Coordinated control of a mechanical hybrid driveline with a continuously variable transmission

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Abstract

Requirements for a driveline with respect to fuel economy and driveability can generally not be met simultaneously even with a continuously variable transmission (CVT). A new driveline concept with an additional flywheel makes it possible to compensate for the engine inertia in transient situations. This allows the driveline to be operated for optimal fuel economy without losing driveability. However, to realize this objective it is inevitable to control the driveline in a coordinated way. In light of the system uncertainties and the unknown load as well as nonlinearities a nonlinear robust controller is proposed to generate the setpoints for engine torque and CVT ratio. The focus is on the control law for the CVT ratio setpoint. The controlled CVT regulates the power flow from the engine and flywheel to the wheels while compensating for the internal inertias. © 2001 Society of Automotive Engineers of Japan, Inc. and Elsevier Science B.V. All rights reserved.

1. Introduction

Increasing demands for fuel economy and driveability as well as growing strict government emission regulations are forcing researchers to continuously introduce advanced new technologies into the driveline. Vehicular drivelines with a CVT and a drive-by-wire (DBW) system have a great potential for improvements both in fuel economy and exhaust emissions. The CVT makes it possible to decouple, at least in a wide range, the engine speed $\omega_e$ from the wheel speed $\omega_w$. On the other hand, the DBW system decouples the engine throttle from the accelerator pedal. Therefore, with the combination of a CVT and a DBW system the required power at the wheels, which is interpreted according to the driver pedal position, can be generated in a wide range of engine operating points $(\omega_e, T_e)$. There has been a lot of investigations on how to make the most of this freedom [1,2]. One obvious choice is to make the engine operate in a point on or as close as possible to the so-called E-line, i.e. to the locus of operating points $(\omega_e, T_e)$ where the engine output power $P_e$ is delivered with optimal fuel efficiency. Then, for a given power $P_w$ at the wheels, the corresponding point $(\omega_e, T_e)$ on the E-line is determined, which will be used as setpoints for the CVT controller, respectively the engine controller. This strategy will, in general, not result in an acceptable vehicle performance if the driver requires fast transients. The main reason is that an engine, which is constrained to its E-line, can deliver more power only if $\omega_e$ increases. Hence, in response to a fast rise in the driver’s power demand, it is desired to increase $\omega_e$ as fast as possible by shifting the CVT down quickly. This, however, has a reversed effect on the obtainable vehicle acceleration because a large portion of the extra power, which becomes available if $\omega_e$ increases, is needed to accelerate the engine and hardly any extra power remains for the vehicle. In fast downshifting, it is even possible to withdraw energy from the vehicle, resulting in a large engine acceleration and a short vehicle deceleration. It is well known as a non-minimum phase response [3]. This response on a command to accelerate is clearly unacceptable. Modified strategies, such as tracking the E-line in stationary situations and leaving the E-line in transients, only partly solve this driveability problem [4]. As a mechanical solution for this driveability problem (see Fig. 1), a power assist unit is developed, consisting of a compact flywheel and a planetary gear set in parallel to the CVT [5]. The developed driveline is called zero inertia or ZI driveline since the engine inertia can be cancelled by the flywheel inertia. The planetary set is
incorporated to have opposite signs of engine and flywheel accelerations. As a result, the flywheel boosts the driveline during downshifting and absorbs energy during upshifting. This purely mechanical hybrid driveline can greatly improve the driveability while the system focuses on optimal fuel economy. However, the introduction of extra dynamics and nonlinearities in the driveline by this power assist unit complicates the design of a suitable driveline management system (DLM).

The remainder of this paper is structured as follows. In Section 2 a dynamical model for the ZI driveline is presented and the main idea behind this driveline is elucidated. Section 3 concentrates on the design of the DLM, using the model from Section 2. Some simulation results with the proposed DLM are presented, and the main idea behind this driveline is presented. Some simulation results with the proposed DLM are presented, and conclusions and some suggestions for future research are given in Section 4.

2. System description

The considered driveline (see Fig. 2) consists of the usual components (internal combustion engine, torque converter, DNR set, CVT, final reduction, differential gear and drive shafts) plus a planetary set and a flywheel. Throughout this paper it is assumed that the torque converter is locked and that the DNR set is in drive mode. Hence, the angular velocity of the primary CVT pulley is equal to the angular velocity of the engine. For each \( \omega_k \), the induced engine torque \( T_e \) is upper bounded by the wide open throttle (WOT) torque \( T_{\text{wot}}(\omega_k) \). For \( T_e < T_{\text{wot}}(\omega_k) \) it is assumed that the dynamical behaviour of the engine torque can be described by a first order system with the desired torque \( T_{e,d} \) (the setpoint for the local engine controller) as the input:

\[
T_e = k_e \cdot (T_{e,d} - T_e), \quad k_e > 0. \tag{1}
\]

The model for the CVT is given by

\[
\omega_s = \frac{i_c \omega_p}{k_f \eta}, \quad T_s = \frac{\eta}{k_c} T_p, \tag{2}
\]

where \( i_c \) is the continuously variable transmission ratio, \( \omega_s \) is the angular velocity of the secondary pulley, \( T_p \) is the torque from the belt on the primary pulley, \( T_s \) is the torque from the belt on the secondary pulley and \( \eta \) is the efficiency of the CVT. The CVT ratio \( i_c \) is upper and lower bounded by the so-called overdrive ratio \( i_{od} \) and respectively the low ratio \( i_{low} \). The efficiency depends, amongst others, on \( i_c \) and \( T_p \). Nevertheless, variations in \( \eta \) are neglected. It is assumed that the dynamical behaviour of the CVT can be described by a first order system with the desired ratio \( i_c,d \) (the setpoint for the local CVT controller) as the input:

\[
\frac{d i_c}{d t} = k_{cvt}(i_{c,d} - i_c), \quad k_{cvt} > 0. \tag{3}
\]

The flywheel (angular velocity \( \omega_f \)) is rigidly connected to the sun gear of the planetary set. The annulus gear (angular velocity \( \omega_a \)) is connected to the primary pulley and the engine via a gearing with fixed transformation ratio \( i_p \), thus \( \omega_k = i_p \omega_a \). Similarly, the carrier (angular velocity \( \omega_c \)) is connected to the secondary pulley via a gearing with fixed transformation ratio \( i_f \), thus \( \omega_k = i_f \omega_c \). The transmission ratio of the combination of final reduction and differential gear is \( i_d \), so \( \omega_d = i_d \omega_a \), where \( \omega_d \) is the angular velocity of the drive shaft at the output side of the differential. The planetary set enforces a kinematical relation between \( \omega_a, \omega_c \) and \( \omega_f \),

\[
\omega_f = (1 + z) \omega_c - z \omega_a, \tag{4}
\]

where \( z \) is the quotient of the radii of the annulus and the sun. With \( \omega_k = \omega_c/i_p \), \( \omega_a = \omega_c/i_s \), \( \omega_a = i_k \omega_a \) and the abbreviations \( x_c = z/i_s \) and \( x_k = (1 + z)/i_c \) it follows that

\[
\omega_f = (x_k - x_c) \omega_c. \tag{5}
\]

Hence, the flywheel is at rest for each \( \omega_k \) if \( i_c \) equals to the geared neutral ratio \( i_{gn} \):

\[
i_{gn} = \frac{x_c}{x_k}. \tag{6}
\]

The ratio \( z, i_s \) and \( i_k \) can be optimized such that \( i_{gn} \) has a desired quantity. The torques \( T_a \) on the annulus, \( T_c \) on the carrier and \( T_f \) on the sun are related by

\[
T_a = -z T_f, \quad T_c = -(1 + z) T_f. \tag{7}
\]

The moment of inertia of the subsystem flywheel and sun is \( J_s \). The power dissipation in this subsystem is modeled with a viscous damper \( b_f \), so the equation of motion becomes

\[
J_s \ddot{\omega}_f = T_f - b_f \dot{\omega}_f. \tag{8}
\]
The inertia of the driveline part from the engine up to and including the annulus (as seen at the engine) is lumped into the moment of inertia $J_e$. The equation of motion for this part is

$$J_e \ddot{\omega}_e = T_e - T_p - \frac{1}{i_a} T_a$$  \hspace{1cm} (8)

Similarly, the inertia of the driveline part from the secondary pulley up to and including the differential gear (as seen at this pulley) is lumped into the moment of inertia $J_s$. The equation of motion for this part is

$$J_s \ddot{\omega}_s = T_s + \frac{1}{i_c} T_c - T_a i_c$$

where $T_a$ is the torque of the drive shaft on the differential.

The external load on the vehicle is taken into account by the road-load torque $T_c$, consisting of a constant term $T_{roll}$ due to a rolling resistance, an air drag term $c_d \omega_w^2$ and a disturbance term $T_{\text{dist}}$ due to hill climbing/descending, wind gusts, etc.:

$$T_c = T_{\text{roll}} + c_d \omega_w^2 + T_{\text{dist}}$$  \hspace{1cm} (9)

Let $J_w$ be the equivalent moment of inertia of the vehicle (as seen at the wheels). Then the equation of motion for the vehicle is

$$J_w \ddot{\omega}_w = T_d - T_f.$$  \hspace{1cm} (10)

It is assumed that the drive shaft is flexible with stiffness $k_d$ and viscous damping $b_d$. Its inertia is neglected, so the model equation for this element is given by

$$\dot{T}_d = k_d (\omega_d - \omega_e) + b_d (\dot{\omega}_d - \dot{\omega}_e).$$  \hspace{1cm} (11)

With Eqs. (6) and (7) it is possible to eliminate $T_a$ and $T_c$ from Eq. (8). Next, the obtained relations can be combined by elimination of $T_p$ and $T_s$, using Eq. (2). Finally, substitution of Eq. (4) and of $\omega_d = i_c \omega_a = i_i \omega_e$ results in

$$J_{\text{drv}} \ddot{\omega}_e = T_e - \frac{i_i \dot{i}_c}{\eta} T_d - b_{\text{drv}} \omega_e - M_{\text{drv}} \omega_e \frac{\dot{i}_c}{\dot{r}},$$  \hspace{1cm} (12)

where $J_{\text{drv}}$ and $b_{\text{drv}}$ are the moment of inertia and the damping coefficient of the driveline (as seen at the engine). Both quantities are quadratic functions of $i_c$ whereas $M_{\text{drv}}$ is a linear function of $i_c$:

$$J_{\text{drv}} = J_e + (\chi_s i_c - \chi_e) \left( \frac{2 i_c \dot{i}_c}{\eta} - \chi_e \right) J_f + \frac{i_c^2}{\eta} J_e,$$  \hspace{1cm} (13)
2.1. ZI driveline analysis

To elucidate the main idea behind the proposed driveline with CVT, planetary set and flywheel, it is assumed that the flexibility of the drive shaft and the viscous damping may be neglected (i.e. \( \omega_k = \omega_d \), \( b_t = 0 \) and \( b_d = 0 \)). Then the behaviour of the driveline in this situation is described by

\[ (T_e - J_{eq} \dot{\omega}_e) \omega_e = (T_r + J_{wq} \dot{\omega}_w) \omega_e, \]

where \( J_{eq} \) and \( J_{wq} \) are given by

\[ J_{eq} = J_e - \omega_c \left( \frac{\omega_c}{\eta} - \omega_c \right) J_t, \]
\[ J_{wq} = J_w + \omega_c \left( \frac{\omega_c}{\eta_2} - \frac{\eta_2}{\omega_c} \right) J_t. \]

It follows that \( J_{eq} = 0 \) if \( \dot{\omega}_c = \dot{i}_{eq} \), where the so-called zero inertia ratio \( i_{eq} \) is defined by

\[ i_{eq} = \eta_{ign} \left( 1 + \frac{J_e}{\omega_c J_t} \right). \]

The conclusion is that the engine inertia is compensated by the flywheel inertia if \( \dot{\omega}_c = \dot{i}_{eq} \), that it is more than compensated if \( \dot{\omega}_c > \dot{i}_{eq} \) and that it is partly compensated if \( \dot{\omega}_c < \dot{i}_{eq} \). This explains the name zero inertia driveline for the proposed driveline.

Analyzing the power at wheels, it is clear that the additional flywheel has an important effect on vehicle driveability.

\[ P_w = T_e \omega_k - J_{eq} \omega_k \dot{\omega}_k = P_e - P_{dc}. \]

For a conventional powertrain, the power consumed by engine inertia expresses as \( P_w = J_e \omega_k \dot{\omega}_k \) whereas it is given by \( P_{dc} = J_{eq} \omega_k \dot{\omega}_k \) for the ZI driveline. Since \( J_e \) is always positive, it implies that the engine first uses part of \( P_e \) to accelerate itself while the rest can be used for accelerating the vehicle. This fact reveals how the engine inertia \( J_e \) influences driveability. In the extreme case, where the CVT ratio is regulated such that \( \omega_k \) is large enough to render the \( P_w \) negative shortly, the engine withdraws energy from the vehicle. The serious problem of driveability arises. However, for the driveline with flywheel its equivalent inertia at the engine, i.e., \( J_{eq} \), can be zero and even negative, at least smaller than that of without. For the case of \( J_{eq} = 0 \) it improves vehicle driveability greatly since power delivered by the engine can reach the wheels immediately while the flywheel supplies the power to accelerate the engine. Part of the flywheel energy may even be used for \( (P_w < 0 \) leads to \( P_w > P_e \)) accelerating the vehicle if \( J_{eq} < 0 \).

2.2. Model for control

The model for control design is a simplified version of the simulation model from the previous section. First of all it is assumed that the viscous damping terms may be neglected. Furthermore, it is noted that control of the engine is not the subject of this paper. Therefore, the dynamics of the engine torque is neglected and Eq. (1) is replaced by \( T_e = T_{ed} \). The relevant equations then can be written in state space form as

\[ \dot{x} = f(x) + g_1(x)u_1 + g_2(x)u_2 + qw, \]

where the state \( x \), the inputs \( u_1 \) and \( u_2 \), the disturbance \( w \) and the columns \( f \), \( g_1 \), \( g_2 \) and \( q \) are given by

\[ x = \begin{bmatrix} \alpha e_1 \\ \alpha v \\ T_d \\ i_c \\ \end{bmatrix}, \quad u_1 = T_{e,d}, \quad u_2 = i_{c,d}, \quad w = T_{dist}, \quad \Rightarrow (19) \]

\[ f = \begin{bmatrix} \frac{\eta}{\alpha e_1} (-\frac{\eta}{\alpha e_1} x_3 + M_{drev} k_{cvt} x_1) \\ \frac{1}{\frac{1}{T_{drev}}} (-T_{roll} - \epsilon_d x_2 + x_3) \\ \frac{1}{k_{cvt}} (i_c x_3 - x_2) \\ -k_{cvt} x_4 \end{bmatrix}, \quad \Rightarrow (20) \]

\[ g_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad g_2 = \begin{bmatrix} -M_{drev} k_{cvt} x_1 \\ 0 \\ 0 \end{bmatrix}, \quad \Rightarrow (21) \]

\[ q = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \quad \Rightarrow (22) \]

3. Driveline control

The input quantities for the driveline are the engine torque \( T_e \), the CVT ratio \( i_c \) and the road-load torque \( T_r \). \( T_e \) and \( i_c \) can be controlled and the DLM has to generate setpoints \( T_{e,d} \) and \( i_{c,d} \) for these inputs. Here, a feedback strategy is used to determine \( T_{e,d} \). The determination of \( i_{c,d} \) is based on a feedback strategy because the driveline model is not very accurate and because the long term variations of the road-load can be fairly large. This motivates the choice for a feedback approach to determine \( i_{c,d} \). The DLM is depicted in
Fig. 3. In this figure, PI interprets the pedal position as the desired power\(^1\) at wheels and then PF translates it into the desired power at the engine, FC determines the desired engine operating point, P represents the driveline model, NO is a nonlinear observer, NC is a nonlinear control block and NT is a nonlinear translator which interprets the output of NC as the setpoint for the CVT ratio. The feedforward strategy for \( T_{e,d} \) is outlined in the next subsection. After that an estimator for the road-load and a state observer are presented. Finally, the determination of the setpoint for the CVT ratio is discussed in the last subsection.

3.1. Engine steering

The target of the DLM is to make the power at the wheels track the desired power \( P_{w,d} \), where \( P_{w,d} \) is commanded by the driver. First the normalized position \( a \in [0, 1] \) of the accelerator pedal is translated into a desired power, using \( P_{w,d} = f(a) \), where \( f \) is a smooth, strictly increasing function with \( f(0) = 0 \) and \( f(1) = P_{\text{max}} \). Next the resulting \( P_{w,d} \) is interpreted as the required power at the engine in a stationary situation. This interpretation means that the desired engine power \( P_{e,d} \) can be determined from

\[
T_{e,d} = \frac{P_{w,d}}{\eta}
\]

In stationary situations, \( P_{e,d} \) has to be delivered with maximum fuel efficiency, i.e., in an engine operating point on the E-line. (See Fig. 4a. In this map, the power levels, WOT line and the brake specific fuel consumption lines are depicted as well.) This line can be represented in a mathematical form by

\[
\omega_e = \Omega_e(P_e), \quad T_e = \Gamma_e(P_e),
\]

where naturally \( \Omega_e(P_e)\Gamma_e(P_e) = P_e \). In transient situations, any operating point that is advantageous for the performance of the driveline can be used. If \( P_{e,d}(t) \) at time \( t \) is smaller than \( P_e(t) \) then the engine controller has to take care that \( T_e \) is decreased as fast as possible to the setpoint \( T_{e,d}(t) = P_{e,d}(t)/\omega_e(t) \). The determination of \( T_{e,d}(t) \) for the case that \( P_{e,d}(t) > P_e(t) \) is less trivial. The strategy that is adopted here is given by

\[
T_{e,d} = k_p T_{\text{wot}}(\omega_e) + (1 - k_p) \Gamma_e(P_{e,d}),
\]

where \( k_p \in [0, 1] \) is a strictly increasing function of \( \Delta P = P_{e,d}(t) - P_e(t) \) with \( k_p(0) = 0 \) and \( k_p(\Delta P) = 1 \) for large \( \Delta P \). Hence, if \( \Delta P \) is large then \( T_{e,d} \) will be equal to the maximum engine torque \( T_{\text{wot}}(\omega_e) \) whereas \( T_{e,d} = \Gamma_e(P_{e,d}) \) if \( \Delta P = 0 \). Such a engine strategy is given in Fig. 4b.

3.2. Road-load estimator

According to Eq. (9) the road-load torque mostly consists of disturbance \( T_{\text{dist}} \). The estimator to determine
the unknown disturbance is based on the assumption that \( T_e, \omega_e, \omega_w \) and the rate of ratio change are measured or determined with sufficient accuracy from measurements. The estimator model is a simplified version of the model from Section 2: the viscous damping and the flexibility of the drive shaft are neglected, so \( \omega_w = \omega_c \). Substitution of this relation in Eq. (10) and elimination of the drive shaft torque \( T_d \) from Eqs. (10) and (12) result in

\[
J_{tot} \dot{\omega}_e = T_e - \frac{i_1 k_c}{\eta} T_r - M_{tot} \omega_e \frac{di_c}{dt},
\]

where \( J_{tot} \) and \( M_{tot} \) follow from Eqs. (13) and (15) if \( J_s \) is replaced by \( J_s + J_v^2 \). It is assumed that the estimated values \( \hat{\omega}_e \) and \( \hat{T}_{dist} \) for \( \omega_e \) and \( T_{dist} \) are related by

\[
J_{tot} \dot{\hat{\omega}}_e = T_e - \frac{i_1 k_c}{\eta} T_r - M_{tot} \omega_e \frac{di_c}{dt},
\]

Let \( \varepsilon_e = \omega_e - \hat{\omega}_e \) be the difference between the measured value \( \omega_e \) and the estimated value \( \hat{\omega}_e \). Then (linearized around equilibrium point)

\[
J_{tot} \dot{\varepsilon}_e = -A \varepsilon_e - \frac{i_1 k_c}{\eta} (T_{dist} - \hat{T}_{dist}),
\]

\[
A = 2c_d \frac{i_1^2}{i_1^2} \omega_e + M_{tot} \frac{di_c}{dt}.
\]

Since the engine speed is measurable, the error between the measured and the observed is applied for estimating the road load. Based on this relation the estimator for \( T_{dist} \) is chosen as

\[
\hat{T}_{dist} = \frac{\eta}{i_1 k_c} (A - J_{tot} k_{ed}) \varepsilon_e,
\]

resulting in the error equation

\[
J_{tot} \dot{\varepsilon}_e = -J_{tot} k_{ed} \varepsilon_e - \frac{i_1 k_c}{\eta} T_{dist}.
\]

If the gain \( k_{ed} \) is large enough then \( \varepsilon_e \) and \( T_{dist} - \hat{T}_{dist} \) will become small. Some results, obtained with this estimator for a sinusoidal and a noisy disturbance, are shown in Fig. 5.

![Fig. 5. Estimated results.](image-url)
3.3. Nonlinear observer

The drive shaft torque $T_d$ is not measured and has to be determined, using a (nonlinear) observer. The measured quantities are $\omega_s$, the secondary pulley speed $\omega_s$ and $\omega_w$. The CVT ratio follows from $i_c = \omega_s/\omega_w$ and $T_{\text{dist}}$ is replaced by the estimate $\hat{T}_{\text{dist}}$. With the state $x := [\omega_s, \omega_w, T_d]^T$, the co-state $\zeta = [i_c, \hat{T}_{\text{dist}}]^T$, the input $u := [T_{\text{dist}}, i_{cd}]^T$ and the measurement column $y := [\omega_c, \omega_w]^T$ the model for observer design is given by
\[
\dot{x} = f(x, \zeta) + g_1(x, \zeta)u_1 + g_2(x, \zeta)u_2, \\
y = Cx,
\]
where $f$, $g$ and $C$ are given by
\[
f(x, \zeta) = \begin{bmatrix} \frac{J_{\text{drv}}}{M_{dc}} (M_{dc} k_{\text{cv}} x_1 - \dot{\omega}_w) \\ \frac{1}{c_1} (x_3 - c_2 x_2 - T_{\text{roll}} - Z_2) \\ k_a (i_l x_1 - x_2) \end{bmatrix}, \\
g_1(x, \zeta) = \begin{bmatrix} \frac{1}{J_{\text{drv}}} \\ 0 \\ 0 \end{bmatrix}, \\
g_2(x, \zeta) = \begin{bmatrix} -\frac{M_{dc} k_{\text{cv}} x_1}{I_{\text{drv}}} \\ 0 \\ 0 \end{bmatrix}, \\
C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}.
\]
The observer is given by
\[
\dot{x} = f(\hat{x}, \zeta) + g(\hat{x}, \zeta)u + L(\hat{x}, \zeta) \cdot (y - \hat{y}), \\
\dot{\hat{y}} = C\hat{x}.
\]
The components of the $(3 \times 2)$ gain matrix $L$ are chosen as
\[
L_{11} = -\frac{M_{dc}}{J_{\text{drv}}} k_{\text{cv}} (\dot{\omega}_1 - u_2) + \kappa, \\
L_{12} = 0, \\
L_{21} = 0; \\
L_{22} = 2c_4 \dot{x}_2 + \kappa, \\
L_{31} = -k_d i_l \dot{x}_1 - \kappa_l, \\
L_{32} = k_d + \kappa_l.
\]
With this choice the observation error $e := x - \hat{x}$ satisfies $\dot{e} = Ae$,
\[
A = \begin{bmatrix} -\kappa & 0 & -\frac{i_l}{J_{\text{drv}}} \\ 0 & -\kappa & \frac{1}{J_w} \\ \kappa_l & -\kappa_l & 0 \end{bmatrix}.
\]
$\kappa$ and $\kappa_l$ are used to place one eigenvalue of $A$ at $-\kappa$ ($\kappa > 0$) and two eigenvalues at $-\kappa/2$ if $\kappa$ and $\kappa_l$ are designed to satisfy:
\[
\kappa^2 = 4 \left( \frac{i_l}{J_{\text{drv}}} + \frac{1}{J_w} \right) \kappa_l^2.
\]
The final observer is a modification of the outlined one. The essential difference is that the obtained values for $L_{11}$, $L_{22}$, $L_{31}$ and $L_{32}$ are modified with a sliding mode term to obtain more robustness for system uncertainties and disturbances.

The results from the nonlinear observer are shown in Fig. 6. From the results, it is seems that the errors between the observed and real value for the shaft torque are much higher than those for the engine speed and wheel speed. This is due to the fact that the measured engine and wheel speed are directly fed back to the observer to improve its precision.

3.4. The CVT ratio setpoint

The starting point to determine the CVT ratio setpoint $i_{cd}$ is the control model from Section 2. However, the input $u_1 = T_e$ is replaced by the known value $T_{\text{cd}}$ whereas the disturbance $w = T_{\text{dist}}$ is replaced by $\hat{T}_{\text{dist}}$ taken as an internal variable. Hence, the term $g_1(x)u_1 + qw$ can be taken into account in the column $f$, resulting in
\[
\dot{x} = f(x) + g(x)u. \\
\]
Here $f$ is equal to the column according to Eq. (20) plus $g_1(x)u_1 + qw$. Furthermore, $g(x)$ is equal to $g_2(x)$ according to Eq. (21) and $u$ is equal to $u_2 = i_{cd}$. The output $y$ is the power $P_w$ at the wheels, so
\[
y = h(x), \\
h(x) = x_2 x_3.
\]
It is assumed in the sequel that the time derivative of $\hat{T}_{\text{dist}}$ may be neglected. First, input–output linearization is applied to this system [6]. With the Lie operator $\mathcal{L}$ it can be shown that
\[
\dot{y} = \mathcal{L}_f h(x), \\
y = \mathcal{L}_g^2 h(x) + \mathcal{L}_g \mathcal{L}_f h(x)u
\]
with $\mathcal{L}_g \mathcal{L}_f h(x) \neq 0$ if $i_c \neq i_{cd}$. Hence, the relative degree of this system equals 2 if $i_c \neq i_{cd}$. With the introduction of a new input $v$, defined by
\[
v = \mathcal{L}_f^2 h(x) + \mathcal{L}_g \mathcal{L}_f h(x)u
\]
and a new state $[y, \dot{y}]^T$ the linearized input–output equation becomes
\[
\begin{bmatrix} \dot{y} \\ \ddot{y} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} y \\ \dot{y} \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} v.
\]
The objective is then to find a feedback control law for the new input $v$, such that $y$ can track the desired output $y_d = P_{w, d}$ under the requirements, even in the presence of system disturbances and uncertainties. If the control law is given by:
\[
v = f_d + \lambda (y_d - y),
\]
then the error dynamics \(e = y - y_d\) becomes,
\[
\dot{e} + \lambda e = 0.
\]

For \(\lambda > 0\) the error dynamics has a steady state at \(e = 0\). The control law is modified with a sliding mode term to obtain more robustness for system uncertainties and disturbances.

\[
v = \dot{y}_d + \lambda \text{sat}\left(\frac{y_d - y}{\Phi}\right)
\]

where \(\Phi\) is a boundary layer for the sliding mode. In summary, the desired CVT ratio \(i_{c,d}\) is determined by Eqs. (28) and (29).

4. Results and conclusions

Fig. 7 shows the results of the proposed controller for an increasing the desired power at the wheels from 8 to 28 and 75 kW respectively. Around \(t = 2\) s the fact that wheel power of the flywheel assisted powertrain is higher than that of the normal powertrain represents the potential improvement of the vehicle driveability.

For the flywheel assisted powertrain, Fig. 7a reveals that the realized power at wheel vibrates between \(t = 4\) and \(8\) [s]. This is introduced by the fact that the \(J_{eq}\) changes sign if the CVT ratio equals \(i_{z}\), which causes \(i_{z}\) to become a strong attractor from the view of system dynamics. For the same reason, a peak response happens in Fig. 7b.

The results of Fig. 8 is given by a modified controller for the purpose of reducing the influence introduced by \(i_{z}\), which replaces the term of \(1/J_{eq}\) in \(\mathcal{L}_p \mathcal{L}_f h(x)\) of Eq. (28) with a nonlinear gain \(k(i_c)\). The gain \(k(i_c)\) depends on \(i_c\) and is always negative (since shift down normally contributes to accelerate the vehicle).

The flywheel assisted powertrain was discussed. It is shown that the vehicle can be controlled focused on fuel economy without losing its driveability. In a meanwhile,
a vehicle load estimator, an engine steering strategy, a nonlinear observer and a CVT ratio robust controller have been evaluated. Some first-step results for robust integrated powertrain control are given. The control problem around $i_c = i_{ci}$ is discussed however still be a topic of future research.

References