frac{s+2\pi f_1}{s} \), have a zero \( f_1 = 1.5 \) Hz. This frequency is chosen to ensure the loop gain is small at frequencies higher than the bandwidth. The frequency is not increased further to limit phase lag at the crossover frequency.

A lead filter is then added to create more phase margin around the crossover frequency. The lead filter, \( \frac{\frac{1}{2\pi f_2} s + 1}{\frac{1}{2\pi f_3} s + 1} \), is given a zero \( f_2 = 2/3 \) Hz. and a pole at \( f_3 = 6 \) Hz. This will give a maximum phase advance around the crossover frequency.

The gain of the controller is then adjusted to locate the crossover frequency at 1 Hz.

\[
C(s) = \frac{s + 2\pi f_1}{s} \frac{s + 2\pi f_1}{s} \frac{\frac{1}{2\pi f_2} + 1}{\frac{1}{2\pi f_3} + 1}
\]  \hspace{1cm} (5.40)

The loop gain is depicted in Figure 5.16. The controlled model has a -2 slope, with a -1 slope around the crossover frequency. The phase margin of the controller is about 80 degrees.

The sensitivity is plotted in Figure 5.17 for all three operating points. For frequencies smaller than 0.2 Hz, the sensitivity of all the operating points is smaller than 1. Almost no peaking occurs, the sensitivity stays well below 2 at the crossover frequency (1 Hz).

Figure ?? shows that the input sensitivity is larger than one for \( Ah_{gw} = 2.5 \) and \( 1.25 \) W/°K. at lower frequencies. This means actuator saturation may occur, which will be investigated further in Chapter 6.

The Nyquist plot in Figure 5.19 underlines the stability of the controlled model for all the operating points (Section 5.5.3).
Figure 5.15: The designed controller

Figure 5.16: Loop gain for three linearizations
Figure 5.17: Sensitivity of the three operating points

Figure 5.18: Input sensitivity
Figure 5.19: Stability test
Chapter 6

Simulation results

The simulation results of the controller are given and discussed in this Chapter. The simulation is performed using the nonlinear model in the nominal operating point (Table 6.1). If the heat transfer coefficient $Ah_{gw}$ deviates from its nominal value, it will be indicated for the experiments in question.

### 6.1 Validating controller design

First the nominal operating point is investigated. $T_w$ is perturbed using a combination of sinusoidal signals with a frequency content up to $0.2 \text{ Hz}$ and a maximum amplitude of $50 \text{K}$. Figure 6.1 shows the results for the uncontrolled and controlled situation, using the nominal operating conditions. It can be seen that the combustion phasing just exceeds its bounds in the uncontrolled situation for this disturbance input. When the system is closed-loop controlled, the phasing stays well within its bounds. Input saturation occurs half way and at the end of the experiment. Some saturation was expected from the results in Section 5.5.4. Although the combustion phasing in this situation stays within its bounds, the results are not optimal. A solution to avoid actuator saturation will be given in Section 6.3.

The simulation results are plotted (Figure 6.2) in the return map of Section 5.1 and the combustion phasing plot of Figure 5.10. Here it can be seen that the disturbance $T_w$ influences the temperature $T_{vc}$ and therefore the combustion phasing. In the controlled situation the results are closer around the nominal operating point. The controller results are asymmetric around the setpoint. This is due to the saturation of the actuator.

Next the same disturbance is regarded using the other two operating points (Figure 6.3). What can be directly noted is that, in the case of the operating point $Ah_{gw} = 3.75 \text{ W/}^\circ\text{K}$, the actuator is well above its saturating bounds. This is due to the fact that the setpoint of the controller is still $6.6^\circ\text{CA atDC},$

<table>
<thead>
<tr>
<th>Parameter $Ah_{gw}$</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_1$</td>
<td>363 K.</td>
</tr>
<tr>
<td>$RBL$</td>
<td>3.65 mm.</td>
</tr>
<tr>
<td>$m_f$</td>
<td>9 mg/cycle.</td>
</tr>
<tr>
<td>$p_0$</td>
<td>1.01e5 Pa.</td>
</tr>
<tr>
<td>$T_w$</td>
<td>400 K.</td>
</tr>
<tr>
<td>$N$</td>
<td>1000 RPM.</td>
</tr>
<tr>
<td>$Ah_{gw}$</td>
<td>2.5 W/\text{C}</td>
</tr>
</tbody>
</table>

Table 6.1: Nominal values of inputs, disturbances and parameters
Figure 6.1: Simulation results, disturbance $T_w$ noise with a frequency content up to 0.2 Hz. amplitude of 50 K. The solid line represents the uncontrolled system, a dashed line is used for the controlled system (both $Ah_{gw} = 2.5 \, \text{W/}^\circ\text{K}$). The boundaries are indicated with a dotted line and the setpoint with a dash-dotted line.

while the operating point has an equilibrium at $10.8 \, ^\circ\text{CA aTDC}$. The controller therefore increases RBL. The saturation limit is temporarily lifted in order to compare the results. The results show a settling time in the beginning, because the controller needs some time to account for the different operating conditions. After the settling time, the combustion phasing stays well within its limits for both operating points.
Figure 6.2: Simulation results plotted in the return map (upper) and combined with the combustion phasing (lower). The uncontrolled situation (left), the controlled situation (right). The minimum and maximum temperature $T_{ivc}$ are indicated with black solid lines.
Figure 6.3: Simulation results, disturbance $T_w$ noise with a frequency content up to 0.2 Hz. amplitude of 50 K. The solid line represents the controlled system with $Ah_{gw} = 1.25$ W/$\degree$K, a dashed line is used for the controlled system with $Ah_{gw} = 3.75$ W/$\degree$K. The boundaries are indicated with a dotted line and the setpoint with a dash-dotted line.
6.2 Additional operating point

The results of the previous experiments where all located far from the stability bound. The benefit of control in these situations is that the combustion phasing stays closer to the setpoint. The model does not describe power output or efficiency, nor does it give quantitative information about emissions. This means there is no direct information about the improvement due to control. To illustrate the need for disturbance rejection, an additional operating point is investigated. We consider the nominal parameters, but use a heat transfer coefficient of $A h_{gw} = 0.75 \text{ W/} ^\circ\text{K}$. Of the two equilibrium points, the stable equilibrium point is considered 5.1.3. Under these conditions, the stable and unstable equilibria are closer together. Here, a disturbance can cause the system to become unstable (Figure 6.4). Again a controller is designed using the design method discussed in Section 5.5. The two integrators have zeros at 1.5 Hz, the lead filter has a zero and pole at $2/3$ and 6 Hz, respectively. The gain of the controller is

In this operating point, the disturbances $p_0$, $m_f$ and $T_w$ have less influence on the combustion phasing. The temperature in the inlet manifold $T_1$ however, has a big influence on the combustion phasing. A step disturbance of 5 K. is applied, using an exponential function to simulate the thermal inertia of the system ($T_1 = 5(1 - e^{-2.5t})$). It can be seen that, when the disturbance is applied to the uncontrolled system, it crosses the stability bound and becomes unstable. When the system is controlled, the temperature $T_{ivc}$ stays within the stable region.

![Figure 6.4: Simulation results, disturbance $T_1$. The solid line represents the uncontrolled system with $A h_{gw} = 0.75 \text{ W/} ^\circ\text{K}$, a dashed line is used for the controlled system with $A h_{gw} = 0.75 \text{ W/} ^\circ\text{K}$. The boundaries are indicated with a dotted line and the setpoint with a dash-dotted line.](image)

Again, the simulation results are plotted in the return map of Section 5.1 and the combustion phasing plot of Figure 5.10. This results in four different Figures 6.5. In the return map, the breathing characteristics for $T_1 = 363$ and $T_1 = 368$ are given. In the uncontrolled situation, there is no equilibrium point for $T_1 = 368$ and the system becomes unstable. In the controlled situation, the rebreathing lift is reduced and the results stay near the stable equilibrium.
Figure 6.5: Simulation results plotted in the return map and in combination with the combustion phasing. The breathing characteristic is plotted for the nominal temperature ($T_1 = 363$ K.) and for the maximum deviation ($T_1 = 368$ K.).
6.3 Discussion

The nominal operating point is controlled in Section 6.1. The influence of the disturbance on the error is reduced, which will lead to emission reduction and efficiency improvement. The nominal operating point is far from the stability bound. Therefore an additional operating point is investigated in Section 6.2. This shows the disturbance rejection can be used to avoid instability, when operating closer to the stable bounds of the system. The results in Section 6.1 show actuator saturation, which is not wanted. The nominal and maximum input of the unscaled plant is 3.65 mm. and 4 mm. respectively. This leaves little room for control purposes. This can be avoided by changing the nominal operating conditions. When the temperature $T_1$ is increased, the stable equilibrium shifts towards higher $T_{ivc}$ (Section 5.1.3). This allows the rebreathing lift to be decreased (Appendix C) in order to stay in the chosen setpoint and there is more actuator capacity left for control.
Chapter 7

Conclusions and Recommendations

7.1 Conclusion

The first goal of this study is to select the best sensor and actuator for controller design. Then a model is chosen to simulate the combustion process for which a controller will be developed. A controller is designed for disturbance rejection to guarantee good performance of the HCCI engine, while avoiding actuator saturation and being robust for modeling errors.

Various sensors and actuators are used in literature. The most favorable actuator found is VVA (variable valve actuation). It is one of the fastest actuators, able to control the combustion process on a cycle-to-cycle basis. The most favorable sensor is the piezoelectric pressure transducer, although its price and lifespan are still limiting factors.

A nonlinear model described in [4] is implemented in Chapter 4. The results of the implemented model deviate from the results found in the paper. This originates from uncertainty which exact equations and parameters are used in [4]. Nevertheless, the results are useful and the model is used as a platform to design the controller on. The model does not provide information on emissions or power output, which is required when selecting a setpoint. Therefore a simplification of the engine efficiency is used.

Actuator selection of the model is done using Hankel Singular Values. The most favorable actuator of the model is RBL (rebreathing lift), a form of VVA.

The designed controller is successfully implemented on the nonlinear model. The controller shows good performance regarding disturbance rejection. The controller does not increase the stable operating region of the system, but avoids the system from reaching unstable operating points. Actuator saturation is present in the experiments. This can be avoided by increasing the temperature in the inlet manifold. When the heat transfer in the model was changed, the controller still produced good results. This indicates robustness on modeling errors. When operating close to the stability bound, the controller is able to compensate for the disturbances and keep the system in the stable operating region.

7.2 Recommendations

Experimental data should be provided in order to fit the model on an existing engine. This will take away the uncertainty which exact equations and parameters should be used in the model. The parameters of $E_a$, $A$, $n$, $n_c$, $e$, $k$, $\beta_1$, etc. can be fitted and tuned on the experimental data. The model can then be validated for different engine speeds and fueling to make it available for transient operation, which is important for a road-going engine.
To avoid actuator saturation, the temperature in the inlet manifold can be increased. This leads to a smaller nominal rebreathing lift and will increase the magnitude of the input which can be used for control. The plant capacity increases and actuator saturation is avoided.

In this report, the temperature $T_w$ is used to simulate the temperature changes in the exhaust manifold. These disturbances are better modeled by directly influencing for example the rebreathing temperature $T_{RBL}$.

The combustion phasing setpoint in this report is chosen to be the point with the lowest blowdown temperature. This a simplification and it is better to change the setpoint based on performance and limitations on knock and emissions. Therefore it is recommended that the model is expanded to include IMEP (indicated mean effective pressure), knock and emissions.

The linear controller is designed for the nominal operating point. If for example the engine speed is changed, the controller needs to be redesigned, because of the difference in sampling rate. Different controllers could be designed for different engine speeds and combined using gain scheduling.

A different operating temperature $T_{ivc}$ also means the gain of the system is changed and ideally, the controller needs to be redesigned. When the controller is able to stabilize the unstable region for higher $T_{ivc}$, the stability region can be enlarged. Due to the effect the temperature of the engine has on the combustion, transients are difficult when operating an HCCI engine. The first step is disturbance rejection on the combustion phasing, which is done in this study. A next step could be the use of a feedforward controller with information about the fueling. A better option than designing different controllers for different operating ranges is to use a model predictive control (MPC) strategy. This strategy can account for the nonlinearities in the system and therefore is able to control an operating range instead of an operating point. It can also account for fueling changes. Another advantage of model predictive control is that constraints like actuator saturation can be included in the controller design.
Appendix A

Parameters

Tables A.1, A.2, A.3 and A.4 show the parameters used in the model. Several papers ([2], [4], [5]) are compared. Some values are copied from mail correspondence with the author of the papers, these values will be indicated with "[mail]".

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
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<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$R$</td>
<td>gas constant, J/kg-K</td>
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<td>296.25</td>
<td>296.25</td>
<td>296.25</td>
</tr>
<tr>
<td>$C_p$</td>
<td>constant pressure specific heat, J/kg-K</td>
<td>×</td>
<td>1036.9</td>
<td>1036.9</td>
<td>1036.9</td>
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<td>$C_v$</td>
<td>constant volume specific heat, J/kg-K</td>
<td>×</td>
<td>740.625</td>
<td>740.625</td>
<td>740.625</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>ratio of specific heats</td>
<td>×</td>
<td>×</td>
<td>1.40</td>
<td>1.40</td>
</tr>
<tr>
<td>$Q_{LHV}$</td>
<td>lower heating value of gasoline, J</td>
<td>×</td>
<td>×</td>
<td>44e6</td>
<td>44e6</td>
</tr>
<tr>
<td>$E_c$</td>
<td>activation energy, J/mol</td>
<td>185000</td>
<td>185000</td>
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</tr>
<tr>
<td>$p_0$</td>
<td>Ambient pressure, Pa</td>
<td>1.01e5</td>
<td>×</td>
<td>×</td>
<td>1.01e5</td>
</tr>
</tbody>
</table>

Table A.1: General parameters

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_1$</td>
<td>Intake manifold volume, m$^3$</td>
<td>×</td>
<td>×</td>
<td>0.0013</td>
<td>0.0013</td>
</tr>
<tr>
<td>$V_2$</td>
<td>Exhaust manifold volume, m$^3$</td>
<td>×</td>
<td>×</td>
<td>0.015</td>
<td>0.003 [mail]</td>
</tr>
<tr>
<td>$Bore$</td>
<td>Bore of the cylinder, mm</td>
<td>86</td>
<td>×</td>
<td>×</td>
<td>86</td>
</tr>
<tr>
<td>$Stroke$</td>
<td>Stroke of the cylinder, mm</td>
<td>94.6</td>
<td>×</td>
<td>×</td>
<td>94.6</td>
</tr>
<tr>
<td>$Rod$</td>
<td>Rod length of the engine, mm</td>
<td>×</td>
<td>×</td>
<td>×</td>
<td>152.2 [mail]</td>
</tr>
<tr>
<td>$V_d$</td>
<td>Displacement volume of the cylinder, L</td>
<td>0.55</td>
<td>×</td>
<td>×</td>
<td>0.55</td>
</tr>
<tr>
<td>$CR$</td>
<td>Compression ratio</td>
<td>14</td>
<td>14</td>
<td>×</td>
<td>13.75 [mail]</td>
</tr>
<tr>
<td>$\theta_{ivc}$</td>
<td>Location of intake valve closing, deg.</td>
<td>×</td>
<td>×</td>
<td>×</td>
<td>-140 [mail]</td>
</tr>
<tr>
<td>$\theta_{evo}$</td>
<td>Location of exhaust valve opening, deg.</td>
<td>×</td>
<td>×</td>
<td>×</td>
<td>129 [mail]</td>
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Table A.2: Engine specifications

<table>
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<tr>
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<tr>
<td>$T_1$</td>
<td>Temperature inlet manifold, K</td>
<td>363</td>
<td>363</td>
<td>363</td>
<td>363</td>
</tr>
<tr>
<td>$m_f$</td>
<td>Fuel flow rate, mg/cycle</td>
<td>×</td>
<td>9</td>
<td>×</td>
<td>9</td>
</tr>
<tr>
<td>$T_a$</td>
<td>Wall (ambient) temperature, K</td>
<td>×</td>
<td>400</td>
<td>400</td>
<td>400</td>
</tr>
<tr>
<td>$N$</td>
<td>Engine speed, rpm</td>
<td>1000</td>
<td>1000</td>
<td>1000</td>
<td>1000</td>
</tr>
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</table>

Table A.3: Operating conditions
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</thead>
<tbody>
<tr>
<td>$C_d A_{d1}$</td>
<td>Orifice effective area inlet manifold</td>
<td>×</td>
<td>×</td>
<td>×</td>
<td>1.64e-3 [mail]</td>
</tr>
<tr>
<td>$C_d A_{d2}$</td>
<td>Orifice effective area outlet manifold</td>
<td>×</td>
<td>×</td>
<td>×</td>
<td>1.64e-3 [mail]</td>
</tr>
<tr>
<td>$A$</td>
<td>Arrhenius scaling constant</td>
<td>2500</td>
<td>×</td>
<td>0.4167</td>
<td>0.4167</td>
</tr>
<tr>
<td>$E_a$</td>
<td>Arrhenius activation energy</td>
<td>6317</td>
<td>×</td>
<td>1831930</td>
<td>1831930</td>
</tr>
<tr>
<td>$n$</td>
<td>Polytropic constant</td>
<td>×</td>
<td>1.35</td>
<td>1.367</td>
<td>1.367</td>
</tr>
<tr>
<td>$n_c$</td>
<td>Polytropic constant during compression</td>
<td>1.3</td>
<td>×</td>
<td>1.3</td>
<td>1.3</td>
</tr>
<tr>
<td>$n_e$</td>
<td>Polytropic constant during expansion</td>
<td>1.35</td>
<td>×</td>
<td>1.35</td>
<td>1.35</td>
</tr>
<tr>
<td>$A h_{gw}$</td>
<td>Exhaust runner heat transfer, W/K</td>
<td>×</td>
<td>2.5</td>
<td>×</td>
<td>2.5</td>
</tr>
<tr>
<td>$\Delta T_{wr}$</td>
<td>Exhaust runner temperature drop, K</td>
<td>×</td>
<td>35</td>
<td>×</td>
<td>35</td>
</tr>
<tr>
<td>$b_{k0}$</td>
<td>Constant term in k parametrization</td>
<td>0.162</td>
<td>×</td>
<td>×</td>
<td>0.162</td>
</tr>
<tr>
<td>$b_{k1}$</td>
<td>Linear term in k dependance on $\theta_{soc}$</td>
<td>0.005</td>
<td>×</td>
<td>×</td>
<td>0.005</td>
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<tr>
<td>$b_{k2}$</td>
<td>Square term in k dependance on $\theta_{soc}$</td>
<td>0.001</td>
<td>×</td>
<td>×</td>
<td>0.001</td>
</tr>
<tr>
<td>$a_{0}$</td>
<td>Linear term, e dependance on k</td>
<td>1.0327</td>
<td>×</td>
<td>×</td>
<td>1.0327</td>
</tr>
<tr>
<td>$a_{1}$</td>
<td>Square term, e dependance on k</td>
<td>-5.45</td>
<td>×</td>
<td>×</td>
<td>-5.45</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>RBL exponent in $W_{2c}^2$</td>
<td>×</td>
<td>×</td>
<td>0.5794</td>
<td>0.5794</td>
</tr>
<tr>
<td>$\kappa_0$</td>
<td>Constant term of $W_{2c}$</td>
<td>×</td>
<td>×</td>
<td>0.5729</td>
<td>0.5729</td>
</tr>
<tr>
<td>$\kappa_1$</td>
<td>Modulation of $W_{2c}$ by $p_1/p_2$</td>
<td>×</td>
<td>×</td>
<td>-0.52039</td>
<td>-0.52039</td>
</tr>
<tr>
<td>$\beta_0$</td>
<td>Constant term of $p_{ivc}$, Pa</td>
<td>1035</td>
<td>×</td>
<td>1035</td>
<td>1035</td>
</tr>
<tr>
<td>$\beta_1$</td>
<td>Linear term of $p_{ivc}$</td>
<td>1.1568</td>
<td>×</td>
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</table>

Table A.4: Fitted parameters
Appendix B

Equations

Tables B.2 and ?? show the equations used in the model. Again, several papers are compared. The equations which are used in this paper, are given in the last Table in the last column.
<table>
<thead>
<tr>
<th>[2]</th>
<th>[4]</th>
</tr>
</thead>
<tbody>
<tr>
<td>(p_1)</td>
<td>(p_1 = m_1 \frac{R_{\text{T}<em>{\text{LHV}}}}{V</em>{\text{T}} \times T})</td>
</tr>
<tr>
<td>(W_{01})</td>
<td>(\times)</td>
</tr>
<tr>
<td>(W_{1c})</td>
<td>(W_{1c} = \frac{n_c x_c}{m_c} W_c - W_f)</td>
</tr>
<tr>
<td>(W_{2c})</td>
<td>(\times)</td>
</tr>
<tr>
<td>(T_{\text{er}})</td>
<td>(T_{\text{er}}(t + \tau) = T_{\text{bd}(t)})</td>
</tr>
<tr>
<td>(p_{\text{vec}})</td>
<td>(p_{\text{vec}} = \frac{p_{\text{bd}}}{p_{\text{bd}} + 1} p_1)</td>
</tr>
<tr>
<td>(T_{\text{vec}})</td>
<td>(T_{\text{vec}} = T_1(1 - x_r) + T_{\text{er}} x_r)</td>
</tr>
<tr>
<td>(x_r)</td>
<td>(x_r = \alpha_1 (k_0 + \alpha_1 \frac{p_{\text{vec}}}{p_{\text{bd}}^2} V_{\text{T}} (u_{\text{bd}} + \alpha_2 u_{\text{bd}}^2 + \alpha_3 u_{\text{bd}}^3)))</td>
</tr>
<tr>
<td>(m_c)</td>
<td>(m_c = \frac{e_{\text{vec}}}{e_{\text{bc}}})</td>
</tr>
<tr>
<td>(RR)</td>
<td>(RR(\theta) = A p_{\text{vec}}^{n_c, n_c} (\theta) \exp(-E_v n_c^{n_c} / R T_{\text{vec}}))</td>
</tr>
<tr>
<td>(v_{\text{vec}})</td>
<td>(v_{\text{vec}}(\theta) = V_c(\theta) / V_c(\theta))</td>
</tr>
<tr>
<td>(\theta_{\text{g}})</td>
<td>(\theta_{\text{g}} = \theta_{\text{soc}} + \Delta \theta)</td>
</tr>
<tr>
<td>(\Delta \theta)</td>
<td>(\Delta \theta = k(T_{\text{vec}}) - 2/3(T_m)^{1/3} \exp\left(\frac{E_v n_c^{n_c}}{R T_{\text{vec}}}\right))</td>
</tr>
<tr>
<td>(T_m)</td>
<td>(T_m = T_{\text{vec}} + \epsilon(1 - e) \Delta T)</td>
</tr>
<tr>
<td>(\Delta T)</td>
<td>(\Delta T = \frac{R_{\text{LHV}}}{c_v V_{\text{T}}} Q_{\text{LHV}} m_{\text{vec}} + T_{\text{vec}} / p_{\text{vec}})</td>
</tr>
<tr>
<td>(e)</td>
<td>(e = a_0 + a_1 k)</td>
</tr>
<tr>
<td>(k)</td>
<td>(k = b_0 + b_1 \theta_{\text{soc}} + b_2 \theta_{\text{soc}}^2)</td>
</tr>
<tr>
<td>(T_{\text{bc}})</td>
<td>(T_{\text{bc}} = T_{\text{vec}}^{n_c, n_c} (\theta_{\text{c}}))</td>
</tr>
<tr>
<td>(p_{\text{bc}})</td>
<td>(p_{\text{bc}} = p_{\text{vec}}^{n_c, n_c} (\theta_{\text{c}}))</td>
</tr>
<tr>
<td>(T_{\text{ac}})</td>
<td>(T_{\text{ac}} = T_{\text{bc}} + (1 - e) \Delta T)</td>
</tr>
<tr>
<td>(p_{\text{ac}})</td>
<td>(p_{\text{ac}} = p_{\text{ac}}^{n_c, n_c} (\theta_{\text{ac}}))</td>
</tr>
<tr>
<td>(T_{\text{evo}})</td>
<td>(T_{\text{evo}} = T_{\text{ac}}^{n_c, n_c} (\theta_{\text{evo}}))</td>
</tr>
<tr>
<td>(p_{\text{evo}})</td>
<td>(p_{\text{evo}} = p_{\text{ac}}^{n_c, n_c} (\theta_{\text{evo}}))</td>
</tr>
<tr>
<td>(T_{\text{bd}})</td>
<td>(T_{\text{bd}} = T_{\text{evo}}^{n_c, n_c} + \Delta T_{\text{bd}})</td>
</tr>
<tr>
<td>(W_{c2})</td>
<td>(W_{c2}(t + \tau) = W_{1c}(t) + W_f(t) + W_{2c}(t))</td>
</tr>
<tr>
<td>(m_2)</td>
<td>(m_2 = W_{c2} - W_{2c} - W_{2c} - W_{2c})</td>
</tr>
<tr>
<td>(T_2)</td>
<td>(T_2 = \frac{20}{\gamma_2^{2/3}} T_2 = \frac{1}{\epsilon_{\text{vec}}} [c_2 W_{c2} T_{\text{bd}} - (c_2 W_{c2} + R(W_{bc} + W_{20}))/T_2 - A_{\text{vec}}(T_2 - T_0)])</td>
</tr>
<tr>
<td>(p_2)</td>
<td>(p_2 = \frac{20}{\gamma_2^{2/3}} W_{c2} T_{\text{er}} - (W_{20} + W_{21} + W_{2c}) T_2)</td>
</tr>
<tr>
<td>(W_{20})</td>
<td>(\times)</td>
</tr>
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</table>

Table B.1: Equations used in [2] and [4]
Table B.2: Equations used in [5] and this report

<table>
<thead>
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<th></th>
<th>[5]</th>
<th>This report</th>
</tr>
</thead>
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<tr>
<td>$p_1$</td>
<td>$p_1 = \frac{RT}{V_1} (W_{01} - W_{1c})$</td>
<td>$p_1 = \frac{Rf}{V_1} (W_{01} - W_{1c})$</td>
</tr>
<tr>
<td>$W_{01}$</td>
<td>$W_{01} = C_d A_{01} \sqrt{2(p_0 - p_1)p_1}$</td>
<td>$W_{01} = C_d A_{01} \sqrt{2(p_0 - p_1)p_0}$</td>
</tr>
<tr>
<td>$W_{1c}$</td>
<td>$W_{1c} \approx \frac{p_1 V_{R,DC}}{RT} - \frac{T_f}{T} W_{2c}$</td>
<td>$W_{1c} \approx \frac{p_1 V_{R,DC}}{RT} - \frac{T_f}{T} W_{2c}$</td>
</tr>
<tr>
<td>$W_{2c}$</td>
<td>$W_{2c} = \frac{1}{T_f} \left( (\kappa_0 + \kappa_1 \frac{p_1}{T_f}) R^{BL} (\theta) \right)$</td>
<td>$W_{2c} = \frac{1}{T_f} \left( (\kappa_0 + \kappa_1 \frac{p_1}{T_f}) R^{BL} (\theta) \right)$</td>
</tr>
<tr>
<td>$T_{er}$</td>
<td>$T_{er}(t + \tau) = T_{er}(t) \text{bd}(t) + a_{th} T_w$</td>
<td>$T_{er}(t + \tau) = T_2 + \Delta T_w$</td>
</tr>
<tr>
<td>$p_{rec}$</td>
<td>$p_{rec} = \beta_0 + \beta_1 \rho_1$</td>
<td>$T_{bd} = T_2 + \Delta T_w$</td>
</tr>
<tr>
<td>$T_{rec}$</td>
<td>$T_{rec}(k) = x_r(k) T_{rec}(k) + (1 - x_r(k)) T_1$</td>
<td>$T_{bd} = T_2 + \Delta T_w$</td>
</tr>
<tr>
<td>$x_r$</td>
<td>$x_r = \frac{W_{rec}}{W_{rec}}$</td>
<td>$x_r = \frac{W_{rec}}{W_{rec}}$</td>
</tr>
<tr>
<td>$m_c$</td>
<td>$m_c = \frac{p_c V_{R}}{RT_c}$</td>
<td>$m_c = \frac{p_c V_{R}}{RT_c}$</td>
</tr>
<tr>
<td>$RR$</td>
<td>$RR(\theta) = A_p n_{v_{rec}} \exp(-\frac{E_{a_{v_{rec}}}}{RT_c})$</td>
<td>$RR(\theta) = A_p n_{v_{rec}} \exp(-\frac{E_{a_{v_{rec}}}}{RT_c})$</td>
</tr>
<tr>
<td>$\theta$</td>
<td>$\theta_c = \theta_{soc} + \Delta \theta$</td>
<td>$\theta_r = \theta_{soc} + \Delta \theta$</td>
</tr>
<tr>
<td>$\Delta \theta$</td>
<td>$\Delta \theta = k(T_{soc})^{1/3} (T_m)^{1/3} \exp(-\frac{E_{a_{v_{rec}}}}{3R_c T_m})$</td>
<td>$\Delta \theta = k(T_{soc})^{1/3} (T_m)^{1/3} \exp(-\frac{E_{a_{v_{rec}}}}{3R_c T_m})$</td>
</tr>
<tr>
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<td>$T_m = T_{soc} + e \Delta T$</td>
<td>$T_m = T_{soc} + e \Delta T$</td>
</tr>
<tr>
<td>$\Delta T$</td>
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<td>$\Delta T = \frac{M_{c_{soc}}}{M_{c_{soc}}}$</td>
</tr>
<tr>
<td>$e$</td>
<td>$e = b_0 + b_1 \theta_{soc} + b_2 \theta_{soc}$</td>
<td>$e = a_0 + a_1 k$</td>
</tr>
<tr>
<td>$k$</td>
<td>$k = b_0 + b_1 \theta_{soc} + b_2 \theta_{soc}$</td>
<td>$k = b_0 + b_1 \theta_{soc} + b_2 \theta_{soc}$</td>
</tr>
<tr>
<td>$T_{bd}$</td>
<td>$T_{bd} = T_{v_{rec}} (\text{bd} - 1) (\theta_r)$</td>
<td>$T_{bd} = T_{v_{rec}} (\text{bd} - 1) (\theta_r)$</td>
</tr>
<tr>
<td>$p_{bd}$</td>
<td>$p_{bd} = p_{v_{rec}} (\text{bd} - 1) (\theta_r)$</td>
<td>$p_{bd} = p_{v_{rec}} (\text{bd} - 1) (\theta_r)$</td>
</tr>
<tr>
<td>$T_{ac}$</td>
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<td>$T_{ac} = T_{bd} + \Delta T$</td>
</tr>
<tr>
<td>$p_{ac}$</td>
<td>$p_{ac} = p_{bd} (\text{bd} - 1) (\theta_r)$</td>
<td>$p_{ac} = p_{bd} (\text{bd} - 1) (\theta_r)$</td>
</tr>
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<td>$T_{evo} = T_{ac} (\text{bd} - 1) (\theta_{evo})$</td>
</tr>
<tr>
<td>$p_{evo}$</td>
<td>$p_{evo} = p_{ac} (\text{bd} - 1) (\theta_{evo})$</td>
<td>$p_{evo} = p_{ac} (\text{bd} - 1) (\theta_{evo})$</td>
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<tr>
<td>$T_{bd}$</td>
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<td>$T_{bd} = T_{v_{rec}} (\text{bd} - 1) (\theta_{evo})$</td>
</tr>
<tr>
<td>$W_{c2}$</td>
<td>$W_{c2} = W_{1c}(t) + W_f(t) + W_{v_{rec}}(t)$</td>
<td>$W_{c2}(t + \tau) = W_{1c}(t) + W_f(t) + W_{v_{rec}}(t)$</td>
</tr>
<tr>
<td>$m_2$</td>
<td>$m_2 = W_{c2} - W_{2c} - W_{2c}$</td>
<td>$m_2 = W_{c2} - W_{2c} - W_{2c}$</td>
</tr>
<tr>
<td>$T_2$</td>
<td>$T_2 = \frac{1}{C_{m_{v_{rec}}}} (c_p W_{c2} T_{bd} - (c_v W_{c2} + R (W_{2e} + W_{20})) T_2 - \Delta h_{gw}(T_2 - T_w))$</td>
<td>$T_2 = \frac{1}{C_{m_{v_{rec}}}} (c_p W_{c2} T_{bd} - (c_v W_{c2} + R (W_{2e} + W_{20})) T_2 - \Delta h_{gw}(T_2 - T_w))$</td>
</tr>
<tr>
<td>$p_2$</td>
<td>$p_2 = \frac{T_2}{V_2} (W_{c2} T_{rec} - (W_{20} + W_{2e}) T_2) - \frac{1}{V_2} \Delta h_{gw} (T_2 - T_w)$</td>
<td>$p_2 = \frac{T_2}{V_2} (W_{c2} T_{rec} - (W_{20} + W_{2e}) T_2) - \frac{1}{V_2} \Delta h_{gw} (T_2 - T_w)$</td>
</tr>
<tr>
<td>$W_{20}$</td>
<td>$W_{20} = C_d A_{20} \sqrt{2(p_2 - p_0)p_2}$</td>
<td>$W_{20} = C_d A_{20} \sqrt{2(p_2 - p_0)p_2}$</td>
</tr>
</tbody>
</table>
Appendix C

Actuator influence on the return map

The influence of $T_1$ on the return map is shown in Section 5.1.3. The influence of $RBL$ and $m_f$ on the return map are shown in Figure C.1 and Figure C.2 respectively. $RBL$ has influence on the breathing characteristic only. When the rebreathing lift is decreased, the temperature $T_{ivc}$ of the operating point decreases. $m_f$ influences the combustion characteristics, where a higher fueling leads to a higher blowdown temperature.
**Figure C.1:** Influence $RBL$ on the combustion process

**Figure C.2:** Influence $m_f$ on the combustion process
Appendix D

Approximation blowdown temperature and combustion phasing

$T_{bd}$ and $CA_{50}$ are calculated using the Arrhenius rate integral.

$$\int_{\theta_{ivc}}^{\theta_{SOC}} A p_{ivc}\phi_{ivc}^n(\theta)\exp\left(-\frac{E_{a\phi_{ivc}}}{RT_{ivc}}(1-n)(\theta)\right)\,d\theta = 1$$

This integral is not convenient when determining a linearization to create a state space model. In order to implement the calculation of $T_{bd}$ and $CA_{50}$ in the state space model an approximation is made. $T_{ivc}$, $p_{ivc}$, $p_2$ and $m_f$ are varied and $T_{bd}$, as well as $CA_{50}$, is then calculated for all these situations. A least squares fit is performed on the following equations to determine $c_1$ till $c_9$ and $d_1$ till $d_9$.

$$T_{bd} = c_1 T_{ivc}^2 + c_2 T_{ivc} + c_3 p_{ivc} T_{ivc} + c_4 p_{ivc}^2 + c_5 p_{ivc} + c_6 p_2^2 + c_7 p_2 + c_8 m_f + c_9 \quad (D.1)$$

$$CA_{50} = d_1 T_{ivc}^2 + d_2 T_{ivc} + d_3 p_{ivc} T_{ivc} + d_4 p_{ivc}^2 + d_5 p_{ivc} + d_6 p_2^2 + d_7 p_2 + d_8 m_f + d_9 \quad (D.2)$$

The approximations for $T_{bd}$ and $CA_{50}$ are plotted against $T_{ivc}$, together with the combustion characteristics from section 5.1 in figures D.1 and D.2.
Figure D.1: Combustion characteristics combined with the quadratic approximation.  \( p_1 \approx 1.01e5Pa, RBL \approx 3.6mm, m_f = 9mg/cycle. \)

Figure D.2: combustion characteristics combined with the quadratic approximation.  \( p_1 \approx 1.01e5Pa, RBL \approx 3.6mm, m_f = 9mg/cycle. \)
Appendix E

Heat Release

Heat release calculation is a method to determine the progress and completeness of the combustive reactions. Effect like volume change, heat transfer and mass loss are separated from the sensor signal and the amount of fuel energy converted into thermal energy is determined. This is why heat release is commonly used to interpret the sensor signal in order to connect it to your control algorithm.

Heat release analysis is based on the first law of thermodynamics:

\[ dU = dQ - dW + h \cdot dm \quad (E.1) \]

Where \( dU \) denotes the total internal energy of the mass in the system. \( dQ \) is the heat transported into the system, which can be separated into heat transfer across the system boundaries and energy released from the fuel. \( dW \) represents the work performed. \( h \cdot dm \) are all energy leaks due to inflow and outflow of mass, such as leakage from crevice regions or injection of fuel after IVC.

Equation E.1 can be simplified by fixing \( c_p \) and \( c_v \) at a constant value, assuming ideal gas laws, a uniform and homogeneous mixture and no mass flow. Rewriting will result in equation E.2, which can be used to determine the heat release if the pressure and volume of the system are known.

\[
\frac{dQ_{\text{chemical}}}{d\alpha} = \left[ \frac{\gamma}{\gamma - 1} \right] p \frac{dV}{d\alpha} + \left[ \frac{1}{\gamma - 1} \right] V \frac{dp}{d\alpha}
\]

(E.2)
Bibliography


[38] Jan-Ola Olsson, Per Tunestål and Bengt Johansson, Compression Ratio Influence on Maximum Load of a Natural Gas Fueled HCCI Engine, SAE paper 2002-01-0111.


