Severity of Tip-Out Induced Impacts in Drive Line Systems With Backlash

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Numerical simulations are used to study the transient behavior of a four degree-of-freedom, rotational piecewise linear system. This study focuses on the impact between bodies in a system with backlash as a result of a sudden step input and the associated transient response. The subsequent single sided impacts and double sided impacts are studied as a function of the amplitude of the step input and the size of the backlash. Transitions have been observed between double sided impact regions and single sided impact regions, which agree with earlier findings. However, in this paper a more complete overview of the boundaries is given. The severity of the impacts is quantified with the peak-to-peak acceleration of the impacting bodies. The increase in the size of step input increases the severity of the impacts. However, the increase in backlash size leads to an extremum in impact severity. This is a possible explanation for seeming contradictions in literature.

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1 Introduction

The dynamic systems in which moving components make intermittent contact with each other can often be modeled via piecewise approximations. Backlash is one such nonlinearity, which is omnipresent in dynamic systems and often reduces system performance and can increase wear. Gear trains are common power transmission applications where backlash due to the clearance between teeth is evident. Backlash can be modeled as a piecewise linear phenomenon [1]. The backlash between two bodies makes the system discontinuous in the stiffness but the associated elastic forces remain continuous. The backlash also introduces a discontinuity in the damping constants. In this case however, also the associated viscous damping forces become discontinuous. A detailed discussion of frictionless models of impact in rigid body systems is presented in Ref. [2]. Periodic orbits of backlash oscillators have been studied extensively [3–5]. Further research on the transient dynamic behavior of systems with backlash is essential in dealing with this phenomenon in system design [6,7]. An example of a problematic effect of transient dynamics in systems with backlash can be found in the drive line of a vehicle. Sudden steps in drive torque can cause the gears to impact. An example of such a step is an engine tip-out, a sudden change in the applied motor torque. The resulting impact, often referred to as clunk, can be heard as a dull sound and can damage the gears. Knowledge of this phenomenon can help in the design of transient engine control strategies.

In this paper we study the transient dynamic response of a four degree-of-freedom (4-DOF) system with backlash, which resembles a low order model of a drive line system. A numerical model is developed to study the effect of the system parameters on the transient behavior. Specifically, the number, nature, and severity of impacts are studied with respect to the loading conditions and system parameters. This paper is organized as follows. First, in the next section, the model of the system under study will be presented. The numerical simulation approach will be discussed in Sec. 3. Using this approach and the model a numerical investigation will be conducted and presented in Sec. 4 to study the effect of changes in system and loading parameters on transient dynamics. Finally, conclusions will be drawn and suggestions for future directions will be given.

2 System Model

The 4-DOF system studied in this paper is shown in Fig. 1 and represents a lumped mass, low order, torsional model of the drive line dynamics. The model has four rotational degrees-of-freedom resulting in the global coordinate vector \( \theta = [\theta_1 \ \theta_2 \ \theta_3 \ \theta_4]^T \). There is backlash located between the teeth of the gears with inertia \( J_3 \) and \( J_4 \). The vehicle inertia is assumed to be of orders of magnitude larger than the total drive line inertia and hence the drive line system is grounded at the fourth inertia \( J_4 \), by a spring \( k_s \), and a damper \( c_s \).

A time dependent input torque, as applied to inertia \( J_1 \), is considered to study transient dynamics and is represented by

\[
T(t) = T_s + T_d \delta(t)
\]

\[
I(t) = \begin{cases} 
1 & t \leq t_0 \\
0 & t > t_0 
\end{cases}
\]

Initially, both the static part and the dynamic part of the load is applied to the system. This causes inertias \( J_3 \) and \( J_4 \) to be in contact with each other. At \( t = t_0 \) the dynamic load \( T_d \) is released to simulate an engine tip-out. The ratio between the dynamic force and the static force will be referred to as \( \sigma_r \) and is given by

\[
\sigma_r = \frac{T_d}{T_s}
\]

The system model is assumed to be frictionless in order to study the effect of the piecewise linear behavior of the backlash without introducing other nonlinearities. Energy dissipation is accounted for by the introduction of linear viscous damping. Due to the high damping coefficients, a realistic response is acquired, which can be compared with previous experiments [6]. The state in which inertia \( J_3 \) and \( J_4 \) do not make contact is defined as the backlash state. The backlash state is characterized by

\[
|r(\theta_3 - \theta_4)| < 0.5 \phi_r
\]

where \( \phi_r \) represents the translational clearance between the gear teeth, which are located on a radius \( r \). In the backlash state, the
system dynamics break down into two uncoupled subsystems. The equations of motion in the backlash state can be written in matrix form as

\[ \mathbf{J} \ddot{\mathbf{q}} + \mathbf{C} \dot{\mathbf{q}} + \mathbf{K} \mathbf{q} = \mathbf{T} \]  

(5)

with

\[ \mathbf{J} = \begin{bmatrix} J_1 & 0 & 0 & 0 \\ 0 & J_2 & 0 & 0 \\ 0 & 0 & J_3 & 0 \\ 0 & 0 & 0 & J_4 \end{bmatrix}, \quad \mathbf{C} = \begin{bmatrix} c_1 + c_2 & -c_2 & 0 & 0 \\ -c_2 & c_2 + c_3 - c_4 & 0 & 0 \\ 0 & -c_3 & c_4 & 0 \\ 0 & 0 & 0 & c_5 \end{bmatrix}, \quad \mathbf{T} = \begin{bmatrix} T(t) \ 0 \ 0 \ 0 \end{bmatrix} \]

where \( k_2, k_3, \) and \( k_5 \) represent the stiffness coefficients and \( c_1, c_2, c_3, \) and \( c_5 \) are the damping coefficients of the associated links in the drive line system.

The two states in which inertia \( J_1 \) and \( J_2 \) make contact are defined as the sticking states. In the sticking states, the two subsystems couple and for

\[ r(\theta_4 - \theta_1) < -0.5 \phi_r \]

(6)

the equations of motion in the first sticking state can be written as

\[ J_1 \ddot{\theta}_1 = k_2(\dot{\theta}_2 - \dot{\theta}_1) + c_2(\dot{\theta}_2 - \dot{\theta}_1) - c_1 \dot{\theta}_1 + T(t) \]

\[ J_2 \ddot{\theta}_2 = -k_2(\dot{\theta}_2 - \dot{\theta}_1) - c_2(\dot{\theta}_2 - \dot{\theta}_1) + k_1(\dot{\theta}_1 - \dot{\theta}_2) + c_3(\dot{\theta}_1 - \dot{\theta}_2) + r k_3 \left( r \dot{\theta}_4 - r \dot{\theta}_3 + \frac{\phi_r}{2} \right) + r c_4 (r \dot{\theta}_4 - \dot{\theta}_3) - r \dot{\theta}_3 \]

(7)

\[ J_3 \ddot{\theta}_3 = -k_3(\dot{\theta}_3 - \dot{\theta}_2) - c_3(\dot{\theta}_3 - \dot{\theta}_2) + r k_4 \left( r \dot{\theta}_4 - r \dot{\theta}_3 + \frac{\phi_r}{2} \right) - r c_4 (r \dot{\theta}_4 - \dot{\theta}_3) - r \dot{\theta}_3 \]

\[ J_4 \ddot{\theta}_4 = -k_3 \dot{\theta}_4 - c_3 \dot{\theta}_4 - r k_5 \left( r \dot{\theta}_4 - r \dot{\theta}_3 + \frac{\phi_r}{2} \right) - r c_4 (r \dot{\theta}_4 - \dot{\theta}_3) - r \dot{\theta}_3 \]

(8)

The nominal system parameters are shown in Table 1. The coupled system (Eqs. (7) and (9)) has four natural frequencies at 3.2 Hz, 16.2 Hz, 30.8 Hz, and 258.3 Hz. The lowest natural frequency at 3.2 Hz resembles the lowest or shuffle frequency of a drive line system of a car in the first gear [6]. This mode corresponds to the in-phase motion of all the coordinates, which is known to occur in transient response and can cause gears to impact. All the modes have relatively high damping (dimensionless modal damping coefficients \( \zeta = 10\% - 20\% \)). In the next sections, two parameters will be varied.

- The load ratio \( \sigma_r \) will be varied between 0 and 10, i.e., \( T_d \) will be varied between 0 Nm and −10 Nm since \( T_i = -1 \) Nm.
- The rotational backlash defined as \( \phi_\theta = \phi_r / r \) will be varied between 0 rad and 0.35 rad, i.e., \( \phi_r \) will be varied between 0 mm and 35 mm.

### 3 Numerical Simulation Process

A flowchart of the numerical simulation process is shown in Fig. 2. The MATLAB solver ode45 is used with variable time stepping and event detection to trace the exact time points of state changes. The simulation starts in the second sticking state. The initial static deformation \( \theta(t) \) at \( t=0 \) s is determined by solving Eq. (9) for \( \theta(0)=0, \dot{\theta}(0)=0, \) and \( T(0)=T_i+T_p \). Subsequently, using the initial conditions \( \theta(0) = 0 \) and \( \dot{\theta}(0) = 0 \), the equations of motion are integrated until inertia \( J_1 \) and \( J_4 \) lose contact. At this
point, the simulation iterates to the exact time point at which contact was lost. Then, the simulation switches to the backlash state, as in Eq. (5). The uncoupled equations of motion are integrated up to the point at which an impact is detected. Again after the iteration to find the exact switching time, the process switches back to the relevant sticking state. This process continues until the simulation time span of interest is completed.

4 Numerical Results

4.1 Global Response. The global response of the system as depicted in Fig. 1 with the four coordinates $\theta = [\theta_1, \theta_2, \theta_3, \theta_4]^T$ is shown in Fig. 3 with the nominal system and loading parameters ($\sigma_r=5$ and $\phi_b=0.09$ rad). The system is in static equilibrium until $t_s=0.1$ s at which a step in the applied load causes a transient excitation of the system. The lowest resonance frequency of approximately 3 Hz can clearly be recognized. The vibrational response of the lowest mode is the cause of impacting gears [6]. In order to examine the transient behavior with respect to the backlash gap the relative motion between the teeth on inertia $J_3$ and $J_4$ is examined. This relative coordinate will be referred to as the backlash coordinate and is defined as $x_b=r(\theta_3-\theta_4)$. Figure 4 shows the time history of the backlash coordinate with the nominal system and loading parameters. It can be noted that the transitions between the backlash state and the sticking states are evident in this relative coordinate system. Furthermore, as the overall energy of the response decreases due to damping, the system switches from impacts on both sides to impacts on only one side of the backlash gap. Figure 5 shows a close-up of the nominal response ($\sigma_r=5$ and $\phi_b=0.09$ rad) at the first primary impact. This figure shows that in fact multiple impacts occur. This sequence of secondary impacts is often referred to as chatter [8]. After subsequent impacts, inertia $J_3$ and $J_4$ stick together.

4.2 Double and Single Sided impacts. The nominal response in Fig. 4 shows three primary impacts. These primary impacts can qualitatively be classified as a double sided (DS) impact or a single sided (SS) impact. A DS consists of a pair of primary impacts on opposite sides of the backlash gap. In this case the entire backlash gap is crossed. A SS consists of one primary impact and in this case the backlash gap is crossed only partially. The nominal response of Fig. 4 consists of one double sided impact and one single sided impact.

The load ratio ($\sigma_r$) and the size of the backlash ($\phi_b$) have strong influence on the number and nature of the impacts demonstrated.
by the response of the system. There exists a combination of both single and double sided impacts. Figure 6 illustrates the effect of variations in force ratio \( \sigma_r \) and rotational backlash \( \phi_0 \) on the number and nature of impacts. This figure is generated by performing numerical simulations in a two-dimensional parameter space spanned by the force ratio and the rotational backlash. The different regions represent a combination of single sided impacts and double sided impacts. In this figure NI stands for no impacts, SS(1) refers to one single sided impact and DS(2)SS(1) refers to two double sided impacts followed by one single sided impact. The results in this figure agree with earlier findings by Gurm et al. [6]. However, Fig. 6 gives a more extensive overview of the parameter space and shows that the boundaries between the regions are nonlinear. It can be noted that for a given load ratio, double sided impacts change into single sided impacts as the backlash size increases. Similarly, for a given backlash the complexity of impacts increase as the load ratio increases. This increase in complexity starts as either a single sided impact changing into multiple single sided impacts or into a double sided impact. By further increasing \( \sigma_r \), more complex patterns develop.

4.3 Impact Severity Quantification. A number of metrics has been developed to quantify impact responses [9]. These include global metrics that consider the complete time history and metrics that are based on the state just prior to impact such as the relative kinetic energy between the impacting bodies. In this report a metric based on information just after the impact is used: the peak-to-peak acceleration. This metric is solely based on primary impacts and therefore neglects the secondary impacts (chatter). The peak-to-peak acceleration of the third inertia \( J_3 \) is used for the quantification. Figure 7 shows an example of a time history of \( \dot{\theta}_3 \) corresponding to the nominal system response in Fig. 4. The minimum and maximum acceleration of an impact \( i \) are indicated by \( Q_{i,\text{min}} \) and \( Q_{i,\text{max}} \), respectively.

The peak-to-peak acceleration for an impact \( i \) is given by

\[
Q_i = \max(\dot{\theta}_i) - \min(\dot{\theta}_i)
\]  

(10)

The \( n \) peak-to-peak accelerations are combined in one single metric by taking the square root of the sum of squared peak-to-peak accelerations.

\[
Q = \sqrt{\sum_{i=1}^{n} Q_i^2}
\]  

(11)

Figure 8 shows the contour plot of \( Q \) as a function of the force ratio \( \sigma_r \) and the rotational backlash \( \phi_0 \). The increase in load ratio increases the overall impact severity. Furthermore, for small backlash sizes and not too small load ratios, the impact severity increases with increasing backlash size. However, there appears to be an extremum in the severity of impacts with increasing backlash size. This could be an explanation for the contradiction found by Krenz [7] (“clunk severity increases with drive line lash but not necessarily in a linear manner”) and Gurm et al. [6] (“For this particular system the dynamics therefore yield a case where reduced lash is leading to more severe case of clunk(s)”). The observed behavior can explain both these statements. The results can be important in the design of engine control strategies to reduce impact responses.

5 Conclusions and Recommendations

This paper extends the research conducted by Gurm et al. [6] on geared systems with backlash under impulsive loading. A numerical model of a four degree-of-freedom system has been developed. This model can be seen as a drive line system, which is reduced in complexity. The model includes backlash but ignores the effect of friction. The numerical model is used to study the transient dynamics as a result of a reduced drive torque, an engine
tip-out. Focus lies on the impacts that occur between the backlash bodies. These impacts have been qualified into combinations of double sided and single sided impacts as a function of the static/dynamic load ratio and the backlash size. The global trend in the numerical results is that more double sided impacts occur at higher load ratios and smaller backlash gaps. Furthermore, transitions have been observed between double sided and single sided impact regions. The results agree with earlier findings from Gurm et al. [6] but give a more extensive overview of the transitions between different regions. The severity of the impacts is quantified with the sum of squared peak-to-peak accelerations of the colliding bodies. This metric has been determined with changing load ratios and backlash sizes. Increasing load ratios increase the severity of the impacts. However, there appears to be an extremum in the severity of impacts with increasing backlash size. This result provides a possible explanation for seeming contradictions in literature [6,7].

Future work on this topic includes a detailed experimental investigation of the numerical observations including classification of the impact nature and severity. It is also essential to understand the role of secondary impacts in the estimation metric.

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