Numerical simulations of particle migration in a viscoelastic fluid subjected to shear flow

G. D’Avino a,*, T. Tuccillo a, P.L. Maffettone a, F. Greco b, M.A. Hulsen c

a Dipartimento di Ingegneria Chimica, Università di Napoli Federico II, Napoli I-80125, Italy
b Istituto di Ricerche sulla Combustione, IRC-CNR, Naples, Italy

Abstract

Particle migration is a relevant transport mechanism whenever suspensions flow in channels with gap size comparable to particle dimensions (e.g. microfluidic devices). Several theoretical as well as experimental studies have been performed on this topic, showing that the occurring of this phenomenon and the migration direction are related to particle size, flow rate, and the nature of the suspending liquid.

In this work we perform a systematic analysis on the migration of a single particle in a sheared viscoelastic fluid through 2D finite element simulations in a Couette planar geometry. To focus on the effects of viscoelasticity alone, inertia is neglected. The suspending medium is modeled as a Giesekus fluid.

An ALE particle mover is combined with a DEVSS/SUPG formulation with a log-representation of the conformation tensor giving stable and convergent results up to high flow rates. To optimize the computational effort and reduce the remeshing and projection steps, needed as soon as the mesh becomes too distorted, a ‘backprojection’ of the flow fields is performed, through which the particle in fact moves along the cross-streamline direction only, and the mesh distortion is hence drastically reduced. Our results, in agreement with recent experimental data, show a migration towards the closest walls, regardless of the fluid and geometrical parameters. The phenomenon is enhanced by the fluid elasticity, the shear thinning and strong confinements. The migration velocity trends show the existence of a master curve governing the particle dynamics in the whole channel. Three different regimes experienced by the particle are recognized, related to the particle-wall distance. The existence of a unique migration behavior and its qualitative aspects do not change by varying the fluid parameters or the particle size.

© 2009 Elsevier Ltd. All rights reserved.

1. Introduction

Particle cross-streamline migration occurs in many practical utilizations, i.e. fluid–solid separation where different sized particles need to be separated, suspensions in microfluidic devices where migration can lead to non-homogenous particle distributions, cells in blood provoking clots obstructing the flow, removal of pollutants from a gaseous flow, etc. Recently, new applications exploiting the lateral particle motion are arising. Examples are isolation of human leukocytes [1] or focusing of red blood cells in microchannel flows for bio-sensing applications [2]. Due to its importance in applications dealing with solid-particle transport mechanisms, it has been extensively studied over many decades.

From an experimental point of view, the first work focused on particle migration was performed by Segre and Silberberg [3,4]. The authors studied the lift experienced by spheres in a dilute suspension in a Poiseuille flow at low Reynolds number. They found the existence of an equilibrium height in the channel where the particles tend to migrate.

Many analytical theories were also derived in the past, giving expressions for the lift force experienced by a particle in low Reynolds number Newtonian flows and simple geometries. An enlightening paper by Saffman [5] gave an expression for the potential lift force on a sphere in an unbounded shear flow. Such a theory was later generalized by Asmolov [6] and McLaughlin [7] removing some constraints on the flow parameters.

The motion of a sphere in the presence of a flat wall in shear flow was studied by using a perturbative approach by Leighton and Acrivos [8], Cherukat and McLaughlin [9] and Krishnan and Leighton [10]. They found the critical conditions for Reynolds number such that the sphere separates from the wall.

All the references cited above refer to a Newtonian suspending fluid. The non-Newtonian (i.e. viscoelastic) nature of the suspending medium is, however, relevant in many processes (polymer melts with fillers, rubbers, cosmetics, foods, etc.). In this case, few experimental data sets on migration are available.
Old data by Mason and co-workers [11–13] reported on migration of spheres in viscoelastic fluids in Poiseuille and Couette flows. Only recently, a careful and complete analysis by Lormand and Phillips [14] reported on the migration of a sphere in a viscoelastic fluid in a Couette apparatus with small curvature. Their data clearly show the tendency of the particle to migrate towards the walls of the flow cell. The authors also investigated the influence of particle dimension and external flow parameters on the migration.

On the theoretical side, and limiting to simple shear only, which is the case of interest in our paper, we are aware of only two works explicitly addressing the viscoelasticity-induced cross-flow migration, but at vanishing flow rates [15] or at modest flow rates in a 2D approximation [16]. At \( \text{Re} = 0 \), Ho and Leal [15] predict the existence of migration induced by normal stresses whenever there is a lateral variation of the shear rate in the undisturbed flow. The first work showing migration in an inertialess viscoelastic fluid is the pioneer paper of Huang et al. [16] where Direct Numerical Simulations are performed. They reported a particle lateral motion with direction depending on the presence/absence of a rate-dependent viscosity of the suspending liquid. In their numerical simulations, the influence on particle lift of fluid parameters, particle size/channel height ratio were also studied. Huang et al. [16] accounted for shear thinning with an ad-hoc modification of the Oldroyd-B model. They used an elastic–viscous–split-stress (EVSS) formulation to discretize the momentum balance and the solid–fluid coupling is treated by an arbitrary Lagrangian Eulerian (ALE) method with mesh velocities. As in Ho and Leal [15], the main conclusion of Huang et al. [16] is that migration has to be ascribed to the presence of normal stresses.

The purpose of the present work is to perform a systematic numerical study on the migration of a particle suspended in a realistic viscoelastic fluid under simple shear flow. The suspending medium is modeled as a Giesekus fluid [17], which is often capable of accurately describing experimental viscoelastic data. The study is carried out by neglecting fluid and particle inertia. It is anticipated here that our simulation results qualitative agree with the experiments by Lormand and Phillips [14], i.e., migration is predicted to occur towards the walls of the shear cell, although our results are limited to 2D. The analysis is performed through 2D direct numerical simulations to take into account the non-linear nature of the problem.

The momentum balance is discretized through the Discrete-elastic–viscous–split-stress (DEVSS) method that is one of the most robust formulations currently available. The viscoelastic constitutive equation is stabilized by implementing the Streamline-Upwind Petkov–Galerkin (SUPG) technique. Furthermore a log-conformation representation of the conformation tensor is used. Finally, an ALE particle mover [18] is adopted to handle the particle motion. The numerical scheme implemented here leads to stable and convergent results, and allows one to achieve substantial flow rates (as compared with [16]). In order to efficiently manage the particle motion, a trick is used whereby the particle only moves along the \( y \)-direction (i.e., the migration direction). This is achieved through ‘backprojection’ of the computed fields along the \( x \)-direction (the main flow direction) at any time step. In this way, remeshing due to ALE approach is only needed once–twice per run, always preserving the accuracy of the solution.

The influence of flow rate as well as of particle dimension (as compared to the gap size) on the migration velocity is also studied.

2. Governing equations

We consider a single, rigid, non-Brownian, inertialess, circular particle (2D problem) moving in a channel filled by a viscoelastic fluid. The problem is schematized in Fig. 1: a particle with diameter \( D_p = 2R_p \), denoted by \( P(t) \), moves in a rectangular domain, \( \Omega \), with dimensions \( L \) and \( H \) along \( x \)– and \( y \)-axis respectively and external boundaries denoted by \( \Gamma_i \) (\( i = 1 \ldots 4 \)). The Cartesian \( x \) and \( y \) coordinates are selected with the origin at the center of the domain. On the upper and lower boundaries, equal and opposite velocities are imposed that, for an unfilled fluid, would generate the shear flow depicted on the right part of the same figure.

The vector \( \mathbf{x}_p = (x_p, y_p) \) gives the position of the center of the particle \( P \). In order to evaluate particle rotation, an angular information, \( \Theta = \Theta \mathbf{k} \), is also associated with the particle, where \( \mathbf{k} \) is the unit vector in the direction normal to the \( x \)--\( y \) plane. The particle moves according to the imposed flow and its rigid-body motion is completely defined by the translational velocity, denoted by \( \mathbf{U}_p = \frac{d\mathbf{x}_p}{dt} = (U_p, V_p) \), and angular velocity, \( \omega = \frac{d\Theta}{dt} = \omega \mathbf{k} \).

The governing equations for the fluid domain, \( \Omega - P(t) \), neglecting inertia, can be stated as follows:

\[
\nabla \cdot \sigma = 0 \tag{1}
\n\nabla \cdot \mathbf{u} = 0 \tag{2}
\n\sigma = -pI + 2\eta_s \mathbf{D} + \tau \tag{3}
\]

Eqs. (1)–(3) are the equations for the momentum balance, the mass balance (continuity) and the expression for the total stress, respectively. In these equations \( \mathbf{u}, \sigma, p, I, \mathbf{D}, \eta_s, \) are the velocity vector, the stress tensor, the pressure, the \( 2 \times 2 \) unity tensor, the rate-of-deformation tensor and the viscosity of a Newtonian ‘solvent’, respectively. The viscoelastic stress, \( \tau \), is written as (for the constitutive model chosen, see below):

\[
\tau = \frac{\eta_s}{\lambda} (\mathbf{C} - I) \tag{4}
\]
where \(c\) is the ‘conformation tensor’, \(\eta\) is the polymer viscosity, and \(\lambda\) is the relaxation time.

We will model the viscoelastic fluid with the Giesekus constitutive equation (for \(c\)):

\[
\lambda \frac{\partial c}{\partial t} + c - I + \alpha (c - I)^2 = 0
\]  

(5)

where \(\alpha\) is the so-called mobility parameter that modulates the shear thinning behavior. The symbol \((\cdot)^T\) denotes the upper-convected time derivative, defined as:

\[
c \equiv \frac{\partial c}{\partial t} + \mathbf{u} \cdot \nabla c - (\nabla \mathbf{u})^T \cdot c - c \cdot \nabla \mathbf{u}
\]  

(6)

Notice that the zero shear viscosity for the total stress tensor \(\sigma\) is given by \(\eta_0 = \eta_s + \eta\).

The boundary and initial conditions are given by:

\[
\mathbf{u} = \mathbf{U}_p + \omega \times (\mathbf{x} - \mathbf{x}_p) \quad \text{on } \partial \Omega(t)
\]  

(7)

\[
u = \nu_0 = 0 \quad \text{on } \Gamma_1 \text{ and } \Gamma_3
\]  

(8)

\[
\mathbf{u}(-L/2, y) = \mathbf{u}(L/2, y) \quad \forall y \in [-H/2, H/2]
\]  

(9)

\[
\sigma(-L/2, y) = \sigma(L/2, y) \quad \forall y \in [-H/2, H/2]
\]  

(10)

\[
c |_{t=0} = c_0
\]  

(11)

Eqs. (7) and (8) are the rigid-body motion on the particle boundary and the shear flow conditions on the upper and lower fluid boundaries, with \(\gamma\) the applied shear rate. Eqs. (9) and (10) state periodic boundary conditions on \(\Gamma_2\) and \(\Gamma_4\). Since inertia is neglected, no initial condition of the velocity and pressure fields is required whereas the initial conformation tensor condition is necessary (Eq. (11)). In our simulations, we use a stress-free state, i.e. \(c|_{t=0} = I\), as initial condition over the whole fluid domain. Notice that, by considering periodicity along \(x\)-direction, no inflow section exists, therefore we do not need to specify the conformation tensor on any boundary.

The equation governing the rigid-body motion (Eq. (7)) adds (for the 2D case) three additional unknowns, namely, the translational and angular velocities of the particle. So, to obtain particle motion, it is necessary to consider the balance equations for drag force and torque acting on the particle boundary. Under the assumptions of absence of particle inertia, and of no ‘external’ forces and torques (force- and torque-free particle), such balance equations are given by:

\[
F = \int_{\partial \Omega(t)} \sigma \cdot \mathbf{n} d\mathbf{s} = 0
\]  

(12)

\[
T = \int_{\partial \Omega(t)} (\mathbf{x} - \mathbf{x}_p) \times (\sigma \cdot \mathbf{n}) d\mathbf{s} = 0
\]  

(13)

where \(F = (F_x, F_y)\) and \(T = Tk\) are the total force and torque on the particle boundary, respectively, and \(\mathbf{n}\) is the outwardly directed unit normal vector on \(\partial \Omega\).

The particle position and rotation are updated by integrating the following kinematic equations:

\[
\frac{dx_0}{dt} = \mathbf{U}_p, \quad x_{p|_{t=0}} = x_{p_0}
\]  

(14)

\[
\frac{d\Theta}{dt} = \omega, \quad \Theta|_{t=0} = \Theta_0
\]  

(15)

The analysis is carried out by making the above equations dimensionless, using \(\gamma\) as the characteristic time scale and \(\eta \gamma^2\) as the scale for the stress. Then, the Deborah number, \(De = \gamma \gamma^2\), will appear in the equations. The Deborah number is simply a non-dimensional measure of the intensity of the imposed flow rate.

### 3. Weak form and implementation

#### 3.1. Weak form

The system of Eqs. (1)–(6) with initial and boundary conditions (7)–(11) and the balance Eqs. (12) and (13) form a well-posed problem in the unknowns: \(p, \mathbf{u}, c, \mathbf{U}_p, \omega\). At any time step the problem is solved and the flow fields, and rigid-body unknowns are evaluated. The kinematic Eqs. (14) and (15) are then integrated to update the particle position and rotation.

Such a system is solved by the finite element method. A log-conformation representation for the conformation tensor has been used [19,20]. The original equation for the conformation tensor \(c\), Eq. (5), is transformed to an equivalent equation for \(s = \log(c)\):

\[
\frac{ds}{dt} + \mathbf{u} \cdot \nabla s = g(\nabla \mathbf{u}^T, s)
\]  

(16)

An expression for the function \(g\) for a Giesekus fluid can be found in [20]. Solving the equation for \(s\) instead of the equation for \(c\) leads to a substantial improvement of stability for high Deborah numbers.

The finite element method requires the weak formulation of the equations to be solved. Such a weak form for the \(\Omega - \partial \Phi(t)\) domain can be stated as follows: For \(t > 0\), find \(\mathbf{u} \in \mathbf{U}, \ p \in \mathbf{P}, \ s \in \mathbf{S}, \ G \in \mathbf{G}, \ \mathbf{U}_p \in \mathbf{R}^3, \ \omega \in \mathbf{R}, \ \lambda \in \mathbf{L}^2(\partial \Phi(t))\) such that:

\[
\begin{align*}
\int_{\partial \Omega} 2\eta \mathbf{D}(\mathbf{v}) : \mathbf{D}(\mathbf{u}) d\mathbf{A} &- \int_{\partial \Omega} \nabla \cdot \mathbf{v} p d\mathbf{A} + \int_{\partial \Omega} \alpha (\nabla \mathbf{v})^T : \nabla \mathbf{u} d\mathbf{A} \\
&- \int_{\partial \Omega} (\nabla \mathbf{v})^T : \mathbf{G}^T d\mathbf{A} \\
&+ \int_{\partial \Omega} \left[ (\mathbf{v} - (\mathbf{v} + \chi \times (\mathbf{x} - \mathbf{x}_p))) \right] \cdot \lambda d\mathbf{s} = - \int_{\partial \Phi(t)} \mathbf{D}(\mathbf{v}) : \tau d\mathbf{A}
\end{align*}
\]  

(17)

\[
\int_{\partial \Omega} q \nabla \cdot \mathbf{u} d\mathbf{A} = 0
\]  

(18)

\[
\int_{\partial \Omega} \mathbf{H} : \mathbf{G} d\mathbf{A} - \int_{\partial \Omega} \mathbf{H} : (\nabla \mathbf{u})^T d\mathbf{A} = 0
\]  

(19)

\[
\int_{\partial \Omega} (\mathbf{S} + \bar{\tau} (\mathbf{u} - \bar{\mathbf{u}}) \cdot \nabla \mathbf{S}) : \left[ \frac{\partial s}{\partial \tau} + (\mathbf{u} - \bar{\mathbf{u}}) \cdot \nabla s - g(G, s) \right] d\mathbf{A} = 0
\]  

(20)

\[
\int_{\partial \Phi(t)} p \cdot [\mathbf{u} - (\mathbf{U}_p + \omega \times (\mathbf{x} - \mathbf{x}_p))] d\mathbf{s} = 0
\]  

(21)

\[
s = s_0 \quad \text{at } t = 0
\]  

(22)

for all \(\mathbf{v} \in \mathbf{U}, \ q \in \mathbf{P}, \ s \in \mathbf{S}, \ \mathbf{H} \in \mathbf{G}, \ \mathbf{V} \in \mathbf{R}^2, \ \chi \in \mathbf{R} \) and \(\mu \in \mathbf{L}^2(\partial \Phi(t))\), where \(U, \ P, \ S\) and \(G\) are suitable functional spaces. In Eq. (20) \(\mathbf{u}\) is the velocity of the mesh nodes (see below for details).

To improve the numerical stability at high Deborah numbers, a DEVSS-G formulation is implemented [21,22] for the momentum balance, Eq. (17), and the SUPG technique [23] is used for the constitutive relation, Eq. (20).

The \(\tau\) parameter in Eq. (20) is given by \(\tau = \beta h/2U_c\), where \(\beta\) is a dimensionless constant, \(h\) is a typical size of the element and \(U_c\) is a characteristic velocity for the element. In our simulations, we have chosen \(\beta = 1\) and for \(U_c\) we take the average of the magnitude of the velocities in all integration points. In addition, \(\alpha\) in Eq. (17) is chosen as \(\alpha = \eta\). We take the initial value \(s_0 = 0\), corresponding to zero initial stress.

The rigid-body motion on the particle boundary is imposed through Lagrange multipliers \(\lambda\) (see [24]). As a consequence, the particle kinematic quantities are considered as additional unknowns and are recovered by solving the full system of equations.
The particle motion is taken into account by using an ALE formulation [18], whereby at each time step the mesh nodes are moved according to a mesh velocity \( \mathbf{u} \) obtained by solving an extra equation. Notice that in general such node velocities are different from the fluid velocities at the nodes. This can be done until the mesh becomes too distorted. After that a new mesh is generated and the solution on the old mesh is projected onto the new one.

The node mesh movement is taken into account in the discretized form of the governing equations by subtracting the mesh velocity to the fluid velocity in the convective terms. In this regard, note that, due to the fluid inertialless assumption, the convective term only appears in the viscoelastic constitutive equation, so in such a term (and in the test function) the velocity is actually \( \mathbf{u} - \mathbf{u} \). For the latter velocity, we need an extra equation. Following Hu et al. [18], in order to guarantee a smooth mesh motion, we use the Laplace equation:

\[
\nabla \cdot (\epsilon \nabla \mathbf{u}) = 0 \tag{23}
\]

with boundary conditions:

\[
\mathbf{u} = \mathbf{U} + \alpha \times (x - x_p) \quad \text{on } \partial P(t) \tag{24}
\]

\[
\mathbf{u} = \mathbf{0} \quad \text{on } \Gamma_i, \quad (i = 1, \ldots, 4) \tag{25}
\]

In Eq. (23), the parameter \( \epsilon \) is taken equal to the inverse of the local element area in order to let the largest elements adsorb the most part of deformation. Eqs. (24) and (25) assure that the particle boundary nodes move following the particle motion, whereas no movement is prescribed for domain boundary nodes. The weak form for Eq. (23) can be derived in a standard way.

The DEVSS-G/SUPG formulation with a log-representation of the conformation tensor and an ALE particle mover provides a very efficient method in order to handle the system under investigation, giving satisfactory results for all the set of parameters considered in this work.

3.2. Implementation

For the discretization of the weak form, we use triangular elements with continuous quadratic interpolation \((P_2)\) for the velocity \( \mathbf{u} \), linear continuous interpolation \((P_1)\) for the pressure \( p \), linear continuous interpolation \((P_1)\) for velocity gradient \( \mathbf{G} \) and linear continuous interpolation \((P_1)\) for the log-conformation tensor \( s \).

Regarding the time discretization, we decouple the momentum balance and continuity equations from the constitutive one. As shown in the weak form, the mesh velocity, \( \mathbf{u} \), appears in the constitutive equation only. This allows us to solve separately the Laplace equation and the momentum balance.

Initially, the viscoelastic stress is set to zero in the whole domain. Since we neglect inertia, the initial condition for the velocity is not necessary. So, we can solve Eqs. (17)–(19) and the constraint Eq. (21) in order to get the distribution of the fluid velocity and the rigid-body motion of the particles, at the initial time step. Then, at every time step, we follow the following procedure:

**Step 1:** The particle position and rotation are updated. The new configuration is obtained by integrating the kinematic Eq. (14). The explicit second-order Adams–Bashforth method is used:

\[
x_p^{n+1} = x_p^n + \Delta t \left( \frac{3}{2} U_p^n - \frac{1}{2} U_p^{n-1} \right) \tag{26}
\]

The first time step of the simulation is performed with an explicit Euler method:

\[
x_p^{0+1} = x_p^0 + \Delta t U_p^0 \tag{27}
\]

**Step 2:** The mesh nodes, \( x_m \), are updated according to:

\[
x_m^{n+1} = x_m^n + \Delta t \mathbf{u}^n \tag{28}
\]

for the first time step and:

\[
x_m^{n+1} = x_m^n + \Delta t \left( \frac{3}{2} \mathbf{u}^n - \frac{1}{2} \mathbf{u}^{n-1} \right) \tag{29}
\]

when Eq. (26) is used to update particle position.

**Step 3:** The log-conformation tensor at the next time step, \( s^{n+1} \), is evaluated by integrating the constitutive Eq. (20). A second-order Cranck–Nicolson/Adams–Bashforth scheme is used:

\[
s^{n+1} = \frac{s^n + 1}{\Delta t} \left[ (2(\mathbf{u}^n - \mathbf{u}) - (\mathbf{u}^{n-1} - \mathbf{u}^{n-1})) \cdot \nabla s^{n+1} \right]
\]

\[
= \frac{s^n + 1}{2} (\mathbf{u}^n - \mathbf{u}) \cdot \nabla s^n + \frac{3}{2} G(G, s^n) - \frac{1}{2} G(G^{n-1}, s^{n-1}) \tag{30}
\]

where \( g \) is the term which appears in the evolution equation of \( s \) (Eq. (16)). Notice that the velocity \( \mathbf{u} - \mathbf{u} \) in the convective terms is according to the ALE scheme. In the time integration scheme above, \( 2\mathbf{u}^n - \mathbf{u}^{n-1} \) and \( 2\mathbf{u} - \mathbf{u} \) are used as velocity predictions for \( \mathbf{u}^{n+1} \) and \( \mathbf{u} \), respectively.

**Step 4:** The remaining unknowns \((\mathbf{u}, p, \mathbf{G}, U_p, \omega)^{n+1}\) as well as the Lagrange multipliers \((\lambda)\) can be found by solving Eqs. (17)–(19) and (21) using the particle configuration and the viscoelastic stress evaluated in the previous two steps:

\[
\int_{\Omega} 2\eta_1 \mathbf{D}(\mathbf{v}) : \mathbf{D}(\mathbf{u}^{n+1}) \, d\Omega - \int_{\Omega} \nabla \cdot \mathbf{v} \mathbf{p}^{n+1} \, d\Omega + \int_{\Omega} a(\nabla \mathbf{u})^T : \nabla \mathbf{u}^{n+1} \, d\Omega
\]

\[
- \int_{\Omega} a(\nabla \mathbf{u})^T : (G^{n+1})^T \, d\Omega
\]

\[
+ \langle \mathbf{v} - \mathbf{v} \times (x - x_p^{n+1}), \chi \mathbf{u}^{n+1} \rangle_{\partial \Omega} = - \int_{\Omega} \mathbf{D}(\mathbf{v}) : \mathbf{r}^{n+1} \, d\Omega \tag{31}
\]

\[
\int_{\partial \Omega} q \nabla \cdot \mathbf{u}^{n+1} \, d\Gamma = 0 \tag{32}
\]

\[
\int_{\partial \Omega} \mathbf{H} : \mathbf{G}^{n+1} \, d\Gamma - \int_{\partial \Omega} \mathbf{H} : (\nabla \mathbf{u}^{n+1})^T \, d\Gamma = 0 \tag{33}
\]

\[
\langle \mu, \mathbf{u}^{n+1} - (\mathbf{U}_p^{n+1} + \omega^{n+1} \times (x - x_p^{n+1})), \chi \mathbf{u}^{n+1} \rangle_{\partial \Omega} = 0 \tag{34}
\]

**Step 5:** Finally, the Laplace equation is solved:

\[
\nabla \cdot (\epsilon \nabla \mathbf{u}^{n+1}) = 0 \tag{35}
\]

with boundary conditions:

\[
\mathbf{u} = \mathbf{U}^{n+1} + \omega^{n+1} \times (x - x_p^{n+1}) \quad \text{on } \partial P(t) \tag{36}
\]

\[
\mathbf{u} = \mathbf{0} \quad \text{on } \Gamma_i, \quad (i = 1, \ldots, 4) \tag{37}
\]

and the mesh velocities are obtained.

In Step 4, a sparse linear symmetric system needs to be solved whereas Eq. (30) leads to an unsymmetric system. In both cases we use the parallel direct solver PARDISO [25].

3.3. Node mesh updating

In principle the ALE method allows to use the starting mesh until it becomes too distorted. When this occurs, a new mesh cover-
ing the same domain of the old one is generated and the solution is projected from the old on the new mesh. The projection step is generally done by interpolation: the element containing a new node is searched through the old mesh and, by using the nodal element values of the last solution, the fields are interpolated to give new nodal values. Such a procedure is time-consuming and leads to interpolation errors. Therefore it should be reduced as much as possible during the simulation.

Due to the particular geometrical system investigated in this work (a single circular particle in a channel), we may exploit the fact that the mesh velocities, $u$, can be chosen in an arbitrary way, and the remeshing step can be reduced to once–twice per simulation, or even be avoided.

Indeed, as will be shown in Section 5, the particle moves along the $x$-direction, while translating along the $y$-axis very slowly (particle cross-streamline migration). This is equivalent to fix the particle position along the $x$-axis and move the mesh nodes with an $x$-velocity given by the particle $x$-velocity, $U_p$. However, due to the periodicity along the $x$-axis, the new configuration (particle and mesh translated along the $x$-direction by a quantity related to the particle $x$-velocity) is equivalent to the one of the previous time step. It should be only recognized that the $x$-component of the mesh velocity is $\bar{u}_x = U_p$. Of course the particle can still move along the $y$-axis.

Therefore, such a procedure is implemented by solving in the Step 5 only the $y$-component of Eq. (23) and setting $\bar{u} = (\bar{U}_y, \bar{u}_y)$ in Eq. (30). In the Step 2 the mesh nodes are moved along the $y$-direction only and the mesh distortion is limited along the $y$-axis. Our simulations show that at most once–twice remeshing and projection steps are needed.

Finally, regarding the mesh quality, following Hu et al. [18], we use two monitoring parameters:

$$f_1 = \max_{1 \leq e \leq \mathcal{N}_e} (f_1^e) \quad \text{and} \quad f_2 = \max_{1 \leq e \leq \mathcal{N}_e} (f_2^e)$$

where $\mathcal{N}_o$ is the number of elements and:

$$f_1^e = |\log (V^e/V_0^e)| \quad \text{and} \quad f_2^e = |\log (S^e/S_0^e)|$$

with $V^e$ and $V_0^e$ the volume of the element $e$ and its value in the undeformed configuration, respectively; $S^e$ and $S_0^e$ are the aspect ratios of the element $e$ and its value in the undeformed mesh, respectively. The aspect ratio is defined as:

$$S^e = \left( S_0^e \right)^{\frac{3}{2}}$$

with $l^e$ the maximum length of the sides of the element $e$.

As soon as one of $f_1$ or $f_2$ is greater than 1.39 (i.e. element volume or aspect ratio is larger than four times or smaller than 1/4 of its original value) the mesh is considered too distorted and needs to be regenerated.

4. Convergence and code validation

4.1. Convergence in space and time

In this Section, the results about the convergence in space and time of our method are presented. In what follows, we report a limited set of example cases in order to show the sensitivity of the method implemented to mesh resolution and time step size. However it must be remarked that the convergence is checked for any flow and geometrical parameter presented in this work. In this Section as well as throughout the paper, non-dimensional quantities only are reported ($\cdot$ is used as characteristic time scale, and the gap size $H$ as the characteristic length).

In Fig. 2 a typical mesh used in the simulations is shown. The channel length ($L$) is chosen sufficiently large to avoid that the particle can ‘feel’ itself through the periodic boundaries. Notice the refinement around the particle and between the particle and the closest wall, where largest gradients are expected. The spatial convergence is checked by refining the mesh. In Table 1, some parameters of the mesh used are reported. The mesh in Fig. 2 is the one labeled as ‘M1’ (the coarsest one). The meshes in the table refer to a specific particle size and starting position. However, the number of elements is kept almost the same by changing the geometrical parameters.

In Fig. 3 the angular velocity, $\omega$, and the particle $y$-velocity, $V_p$, of a particle starting from $y_p = 0.3$ are reported for the meshes labeled as ‘M1’, ‘M2’, ‘M3’ and ‘M4’. The step size chosen is 0.005 and the other parameters are: $De = 1.0$, $\alpha = 0.2$, $\eta_p/\eta = 0.1$. $D_p/H = 0.1$.

For any mesh considered here similar trends are found: the migration velocity increases during the start-up and becomes almost constant as soon as the stress develop. The convergence is achieved for the mesh ‘M2’, although the comparison with ‘M1’ shows very small deviations ($\approx 1–2\%$ for $V_p$ and $\approx 0.5–1\%$ for $\omega$). A similar situation is found for $De = 2.0$ (not reported). However, we remark that the convergence severely depends on the fluid and geometrical parameters, and on the flow rate. In general, our convergence tests show that a finer mesh is needed as Deborah number increases (for $De = 4.0$ we need the mesh ‘M4’) and as $\alpha$ is re-

---

**Fig. 2.** Typical mesh used in the simulations. The mesh reported is the one labeled as ‘M1’ in Table 1.
duced (with \( \varepsilon \to 0 \) the Giesekus model tends to the Maxwell model, which gives serious convergence problems, as it is well known). On the other hand, for different particle sizes, leading of course to a different spatial distribution of triangles, about the same number of elements are sufficient to achieve the convergence.

Finally, the influence of time step size, \( \Delta t \), is investigated. We consider the mesh ‘M3’ and \( De = 0.1 \). In Fig. 4, \( V_p \) and \( \omega \) by varying \( \Delta t \) are considered. Even for the largest \( \Delta t \) considered, the curves superimpose. However, for \( \Delta t = 0.02 \) (dotted line), slight deviations are found in the start-up phase (see the inset on the right of Fig. 4) that are more and more pronounced as the particle starts close to the wall (such an effect has been reported in [26,27] too).

On the other hand, by changing fluid and geometrical parameters, we do not find any different behavior with respect to the one shown in Fig. 4. For these reasons, in all simulations in this work, \( \Delta t \) is chosen 0.015, giving a good compromise between simulation speed and accuracy.

### 4.2. Code validation

The method proposed (DEVSS/SUPG + ALE) is validated through a comparison with the fictitious domain method (FDM) [28]. In the latter, a regular mesh is used covering the fluid as well as the solid domain. The particle rigid-body motion is imposed inside the particle (or on its boundary if inertia is neglected [29,24]) through Lagrange multipliers. The details of the fictitious domain method used in this work (weak form and implementation) are reported in [24] with the only difference that here we used a weak implementation of rigid-body constraints instead of collocation [30]. We just give a brief description of the mesh used: since the particle moves along the \( x \)-direction because of the external flow field and migrates slowly (approaching the closest wall), the regular mesh used is refined in a narrow channel where the particle is expected to move. As soon as the particle, due to the migration along the \( y \)-direction, is going out the refined channel, a new mesh is generated by translating the channel (that, of course, partially covers the refined region of the previous mesh). The solution is then projected from the old to the new mesh. In such a way, we can concentrate elements where the particle is moving. Considering that the projection is done at most 3–4 times in a simulation, the accuracy is not compromised at all.

In Fig. 5 (left) the comparison between the two methods is done by considering the particle \( y \)-position as function of time. A good quantitative agreement is found for any particle starting position.

<table>
<thead>
<tr>
<th>Mesh label</th>
<th>#el. on the particle boundary</th>
<th>#el. in the mesh</th>
</tr>
</thead>
<tbody>
<tr>
<td>M1</td>
<td>40</td>
<td>4176</td>
</tr>
<tr>
<td>M2</td>
<td>50</td>
<td>6096</td>
</tr>
<tr>
<td>M3</td>
<td>60</td>
<td>8012</td>
</tr>
<tr>
<td>M4</td>
<td>80</td>
<td>12,978</td>
</tr>
</tbody>
</table>

**Fig. 3.** Particle \( y \)-velocity \((V_p)\) (left) and angular velocity \((\omega)\) (right) as functions of time for the meshes reported in Table 1.

**Fig. 4.** Particle \( y \)-velocity \((V_p)\) (left) and angular velocity \((\omega)\) (right) as functions of time for different time step size \((\Delta t)\).
The FDM predictions are always slightly higher than the ALE ones. As indeed is evident in Fig. 5 where $V_p$ versus time are reported (right), whereas a monotonic curve (initially decreasing and then increasing) is found for $V_p$ in the ALE method, oscillations are evident when the FDM is considered. Such oscillations are due to the relative motion of the particle boundary grid over the fixed grid. In general, the error is small but in the problem investigated $V_p$ is small as well and this results in strong oscillations around the exact value. Finally, considering that the oscillations are on average above the curve for ALE, the trajectories reported on the left of Fig. 5 can be justified. To decrease the velocity perturbations a further mesh refinement is needed and the dashed curves are expected to tend to the solid one.

To close the comparison between ALE and FDM, it must be taken into account the computational aspect as well. Our FDM simulations requires a calculation time about 10 times higher than the ALE method. This is quite easy to understand if one considers that the refinement around the particle is more effective when a boundary fitted mesh is implemented and an irregular, coarse mesh can be done far from the particle. On the other hand, the FDM mesh should satisfy some regularity far from the particle as well, limiting its coarseness. In conclusion, the problem of a single particle in a rectangular box is more effectively tackled by using a boundary fitted method. Instead, for a many particle system, where frequent remeshing and projection steps are required, along with the difficulties related to the mesh generation itself, it would be preferable to use a fictitious domain method, maybe together with a proper algorithm deforming and refining the regular mesh along the particle boundaries.

5. Simulation results

In this Section our simulation results are presented and discussed. As stated in Section 4, the time step size $\Delta t = 0.015$ provides time convergent solutions for any case investigated in this work. On the contrary, the spatial convergence is therefore checked for all the situations investigated.

In Fig. 6, the $y$-position of the particle center $y_p$ as function of time is reported for different starting positions $y_p,0$. The parameters chosen are: $De = 1.0$, $\alpha = 0.2$, $\eta_p/\eta = 0.1$ and $D_p/H = 0.1$. Due to the symmetry with respect to the centerline (at $y = 0$), the trajectories are symmetric as well, and only one-half domain ($y \geq 0$) is considered. The shaded region in the upper part delimits the channel zone...
where the particle cannot access because of its finite size: when the particle center is at $y = 0.45$, the particle touches the wall.

The trends in Fig. 6 show that, from any starting position, the particle moves towards the closest wall (the upper one). Of course, due to the symmetry mentioned above, a particle initially located with the center exactly at the channel centerline will stay there and only rotate. Notice also that, close to the upper wall ($y_p > 0.3$), the trajectories are steeper than those in the channel core, i.e., the particle migration is faster near to the walls. What is found here is completely different from the case of shearing in an inertialess Newtonian suspending fluid where no migration occurs, independently of the initial position. Finally, note that the curves do not touch the shaded region because our numerical method breaks down as soon as the particle is about $1/5D_p$ from the wall. The analysis of the situation of a particle touching the wall is beyond the scope of this work and requires different techniques of analysis. Although a direct quantitative comparison with Huang et al. [16] results cannot be carried out due to the different constitutive model used, our simulations qualitative agree with those of their work (of course the case of an inertialess and shear thinning fluid is chosen).

Particle $y$-velocities ($V_p$), are reported in Fig. 7 and the angular velocities, $\omega$, are plotted in Fig. 8. In both figures, after an initial transient due to the development of viscoelastic stresses, the kinematic quantities change slowly in time for particle near to the channel center, and at a slightly faster pace for particles nearer to the walls. In other words, in the system under investigation, no steady state is ever achieved and the particle motion is always transient in nature. Exceptions are a particle located on the channel centerline and a particle that reaches the wall. The outwardly directed migration can then be looked at as an instability of the dynamics at the centerplane: when the particle center of mass is midway between the walls, no transverse motion occurs, but any disturbance in the vertical position triggers the

![Fig. 7. Particle migration velocities ($V_p$) as a function of time ($t$) for different starting positions ($y_{p0}$). The other parameters are: $De = 1.0$, $\alpha = 0.2$, $\eta_s/\eta = 0.1$, $D_p/H = 0.1$.](image)

![Fig. 8. Particle angular velocities ($\omega$) as a function of time ($t$) for different starting positions ($y_{p0}$). The other parameters are: $De = 1.0$, $\alpha = 0.2$, $\eta_s/\eta = 0.1$, $D_p/H = 0.1$.](image)
migration. A very similar instability was very recently experimentally observed by Sullivan et al. [31] in pressure-driven flow of a bubble suspended in a viscoelastic liquid in microchannels.

The dynamics of the particle so far described is determined by the three state variables position, migration velocity, and angular velocity, reported as a function of time in Figs. 6–8 respectively. It is instructive to analyze such dynamics directly in the ‘phase space’ of the particle, i.e., the space formed by the state variables position, migration velocity, and angular velocity, respectively, for different starting positions (see Fig. 9). Concerning the migration velocity shown in Fig. 9, we may note that: (i) all the data collapse onto a single curve, regardless of the starting position, (ii) in the channel core the migration velocity scales linearly with position (see later), (iii) by approaching the wall a faster migration is observed and (iv) the curve passes through a maximum to abruptly decrease very near to the wall. In short, three different dynamics can be distinguished, pertaining to three different zones of the channel.

Concerning the angular velocity $\omega$ in Fig. 10, again data collapse on a single curve (when disregarding the initial transients), and different dynamics are found in different channel regions. In particular, notice the steep decrease of the angular velocity when approaching the wall.

The data reported in Figs. 9 and 10 are well described by the following functional forms:

$$V_p = m_1 y_p + m_2 y_p^2 + m_3 y_p^3$$ (41)

$$\omega = a_0 + a_1 y_p^2 + a_2 y_p^6$$ (42)

Some remarks: (i) the constant term in Eq. (41) is missing because of the symmetry around the channel centerline ($y_p = 0 \Rightarrow V_p = 0$), (ii) the linear term in Eq. (41) governs the dynamics in the core regime and (iii) the constant term in Eq. (42) is the angular velocity of a particle at the center plane under confined shear flow (compare with [27]).

It should be pointed out that the functions (41) and (42) do not come from any theory or physical consideration but are just the simplest polynomial fits of the simulation data. Indeed, the fitting polynomials will be different with different choices of $D_p/H$, $De$ and $x$ (see below), not only in terms of coefficients but even in power laws. Independently from those geometrical and fluid parameters, however, the functional forms in the core region ($y/H \ll 1$) are found to be universal, i.e., it is always $m_0 = 0$, $m_3 = 0$, $a_0 = 0$, $a_3 = 0$. Notice that for fitting the migration velocity we excluded the last four points because Eq. (41) cannot be used to predict the decreasing trend very close to the wall.

The functional form, Eq. (41), can be integrated to recover the particle trajectory over the channel:

$$\frac{dy_p}{dt} = V_p = m_1 y_p + m_2 y_p^2 \Rightarrow y_p = \frac{y_{p,0} m_1^{1/4} \exp(m_1 t)}{[m_1 + y_{p,0}^2 m_3(1 - \exp(4m_1 t))]^{1/4}}$$ (43)
with initial condition \( t = 0 = y_p = y_{p,0} \). The meaning of \( m_1 \) coefficient is apparent from Eq. (43): indeed, it represents the growth rate of the instability from the center line [31]. Eq. (43) is the master curve governing the particle trajectories in the whole channel (excluded the ‘terminal’ part). In other words, the behavior in the core portion of the channel as well as in the not-too-close-to-wall portion is completely defined by Eq. (43), regardless of the starting position. Notice that the core regime is governed by an exponential-like trajectory, as follows by setting \( m_5 = 0 \) in Eq. (43).

In Fig. 11, Eq. (43) is reported with a grey line. The symbols refer to the curves calculated through numerical simulations reported in Fig. 6. The overlap for any starting position \( y_{p,0} \) confirms the validity of Eq. (43) in the whole channel (except the terminal region): in conclusion, each individual trajectory, just translated in time, is part of an unique behavior.

It seems to appropriate to emphasize once more that such a relationship, Eq. (43), is not valid in the initial transient behavior, due to the ‘rapid’ development of fluid stresses. However, as evident in Fig. 7 or 8, 3–4 relaxation times are sufficient to extinguish the initial start-up and entering the ‘slow’ dynamics.

The just discussed master curves depend on \( D_p/H \) ratio and fluid parameters (\( De \) and \( x \)). In the following, the effects of geometrical and fluid parameters are separately investigated.

5.1. Effect of particle size

The influence of the particle size is investigated by varying \( D_p/H \) ratio (dimensionless diameter) at \( De = 1.0 \) and \( x = 0.2 \). In Fig. 12 the particle y-velocity \( V_p \) is reported as a function of particle y-position \( y_p \) for four different \( D_p/H \) values. The full circles refer to \( D_p/H = 0.10 \) and are the same data as in Fig. 9. The three different regimes identified in the previous section, the core, the intermediate and the close-wall regimes, exist even by varying the confinement. We notice that a stronger confinement (higher \( D_p/H \)) leads to a faster migration in the whole channel, in particular, the instability growth rate monotonically increases by increasing the dimensionless diameter, following a quadratic law. For whatever dimensionless diameter, however, simulation data are still very well fitted by the functional form in Eq. (41). The fitting curves (with the two parameters \( m_1 \) and \( m_5 \) evaluated by a least square method) are reported as grey lines in the figure. It is apparent that the curves well describe the particle dynamic except very near to the wall.

The validity of Eq. (41) for any \( D_p/H \) ratio investigated implies that the particle trajectories are in the form of Eq. (43). Such curves are reported in Fig. 13 (for \( y_{p,0} = 0.05 \)) where the faster migration at stronger confinement is evident.

Coming back to the three dynamical regimes mentioned above, Fig. 14 summarizes their sizes, \( (H^*) \), as the dimensionless diameter is varied. The separatrix between the core and intermediate regimes is identified by those data in Fig. 12 showing a deviation of 5% from the linear trend. The separatrix between the intermediate and the close-wall regimes is identified by 5% deviations of data from the grey curves in the same figure. Notice that the core regime covers the most part of the channel, especially as the confinement is weak, whereas the close-wall regime becomes larger and larger as the confinement is stronger.

![Fig. 11. Master curve given by Eq. (43) (grey line) and the simulated trajectories reported in Fig. 6 (symbols). The parameters are: \( De = 1.0, x = 0.2, \eta_s/\eta = 0.1 \). A representation of the particle is reported as well. The shaded area is the channel region unaccessible to the particle due to its finite size. In the figure is also reported a schematic representation of the particle, for \( D_p/H = 0.1 \), to show its dimension compared to the channel size.](image)

![Fig. 12. Particle migration velocities \( (V_p) \) as a function of particle y-positions \( (y_p) \) for different \( D_p/H \) values. The other parameters are: \( De = 1.0, x = 0.2, \eta_s/\eta = 0.1 \). The grey lines are the functions reported in Eq. (41) with the parameters calculated by fitting the simulation data. The shaded area is the channel region unaccessible to the particle due to its finite size (depending on the particle size).](image)
5.2. Effect of fluid rheology

The effect of Deborah number is now investigated, at a fixed \( D_p/H = 0.1 \). In Fig. 15 the migration velocities as a function of the particle position are reported for different De values. The black circles are the same data as in Fig. 9. It is immediately evident that the overall migration behavior is still found, i.e. we distinguish the linear core regime, a faster intermediate regime and a close-wall regime. Furthermore, the transitions among those three dynamics seems to be independent from Deborah number (for example look at the maxima of the curves).

An important effect of the De-value is the non-monotonic behavior of the instability growth rate visible in Fig. 15. Indeed, the slope of the migration velocity curve in the core regime first increases with De. The curve for De = 4.0 (black squares), however, lies in between those for De = 1.0 (black circles) and De = 2.0 (white squares). As a consequence, beyond some critical De-number (larger than unity), a particle moving in the central channel region migrates increasingly slower as De increases.

In the intermediate regime, on the other hand, the migration velocity increases monotonically with De. This latter behavior implies that the functional form in Eq. (41) cannot be used to describe the data in Fig. 15. Indeed, although the linear term is always present, the exponents of the nonlinear terms vary with De.

Another interesting aspect is the effect of the shear thinning on the particle motion. The Giesekus model, Eq. (5), predicts the shear thinning phenomenon in shearing flows modulated by the parameter \( \alpha \). The thinning is more and more pronounced as \( \alpha \) increases and, for \( \alpha = 0 \), the upper convected Maxwell model is recovered (no shear thinning). The influence of thinning on particle migration is then investigated by varying \( \alpha \). In Fig. 18 the particle trajectories...
for three different values of $\alpha$ are reported at fixed $De = 1$. By increasing $\alpha$, the particle moves faster towards the wall, i.e. the migration velocity is higher. Thus, a particle suspended in a Maxwell fluid is expected to experience the slowest migration rate.

Finally, we checked the validity of the behaviors just reported for a different $Dp/H$. By choosing $Dp/H = 0.2$, we found the same effect of $De$ and $\alpha$ on the particle migration.

5.3. Particle translational x-velocity

We finally consider the last state variable in a 2D one-particle system, namely, the particle x-velocity, $U_x$. Indeed, due to the external imposed flow, the particle translates along the x-direction for both Newtonian and viscoelastic suspending fluid, with a velocity that is in general different from the one ($U_{x,\text{unfilled}} = \gamma y$) experienced by the unfilled fluid at the same vertical position as the particle center.

In Fig. 19 the relative particle x-velocity, $U_x - U_{x,\text{unfilled}}$, usually termed the slip velocity, is reported as function of particle position for $Dp/H = 0.1$ for both the Newtonian and a Giesekus fluid with $De = 2.0$, $\alpha = 0.2$. In the whole channel, and for both fluids, the particle moves faster than the unfilled fluid, i.e., the particle ‘leads’ the fluid. For the Newtonian case, such a result was analytically demonstrated long ago [15].

The dashed curve in Fig. 19 is the Newtonian prediction. Due to the absence of the migration phenomenon, a particle in a Newtonian fluid keeps its horizontal velocity during the motion. In other words, the dashed curve only gives the locus of the translational velocities for different vertical positions. On the other hand, a particle in a viscoelastic fluid migrates while horizontally translating, hence its horizontal velocity changes in time: the solid line in Fig. 19 is travelled in time. As a matter of fact, an initial transient behavior characterized by a fast change in $U_x$ is found at start-up (the almost vertical lines departing from the Newtonian curve). The start-up is clearly visible in the inset where $U_x$ versus time is reported for two different starting positions. The dash-dotted line is the Newtonian prediction that is of course constant in time. In the non-Newtonian case, after the extinction of the initial fast transients, the horizontal velocity reaches a master curve from whatever initial starting position. Even with this state variable, therefore, a slow dynamics is attained, similarly to what observed for the migration velocity. The viscoelastic curve in Fig. 19 is higher than the corresponding Newtonian one in a large central part of the channel, whereas the trend is inverted as the particle is close to the wall.

6. Conclusions

The migration of a 2D single, circular particle in a sheared viscoelastic liquid is studied through direct numerical simulations. In order to pick the influence of the viscoelasticity of the suspending fluid, particle and fluid inertia are neglected.

An ALE particle mover is combined with a DEVSS-G/SUPG formulation together with a log-representation of the conformation tensor, in order to easily manage the particle motion and guarantee numerical convergence at finite Deborah numbers. Finally, the rigid-body motion on the particle boundary is imposed through Lagrange multipliers.

Particle trajectories as well as translational and angular velocities are reported for different fluid parameters and particle dimensions. Our simulations show that the particle migrates towards the closest wall regardless of its starting position, fluid and geometrical parameters, i.e. no stable equilibrium position in the channel is found. Indeed, the cross-flow migration scenario is that of an instability of the centerplane dynamics. Migration towards the walls is indeed observed in recent, accurate experimental data by Lormand and Phillips [14].

The main finding of this work is that, for a given set of geometrical and fluid parameters, the migration velocity behavior in the whole channel can be described by a single curve. As a consequence, regardless of the starting position (and after extinction of the initial transients due to the development of stresses), the particle trajectories collapse on a master curve.

By analyzing the migration velocity curves, three regions in the channel are readily identified, characterized by different particle dynamics. When the particle is close to the channel centerline, the migration velocity is found to always be linear with the particle vertical position. The slope of this linear portion of the migration velocity curve is the growth rate of the instability from the centerplane. Approaching the wall, a faster migration sets in, whereas, quite close to the wall, the migration velocity abruptly decreases.

The existence of these three regimes is confirmed by varying the particle dimensions and the fluid rheology. As expected, a stronger confinement leads to a reduction of the linear region and to a faster migration towards the wall. The instability growth rate is found to depend non-monotonically on Deborah number.

The different dynamics depending on the particle positions through the channel and on the particle size can be exploited in
applications where the migration plays a role. For example, in fluid–solid separation devices, the inlet section could be conveniently chosen such that a small particle experiences the linear slow dynamics whereas a larger particle is in the fast regime, optimizing the separation process.

Regarding the particle angular velocity, a decreasing trend is found as the particle approaches the wall and as the Deborah number increases. Finally, the particle horizontal velocity results show, for both Newtonian and viscoelastic fluid, an higher velocity than that of the unfiltered liquid at the same vertical position, i.e., the particle always ‘leads’ the fluid.

References