Vibration control by the use of a piezo actuator

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DCT 2009.089

Traineeship report

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Eindhoven, September, 2009
Contents

1 Introduction 1

2 Modeling of a cantilever beam 3
   2.1 Modeling .................................................. 3
   2.2 Model theory verification .................................. 6
   2.3 Model order reduction ..................................... 6

3 Simulation 9
   3.1 Introduction ............................................... 9
   3.2 Normal modes .............................................. 9
   3.3 Adding damping to the model .............................. 10

4 Designing the study model 13
   4.1 The basic parts ............................................ 13
      4.1.1 The beam .............................................. 13
      4.1.2 The shaker ........................................... 14
      4.1.3 The clamping structure .............................. 14
      4.1.4 The sensor ........................................... 14
   4.2 The actuator ............................................... 15
      4.2.1 Mounting the actuator .............................. 15
   4.3 Choosing the positions of the actuator, shaker and sensor ............... 16
CONTENTS

5 Test results

5.1 Linearity test ......................................................... 19

5.2 Frequency response measurements ................................ 20
Chapter 1

Introduction

The department CAE of the company PDE Automotive offers simulation and analysis services for automobile parts. This work is mainly done to calculate the stiffness and strength of mechanical parts. Modal analysis is applied as well to identify possible unwanted dynamics. Nowadays it is common to re-design parts when unwanted dynamics appear in simulations. A new area of interesse is to reduce or, when possible, totally remove this undesired behavior with vibration control technologies instead of re-designing. Possible benefits of this method are the savings of development and production costs, the interchangeability of car parts without an necessary pre-analysis and the possibility to cope with changing circumstances.

To acquire the necessary acknowledge for vibration control a casestudy is performed. A beam is being clamped at one side and brought into vibration. An controller has to damp this vibration with the use of a piezo-actuator. First this idea is simulated, later an demonstration model build for this study is used. Chapter 2 in this report treats the modeling of the beam. Since the company works with finite elemental programs, it is logical to use these programs (with NASTRAN specifically in mind) for the production of the model. Since a great part of the experience and expertise of the CAE lies within the use of these programs the importance to fully exploit these for further use is emphasized. A part of the study is used for investigating how this program exports the modeldata and how it has been manipulated. Chapter 3 reviews how this data can be processed and used for simulations. Chapter 4 will show how the demonstration model is designed and how other decisions where made regarding the study model. And finally chapter 5 will discuss the results.
Chapter 2

Modeling of a cantilever beam

2.1 Modeling

For modeling a beam that is clamped at one side and free at the other side (see figure 2.1) the Euler-Bernoulli beam theory is used. The Euler-Bernoulli beam model, for time-dependent loading, is governed by the partial differential equation

$$\rho \frac{\partial^2 v(x,t)}{\partial t^2} + \frac{\partial^2}{\partial x^2} \left( EI \frac{\partial^2 v(x,t)}{\partial x^2} \right) = f(x,t)$$

(2.1)

where $v(x,t)$ is the transverse displacement of the beam, $\rho$ the mass density per volume, $EI$ the rigidity, $f(x,t)$ the externally applied pressure loading and $t$ and $x$ indicate time and spatial axis along the beam axis. The following assumptions are made

- There is an axis of the beam which undergoes no extension or contraction. The $x$-axis is located along this neutral axis.

- Cross sections perpendicular to the neutral axis in the undeformed beam remain plane and remain perpendicular to the deformed neutral axis, that is, transverse shear deformation is neglected.

- The material is linearly elastic and the beam is homogeneous at any cross section.

- $\sigma_y$ and $\sigma_z$ are negligible compared to $\sigma_x$.

- The $xy$-plane is a principal plane.

The beam is divided into an infinite amount of smaller beam-elements. One short beam-element is being taken for further consideration (see figure 2.2). This element contains 2 nodes, each able to translate in the $y$-direction ($v_i$) and rotate around the $z$-axis ($\varphi_i$). For continuity each element should have the same deflection and slope comparing to the neighboring elements. The Euler-Bernoulli beam equation is based on the assumption that the plane normal to the neutral axis before deformation remains normal to the neutral axis after deformation. This assumption yields $\varphi = dv/dx$. The method of weighted residual, Galerkin’s method, is being
applied. Because there are four nodal variables for the beam element, assume a cubic polynomial interpolation function as trial function for $v(x)$

$$v(x) = c_0 + c_1 x + c_2 x^2 + c_3 x^3$$  \hspace{1cm} (2.2)

with $0 \leq x \leq l$. Using the assumption $\varphi = dv/dx$, $\varphi$ can now be expressed as

$$\varphi(x) = c_1 + 2c_2 x + 3c_3 x^2$$  \hspace{1cm} (2.3)

Evaluation of the boundary conditions for both nodes yields

\begin{align*}
v(0) &= c_0 = v_1 \\
\varphi(0) &= c_1 = \varphi_1 \\
v(l) &= c_0 + c_1 l + c_2 l^2 + c_3 l^3 = v_2 \\
\varphi(l) &= c_1 + 2c_2 l + 3c_3 l^2 = \varphi_2
\end{align*}  \hspace{1cm} (2.4)

The nodal degrees of freedom of the element are represented as $q = [v_1 \; \varphi_1 \; v_2 \; \varphi_2]^T$. Solving Eq. (2.2) for $c_i$ using $q$ and $H(x) = [H_1(x) \; H_2(x) \; H_3(x) \; H_4(x)]$ results for Eq. (2.2) in

$$v(x) = H(x)q$$  \hspace{1cm} (2.5)

with the so called Hermitian shape functions

\begin{align*}
H_1(x) &= 1 - \frac{3x^2}{l^2} + \frac{2x^3}{l^3} \\
H_2(x) &= x - \frac{2x^2}{l} + \frac{x^3}{l^2} \\
H_3(x) &= \frac{3x^2}{l^2} - \frac{2x^3}{l^3} \\
H_4(x) &= -\frac{x^2}{l} + \frac{x^3}{l^2}
\end{align*}

![Figure 2.1: One-sided clamped beam](image1)

![Figure 2.2: Degrees of freedom of one beam element](image2)
2.1. MODELING

Using this expression for $v$ and the Galerkin’s method (see [3] for further details) the second term of Eq. (2.1) results in the stiffness matrix of the beam element as follows

$$K_e = \int_0^l B^T E I B dx$$  (2.6)

where

$$B = H'' = [ H'_1 H'_2 H'_3 H'_4 ]$$  (2.7)

Assuming a homogeneous beam, the element stiffness matrix becomes

$$K_e = \frac{2EI}{l^3} \begin{bmatrix} 6 & 3l & -6 & 3l \\ 3l & 2l^2 & -3l & l^2 \\ -6 & -3l & 6 & -3l \\ 3l & l^2 & -3l & 2l^2 \end{bmatrix}$$  (2.8)

Comparable with this last solution the first term of Eq. (2.1) results in the element mass matrix as follows

$$M_e = \int_0^l \rho AH^T H dx$$  (2.9)

$$M_e = \frac{\rho l A}{420} \begin{bmatrix} 156 & 22l & 54 & -13l \\ 22l & 4l^2 & 13l & -3l^2 \\ 54 & 13l & 156 & -22l^2 \\ -13l & -3l^2 & -22l^2 & 4l^2 \end{bmatrix}$$  (2.10)

known as the consistent mass matrix. Now damping can be added accordingly to the Rayleigh damping for example like

$$C = \alpha M + \beta K$$  (2.11)

with the parameters $\alpha$ and $\beta$ left to be determined. The element-forces acting on the nodal freedoms can be expressed as $u_e(t) = [ Y_1 \ M_1 \ Y_2 \ M_2 ]^T$ (see figure 2.3).

![Figure 2.3: Forces acting on the element nodes](image)

Instead of dividing the beam into an infinite amount of elements, it is customary to assume that the displacements can be represented with good fidelity by an finite amount of elements. The justification for this assumption is that the bandwidths of actuators and sensors cannot suffice the highest frequency modes. By choosing the amount of elements sufficiently large, the flexible system is described adequately for the frequencies of interest. Adding the element matrixes together, the beam dynamics can now be expressed as

$$M\ddot{q}(t) + C\dot{q}(t) + Kq(t) = Fu(t)$$  (2.12)

with the force $u(t)$ applied to the dof pointed out by the vector F.
2.2 Model theory verification

With the use of the preceding a simple beam consisting of 3 elements is modeled for verification. The mass and damping matrices of this model appeared nearly the same for a similar model exported from Nastran. Small differences are probably mainly caused by data rounding. Therefore it is no surprise that the frequency responses of both models are pretty much identical. A remark on this is that both models are derived with the use of the finite element method and therefore possibly leading to same inaccuracy with respect to reality. Now that it has been shown for the theory to match the Nastran data, for the remainder of this study models exported from Nastran are used in this study.

![Figure 2.4: Bode diagram of the theoretic model and the NASTRAN-model.](image)

2.3 Model order reduction

The Nastran data used in section 2.2 was directly exported without any manipulations. However for more complex and larger models this data will be manipulated before it is exported for further use. As given in paragraph 2.1 the beam dynamics are represented as $M \ddot{q}(t) + C \dot{q}(t) + K q(t) = F u(t)$. Although this differential equation is already an simplification of reality, this system may contain so many degrees of freedom (abbreviated to dof’s) that a dynamic analysis will become very expensive or even impossible. This number of dof’s is unnecessarily large to represent the structural response well in a limited bandwidth. Besides that, only a few degrees of freedom are actually of interesse. The Rubin reduction method is used to reduce the number of dof’s without significantly influence the accuracy of the model.

To reduce the model without losing the dof’s of interest the set of $n$ dof’s is partitioned into $n_b$ boundary dof’s (these will remain unaffected after the reduction is completed) and $n_i$ internal dof’s where no boundary conditions or external forces are applied.

$$q(t) = \begin{bmatrix} q_b(t) \\ q_i(t) \end{bmatrix}$$ (2.13)
2.3. MODEL ORDER REDUCTION

The system structure can be partitioned accordingly into

\[
\begin{bmatrix}
M_{bb} & M_{bi} \\
M_{ib} & M_{ii}
\end{bmatrix}
\begin{bmatrix}
\ddot{q}_b(t) \\
\ddot{q}_i(t)
\end{bmatrix}
+ \begin{bmatrix}
K_{bb} & K_{bi} \\
K_{ib} & K_{ii}
\end{bmatrix}
\begin{bmatrix}
q_b(t) \\
q_i(t)
\end{bmatrix}
= \begin{bmatrix}
F_b(t) \\
O_i
\end{bmatrix}
\] (2.14)

Next an matrix \( B \) (which points out one boundary dof and the related data per column) is defined as

\[
B = \begin{bmatrix}
I_{bb} \\
O_{ib}
\end{bmatrix}
\] (n*nb) (2.15)

Looking at the undamped system the following eigenvalue problem is associated

\[
[K - \lambda M] u = 0
\] (2.16)

with \( n \) possible solutions for \( \lambda \) and \( u \) which are the eigenvalues and eigencolumns. Gathering the eigenvalues in the diagonal matrix \( \Lambda \) and the eigencolumns in the matrix \( U \) gives

\[
\Lambda = \begin{bmatrix}
\lambda_1 \\
\lambda_2 \\
\vdots \\
\lambda_n
\end{bmatrix}; \quad U = [u_1, u_2, \ldots, u_n]
\] (2.17)

Assume that the eigenvalues are sorted as: \( \lambda_1 \leq \lambda_2 \leq \ldots \leq \lambda_n \), so

\[
MU\Lambda = KU; \quad U^T MU = I; \quad U^T KU = \Lambda
\] (2.18)

In case of a model with a large amount of dof’s, the frequencies corresponding to the highest modes may exceed the frequency band of interest by far. For that reason the matrices \( \Lambda \) and \( U \) are partitioned as well into a part of kept-modes (indexed by \( k, k = 1, 2, \ldots, n_k \)) and a part of deleted-modes (indexed by \( d, d = n_k + 1, n_k + 2, \ldots, n \)) which will be compensated. The partitioning leads to

\[
\Lambda = \begin{bmatrix}
\Lambda_{kk} & O_{kd} \\
O_{dk} & \Lambda_{dd}
\end{bmatrix}; \quad U = [U_k, U_d]
\] (2.19)

Assuming the model does not contain any rigid body mode, the residual flexibility modes can be stated as

\[
\Phi = U_d\Lambda_{dd}^{-1}U_d^T B = (K^{-1} - U_k\Lambda_{kk}^{-1}U_k^T) B = G_{res}B
\] (2.20)

with \( G_{res} \) the residual flexibility matrix. Finally the transformation matrix \( R \) is

\[
R = [U_k, \Phi]_{n*(n_k+n_b)}
\] (2.21)

so that

\[
q(t) = R\hat{p}(t)
\] (2.22)

Transformation reduces the model into

\[
M^{red}\ddot{p}(t) + K^{red}p(t) = F^{red}u(t)
\] (2.23)

where

\[
M^{red} = R^T MR; \quad K^{red} = R^T KR; \quad F^{red} = R^T F(t)
\] (2.24)
CHAPTER 2. MODELING OF A CANTILEVER BEAM

The initial dof’s were the physical coordinates $q(t)$ and are now changed to generalized dof’s $p(t)$. For further use of the model the physical boundary dof’s are necessary and has to be recovered with an additional transformation (known as the coupling procedure of Martinez).

\[
\begin{bmatrix} q_b(t) \\ q_i(t) \end{bmatrix} = \begin{bmatrix} U_{bb} & U_{bf} \\ U_{ib} & U_{if} \end{bmatrix} \begin{bmatrix} p_b(t) \\ p_f(t) \end{bmatrix} \tag{2.25}
\]

The partitioned column $p_f(t)$ contains the modal coordinates related to the free-interface modes. The first subset of Eq. (2.25) is

\[
q_b(t) = U_{bb}p_b(t) + U_{bf}p_f(t) \tag{2.26}
\]

which gives

\[
p_b(t) = U_{bb}^{-1}q_b(t) - U_{bb}^{-1}U_{bf}p_f(t) \tag{2.27}
\]

This results in the following transformation

\[
p(t) = \begin{bmatrix} p_b(t) \\ p_f(t) \end{bmatrix} = \begin{bmatrix} U_{bb}^{-1} & -U_{bb}^{-1}U_{bf} \\ Q_{fb} & L_{ff} \end{bmatrix} \begin{bmatrix} q_b(t) \\ p_f(t) \end{bmatrix} = Tp^*(t) \tag{2.28}
\]

which completes the reduction into

\[
q(t) = \begin{bmatrix} q_b(t) \\ q_i(t) \end{bmatrix} = RT \begin{bmatrix} q_b(t) \\ p_f(t) \end{bmatrix} = R_{tot}p^*(t) \tag{2.29}
\]

where $q_b(t)$ are the necessary physical dof’s and $p_f(t)$ are the compensating residual dof’s.
Chapter 3

Simulation

3.1 Introduction

In the FEM program NASTRAN the beam is modeled and the system matrixes $M$, $C$ and $K$ are exported to MATLAB into equation

$$M\ddot{q}(t) + C\dot{q}(t) + Kq(t) = Fu(t) \quad (3.1)$$

For the simulation and further analysis of the beam dynamics, this equation has to be rewritten into a first order state space formulation. The state vector is introduced as

$$\bar{x}(t) = \begin{bmatrix} q(t) \\ \dot{q}(t) \end{bmatrix} \quad (3.2)$$

Equation (3.1) can now be expressed like

$$\ddot{x}(t) = \begin{bmatrix} \dot{q}(t) \\ \ddot{q}(t) \end{bmatrix} = \begin{bmatrix} 0 \\ -M^{-1}K -M^{-1}C \end{bmatrix} \begin{bmatrix} q(t) \\ \dot{q}(t) \end{bmatrix} + \begin{bmatrix} 0 \\ M^{-1}F \end{bmatrix} u(t) \quad (3.3)$$

In this equation the inverse of $M$ is taken. This calculation unfortunately lacks numerical accuracy because of the ill-conditioned matrix $M$. This problem is solved using the normal modes described next. The exported data from NASTRAN is reduced using the Rubin method described before. The resulting added compensating residual dof’s are undamped which unfortunately brings another numerical problem with it. This will be discussed in section 3.3.

3.2 Normal modes

The Rubin reduction method, described in section 2.3, is similar for a part to the normal modes. Hereby the eigenvalue problem is addressed as well. A structure without rigid body modes is assumed without damping to begin with.

$$[K - \lambda M] u = 0 \quad (3.4)$$
which lead to

$$\Lambda = \begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \vdots \\ \lambda_n \end{bmatrix}; \quad U = [u_1, u_2, \ldots, u_n]$$ (3.5)

The normal mode matrix $U$ can now be used as a coordinate transformation matrix as

$$q = U \eta$$ (3.6)

The normal mode vectors $u_n$ are generalized orthogonal, which means that the mass, stiffness and damping matrices can be diagonalized with an congruence transformation. A congruence transformation is performed by pre-multiplying by the transpose of the coordinate transformation matrix and then post-multiplying by same transformation matrix. This leads for the mass matrix to

$$U^T M U = \begin{bmatrix} \ddots & \vdots \\ \vdots & d_n & \ddots \\ \vdots & \vdots & \ddots \end{bmatrix}$$ (3.7)

By orthonormalize the normal mode vectors $u_n$ with respect to the mass, the corresponding $d_n$ becomes equal to one. Pre-multiplying with $U^T$ and transformation of Eq. 3.1 now leads to the new system equation

$$\ddot{\eta}(t) + C_n \dot{\eta}(t) + \Lambda \eta(t) = U^T F u(t)$$ (3.8)

with diagonal matrixes

$$U^T M U = I; \quad U^T C U = \Omega; \quad U^T K U = \Lambda$$ (3.9)

where

$$\Omega = diag(2\xi_n \omega_n); \quad \Lambda = diag(\omega_n^2)$$ (3.10)

$\omega_n$ stands for the natural frequency of each mode, $\xi_n$ for the viscous damping factor. The new system equations are completely uncoupled and the mass matrix is reduced to an identity matrix containing only ones so that a new first order state space can be formulated by

$$\dot{s}(t) = \begin{bmatrix} \ddot{\eta}(t) \\ \dot{\eta}(t) \end{bmatrix} = \begin{bmatrix} 0 & I \\ -\Lambda & -\Omega \end{bmatrix} \begin{bmatrix} \eta(t) \\ \dot{\eta}(t) \end{bmatrix} + \begin{bmatrix} 0 \\ U^T F \end{bmatrix} u(t)$$

$$= A_n s(t) + B_n u(t)$$ (3.11)

This formulation has the advantage that the inverse of $M$ does not have to be calculated, making the result numerical more reliable. The resulting states of this equations are generalized. To result in physical dof’s the generalized dof’s $\eta(t)$ have to be transformed back into the original dof’s $q(t)$. After this transformation the model represents the original model again without modifying $\eta$, see figure 3.1.

### 3.3 Adding damping to the model

The system model exported from NASTRAN is reduced with the Rubin method to simplify this model without reducing the accuracy significantly. The exported model consist partially of physical dof’s and for a part of compensating residual dof’s. The resonances that belong with these
added dof’s are however not damped at all and will cause problems with the simulation. The
gains at the natural frequency of the free interface modes go theoretically to infinitely. To solve
this problem for the simulation, damping has to be added by hand. This is done by adding
Rayleigh damping for the free interface modes. The general expression is $C = \alpha M + \beta K$ which
results for a critically damped modal expression in:

$$\Omega_n = 2 \sqrt{M_n K_n}$$

(3.12)

To influence the exported model from NASTRAN as minimal as possible a small fraction (0.1%) of
this critically damping is added (see figure 3.2).

Figure 3.1: Bode diagram with and without modal notation.

Figure 3.2: Bode of an exported model before and after adding damping.
Chapter 4

Designing the study model

For the production of the study model a few parts had to be designed and produced, for example the beam and the structure that will hold the beam. Some other more standard parts could be picked out of stock. The shaker, to bring the beam in vibration, and the sensor where already available at PDE and could be used well for this research. The piezo actuator had been delivered by a third party. In this chapter more details from these parts are being explained.

4.1 The basic parts

4.1.1 The beam

For designing the demonstration model, the cantilever beam, it’s obvious desired that the effect is taking place in a range of frequencies visible for the human eye. Therefore it has been chosen for the beam to have its first resonance at around 30 Hz. Working with standard metal beams available out of stock at PDE some dimensions and parameters are already determined with the length of the beam still unknown.

\[ f_n = \frac{K}{2\pi} \sqrt{\frac{EI}{\rho Al^3}} \]  

(4.1)

With \( K \) an coefficient, which is \( K = 3.52 \) for a cantilever in its first eigenfrequency. Now it can be calculated that the beam should have a length of 0.237 m in order to fulfill the desired dynamical behavior.
4.1.2 The shaker

The beam will be vibrated simply by the use of an electro-mechanic shaker connected to the beam. The shaker will be used to create unwanted vibrations in the beam. The idea is of course is to compensate for these unwanted vibrations with an actuator. The major drawback of using a shaker is that it needs to be connected to the beam somehow. In this case it is connected together with a thin bar. Obviously this is going to influence the dynamic behavior of the beam making it more difficult to model. In the case that its omitted in the model it will make it more difficult to compare results from the model with the study model. As long as the amplitude of the vibration of the beam is not extremely large, the influence of the connecting bar is not too much of a concern. The intention is to stick to the elastic deformation of the beam anyway to prevent any damage and keeping the dynamical behavior predictable.

4.1.3 The clamping structure

The beam first has to be attached to a very rigid structure preventing non-modeled dynamical behavior. In figure 4.1 the final test object can be seen with this structure. Furthermore it can be seen in the figure that the structure itself is placed on rubber-parts isolating it from surrounding vibrations. The right end of the beam is enclosed by two small plates preventing the beam from over-stretching. This could damage the actuator, and the beam possibly, if unprotected (see section 4.2 for more information about the actuator).

![Figure 4.1: The test object with the beam clamped onto a rigid structure.](image)

4.1.4 The sensor

For monitoring the results and making a closed loop control system possible, a sensor is being used. For this application an acceleration sensor is used simply because it was already available at PDE. Although this is the most cost beneficial available option it is also a less desirable one. The actuator (see next section) has already been determined and its force acts on a different manner on the beam than the acceleration sensor can detect. In other words, the chosen sensor detects
the reaction of the actuator indirectly making this combination non-collocated. This will effect
the maximum possible bandwidth of the controlled system and its stability. A more favorable
option would be another piezo acting as a sensor. In that case the sensor could for example be
placed on the other side of the beam making it collocated.

4.2 The actuator

In this study a stacked ceramic multi-layer actuator (SCMA) is used for damping the beam. The
actuator has a cross section of 5 mm x 5 mm and a length of 8 mm. It has a maximum free
displacement of 7 µm, an estimated blocking force of 1000 N and operates between 0 V and
60 V. Mounting the actuator on the beam without any precaution would result in positive and
negative axial stress during the tests. But SCMA’s are sensitive to pulling forces so this has to
be prevented. Furthermore the displacement of the actuator would result in a force acting on the
beam in only one direction. A force acting in both direction is to be preferred for the best control
effectiveness and results. Therefore the actuator is being mounted on the beam under a pre-load.
The amount of pre-load is taken half the maximum force the actuator can deliver. By taking an
offset voltage on the actuator the pre-load can be compensated for and the actuator is now able
to deliver a force on the beam in two directions. The static performances are influenced by the
available stroke that is still left due to the offset, hysteresis, creep, stiffness and load capability.
The maximum dynamic operating conditions are limited by the heating of the actuator caused
by dielectric losses. Since the focus is mainly on the lower frequencies and by sticking to the
maximum voltage, given by the manufacturer, this should not be of much influence.

4.2.1 Mounting the actuator

By using a laminar piezoelectric actuator it could be mounted onto the beam simply as in the fol-
lowing figure (see figure 4.2). The actuator is glued to ensure its bonded well with the structure.
More than that is not necessary, keeping it as simple as possible and reducing the influence of
the added material to a minimal.

![Figure 4.2: Laminar piezoelectric actuator glued to a beam.](image-url)
In this study a stacked piezoelectric actuator is used which can’t be glued directly onto the beam. The actuator delivers its force through its surface and it must only be stressed axially. Clamping or gluing the actuator on the sides could result in a significant loss of performance. Tilting and shearing forces have to be avoided as well (see figure 4.3).

Figure 4.3: The prescribed loading method for a stacked piezoelectric actuator.

Keeping these conditions in mind the following solution has been realized. The actuator is clamped and glued in between 2 small metal blocks to ensure a good load distribution (see figure 4.4). The metal blocks and the beam are hold together with the use of a bolt and wrench. This is accomplished by drilling a hole through the metal blocks and the beam. As can be seen in the figure, the actuator is not mounted directly against the beam. This is done because of the possibility of short circuiting it otherwise and other unwanted side-effects. A non conductive material has been put in between the actuator and the beam for extra security.

Figure 4.4: (Left) The actuator mounted on the beam. (Right) A schematic representation with a: metal blocks, b: the piezo and c: the beam.

Again for comparing results between models and practise these added materials is not ideally. The added metal blocks make the beam locally stiffer and the actuator will transduce its force on the beam on a different manner than modeled.

4.3 Choosing the positions of the actuator, shaker and sensor

At this stage in the project the positioning of the actuator, shaker and sensor had to be chosen to be able to finish the test object. With the use of the model of the beam (see chapter 2.1 Modeling for more info about this) figure 4.5 can be made. The first eigenfrequency of the beam is of the biggest interest but for maximizing results the first three eigenfrequencies are taken into account. Since the shaker can only deliver a force in the transverse direction of the beam this could cause problems for some frequencies. It can be seen in the figure that the second and third eigenfrequency collide with the neutral axis at some positions. If the shaker would be mounted at 120 mm from the left of the beam for example, it would be very difficult to bring the beam in its second eigenfrequency. Therefore it is logical to choose the position for the shaker somewhere between 0 and 120 mm for the best results.
4.3. **CHOOSING THE POSITIONS OF THE ACTUATOR, SHAKER AND SENSOR**

Since the actuator transmits its force in a different manner than the shaker it does not have this problem. In theory it could be placed anywhere on the beam. Only when considering the static situations, the position of the actuator is of influence. The vibrations are mainly visible at the tip of the beam. Placing the actuator as far as possible to the left would make its effect better observable than it would if placed near the tip. On top of that, the extra added weight to the beam near the tip would influence the dynamics of the beam more than it would when placed near the clamping. This is mainly because the excitation of the beam near its first eigenfrequency will be of much larger amplitude than with the next eigenfrequencies. Because it is desirable to minimize the influence of the dynamical behavior of the beam, it is beneficial to place the actuator, shaker and sensor together. For the installation of the actuator bolts are needed which can be used for mounting the shaker and sensor as well (see figure 4.4). This prevents adding more material and weight than necessary.

After these considerations, still a position for mounting the parts have yet to be chosen. To acquire more insight on the effect of the actuator placement, four different models are made in NASTRAN. Each model has the actuator placed on a strategic place on the beam with the shape of the beam in mind for the first three eigenmodes. With the use of the models, the frequency response are plotted (see figure 4.6) from actuator to the tip (expressed in lateral acceleration). The model with the most desirable dynamic behavior is chosen and the position of the actuator is determined.

![Figure 4.5: The modeled normalized eigenmodes of the beam.](image-url)
Figure 4.6: Frequency response from actuator to the tip-acceleration for different actuator placements.
Chapter 5

Test results

5.1 Linearity test

Standard feedback control normally contains both positive and negative feedback signals. In this case the actuator can only extend. It is not able to shrink from a static situation nor can it cope with pulling forces. This problem was solved by applying an offset voltage on the actuator combined with a build in pre-load (to prevent any pull force, see section 4.2). A simple test is performed to verify the output and linearity over the input range. The maximum allowed voltage on the actuator is divided into ten parts and for each part the step response for both increasing and decreasing voltage is measured and compared (see figure 5.1).

Figure 5.1: The test object with the beam clamped onto a rigid structure.

It can been seen that for almost the entire range the actuator reacts almost identical.
5.2 Frequency response measurements

Frequency response measurements have been done for further analysis. As can be seen in the next figures the coherence of the measurements from the shaker to the sensor are decent. The coherence from the piezo are not acceptable however. For further control applications and tests to be successful this has to be improved first.

![Coherence shaker](image1.png)

**Figure 5.2: Coherence shaker.**

![Coherence piezo-sensor](image2.png)

**Figure 5.3: Coherence piezo.**
5.2. FREQUENCY RESPONSE MEASUREMENTS

Figure 5.4: Transfer function estimate shaker.
Bibliography


