LPV control of an active vibration isolation system

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Abstract—The closed-loop performance of motion systems that suffer from non-stationary disturbances could benefit from knowledge of these disturbances. Indeed, if the disturbance could be measured, this measurement could be used to enable a linear parameter varying (LPV) controller to adapt itself to the current operating condition, resulting in a closed-loop system with an overall increased performance. In this paper, this idea is applied to an active vibration isolation system which suffers from non-stationary disturbances.

Index Terms—LPV, vibration isolation, frequency estimator

I. INTRODUCTION

In high-precision motion systems, vibrations of unknown frequency and amplitude can limit the performance. These vibrations can for instance be induced by floor vibrations that are transferred to the system or by forces that directly act on the system such as airflow from the air conditioning. Examples of processes where this occurs include high-resolution measurement equipment such as scanning electron microscopes [1] used for sub-micron imaging, and photolithographic wafer steppers and scanners [2] used to fabricate integrated circuits. For these systems, the typical amplitudes of the induced vibrations are of the same magnitude as the dimensions of the measured or manufactured objects, and therefore these vibrations limit the performance.

To provide such systems with a vibration free platform, vibration isolation systems are used, where disturbance isolation is achieved passively, actively, or both. Figure 1 shows a schematic representation of such a vibration isolation system. Herein, \( k, b \) are the isolator stiffness and damping, respectively, \( d \) represents the floor vibrations and \( f_d \) denotes the disturbance forces that act directly on the payload. The payload displacement in vertical direction is denoted by \( y \), and \( f_a \) is the force generated by the actuator. The goal of a vibration isolation system is to minimize the effect of the environmental disturbances \( d \) and \( f_d \) on the vertical velocity \( v = \dot{y} \) of the payload. The passive part of the vibration isolation system consists of a heavy payload supported by elastic springs and damping units, resulting in a mechanical low-pass system with typical resonance frequencies between 2 and 5 Hertz. Since the passive system is generally weakly damped, disturbance amplification at the natural frequency is often observed. Increasing the structural damping of the system may offer a solution, however, a major disadvantage of passive damping is that the disturbance rejection properties deteriorate. This is illustrated in Figure 2, where a typical frequency response of the transfer from \( f_d \) to \( y \) is depicted. Moreover, the passive type is, in principle, unable to attenuate disturbances \( f_d \) that directly act on the payload such as an exiting force generated in a mounted operating machine, airflow from the air conditioning, and acoustic excitations. By applying active damping near the natural frequency of the passive system, i.e., by active vibration isolation, a significant benefit in vibration isolation can be obtained.

In the active vibration isolation system (AVIS) we consider, active vibration isolation is achieved by controlled actuation of the payload, based on feedback of its absolute velocity. The absolute velocity is obtained via geophones while the actuation is performed by means of Lorentz actuators. A picture of the AVIS is shown in Figure 3. The schematic of one of the controlled vibration isolators is depicted in Figure 4 where \( u = f_a \) is the controller output, \( v = \dot{y} \) is the payload velocity in vertical direction, \( G \) denotes the transfer function of an isolator module, and \( C \) is the controller. A fourth-order model of an isolator of the AVIS is given by the transfer function [3]

\[
G(s) = \frac{m_2 s^2 + b_12 s + k_{12}}{m_1 m_2 s^4 + (m_1 + m_2)(b_12 s^3 + k_{12})},
\] (1)
with $m_1 = 950$ kg, $m_2 = 50$ kg, $b_{12} = 3 \cdot 10^2$ Ns$^{-1}$, and $k_{12} = 1.75 \cdot 10^6$ Nm$^{-1}$. The isolator’s passive stiffness and damping are $k = 4.25 \cdot 10^5$ Nm$^{-1}$ and $b = 2 \cdot 10^5$ Ns$^{-1}$, respectively. To reduce the effect of disturbances near the resonance frequency, controller $C$ is designed as the complex valued damper [3]

$$C(s) = k_d \left( \frac{s}{s + \omega_{lp}} \right)^2 \left( \frac{\omega_{lp}}{s + \omega_{hp}} \right)^2,$$  

(2)

which consists of a gain $k_d$, combined with a low-pass filter with cut-off frequency $\omega_{lp}$, and a high-pass filter with cut-off frequency $\omega_{hp}$. The choices for $\omega_{hp}$ and $\omega_{lp}$ are related to the limitations of the sensors and actuators, respectively. Since the geophones produce unreliable output below 0.1 Hz, and since actuator limitations occur beyond 100 Hz, the cut-off frequencies are chosen as $\omega_{hp} = 0.2\pi$ rad/s and $\omega_{lp} = 200\pi$ rad/s, and the gain is $k_d = 3 \cdot 10^4$ Ns$^{-1}$. This controller is designed to preserve the desirable properties at high frequencies of the passive isolator, while it significantly reduces the effect of disturbances around the resonance frequency. In Figure 5 the Bode diagram of the uncontrolled plant (i.e., of the passive vibration isolation system)

$$P_p(s) = \frac{v(s)}{f_d(s)} = \frac{sG(s)}{1 + (bs + k)G(s)},$$  

(3)

is compared with that of the the controlled plant (i.e., of the active vibration isolation system)

$$P_a(s) = \frac{v(s)}{f_d(s)} = \frac{sG(s)}{1 + (bs + k)G(s) + sC(s)G(s)}.$$  

(4)

The maximum amplitude occurs around the resonance frequency, which implies that the plant still is most sensitive to disturbances in this frequency region. However, the sensitivity in this region is reduced by about a factor 17. The price paid is a slightly increased amplitude between 10 and 20 Hz.

In this paper, the payload of the AVIS is used as an experimental benchmark representing a metrology frame that needs to be isolated from environmental disturbances. This means that the amplitude of the vertical velocity $v$ should be controlled to be as low as possible. A machine is mounted to this metrology frame that performs periodic tasks, which result in non-stationary environmental disturbances $f_d$ with frequency content between 4 and 10 Hz, depending on the specific task that is performed. This machine is represented by a rotational imbalance, depicted in Figure 6. The induced disturbances $f_d$ are not known beforehand and cannot be measured directly. As a consequence, direct disturbance compensation via feedforward cannot be applied. Although other disturbance sources are present as well, the periodic disturbance generated by the mounted machine is expected to dominate the measured output. All that is known about the induced disturbance is that it consists of a single, time-varying frequency in the range 4 to 10 Hz. Although controller (2) reduces the effect of disturbances in this range, the achieved isolation performance is considered not to be adequate. To improve the isolation performance, controller (2) will be adjusted in order to provide additional disturbance reduction.

A classical solution to reduce disturbances at specific frequencies is the use of an inverted notch filter in the controller to increase the gain at that frequency. Such a notch filter results in a decreased sensitivity for disturbances in a small frequency range around the center frequency of the notch. Unfortunately, a classical notch filter cannot be applied to the posed problem for two reasons. Firstly, the frequency of the expected periodic disturbance varies in an interval between 4 and 10 Hz, while a fixed notch is only effective around a single frequency. Secondly, even if the disturbance signal would be stationary instead of time-varying (for instance when the mounted machine is operating in a stationary condition), the actual disturbance frequency, and hence the target

![Figure 3](image1.png)

Fig. 3. Active Vibration Isolation System (AVIS).

![Figure 4](image2.png)

Fig. 4. schematic of the controlled isolation system.

![Figure 5](image3.png)

Fig. 5. The Bode diagram of the transfer function from the environmental disturbance $f_d$ to the measured output $v$ of the passive isolation system (black) and of the active isolation system (grey).

II. PROBLEM SETTING AND PROPOSED APPROACH

The isolator's passive stiffness and damping are $k = 10 \text{Nm}$ and $b = 10 \text{Ns}$, respectively. To reduce the effect of disturbances near the cut-off frequencies are chosen as $\omega_{lp} = 10 \text{Hz}$, and since actuator limitations occur beyond $100 \text{Hz}$, the cut-off frequencies are chosen as $\omega_{hp} = 0.2\pi$ rad/s and $\omega_{lp} = 200\pi$ rad/s, and the gain is $k_d = 3 \cdot 10^4$ Ns$^{-1}$. This controller is designed to preserve the desirable properties at high frequencies of the passive isolator, while it significantly reduces the effect of disturbances around the resonance frequency. In Figure 5 the Bode diagram of the uncontrolled plant (i.e., of the passive vibration isolation system)
center frequency of the notch, is not known beforehand. In this paper, we propose a solution to improve the isolation performance that circumvents these two problems. The problem of the unknown frequency will be tackled by devising an algorithm that identifies the dominating disturbance frequency from available measured data from the AVIS. The problem of frequency variation can be solved by extending controller (2) with a notch of which the center frequency can be varied. This way, the controller can adapt the center frequency of the notch to the identified disturbance frequency, resulting in an improved isolation performance at that frequency. The problem requires a frequency identifier that is able to determine the actual frequency of the disturbance from the measured signals of the AVIS. Since the proposed LPV controller is required to adapt itself to the disturbance that acts on the system, ideally the exact disturbance frequency at the current time instant should be available. Unfortunately, the Heisenberg uncertainty principle [4] implies that it is not possible to have both a high time resolution \( \Delta t \) (accuracy with which the moment in time where the frequency is present can be determined), and a high frequency resolution \( \Delta f \) (accuracy with which the frequency can be determined). The uncertainty principle states that the product of \( \Delta t \) and \( \Delta f \) is lower bounded. This means that it is possible to identify a frequency component \( f_n \) with either a high frequency resolution and a low time resolution, or with a high time resolution and a low frequency resolution, but not with both. Since in practical applications low frequency components often appear as short bursts, a so-called multiresolution spectrum is desirable. Such a multiresolution spectrum combines a high frequency resolution (with a corresponding low time resolution) for low frequencies with a high time resolution (and hence a low frequency resolution) for high frequencies. To obtain the frequency spectrum of a measured signal, various methods are available, of which the Fourier transform (FT) [4] is probably the most widely used. Unfortunately, the FT offers only the global frequency content of the signal without any time information. An adapted version of the FT, called the short time Fourier transform (STFT) [5], is able to retrieve both frequency and time information of the signal. The major disadvantage of the STFT, however, is that it has a fixed resolution, i.e., the desired multiresolution spectrum cannot be obtained by this transform. The wavelet transform [4] was developed as an alternative approach to the short time Fourier transform to make a multiresolution analysis possible.

**III. CONTROLLER DESIGN**

The frequency of the dominating disturbance that acts on the system can be identified by using a wavelet-based algorithm as discussed in the previous section. This information can be used to adapt controller (2) in order to improve the isolation performance at that frequency. Such a controller can be designed via the LPV synthesis framework [6] using frequency and parameter dependent weightings expressing the desired performance. For this specific application however, it is expected that the desired performance can be achieved by extending controller (2) with an LPV notch filter, the center frequency of which is adapted to the identified disturbance frequency. Therefore, in this section we will design the controller by hand and analyze the closed-loop stability afterwards.

An LTI notch filter is described by the transfer function

\[
H_n(s) = \frac{s^2 + 2\beta_1(2\pi f_n)s + (2\pi f_n)^2}{s^2 + 2\beta_2(2\pi f_n)s + (2\pi f_n)^2},
\]

where \( f_n \) is the center frequency in Hz, and \( \beta_1 \) and \( \beta_2 \) are parameters that can be used to alter the reduction factor and the width of the notch. Although ideally all three parameters \( f_n, \beta_1, \) and \( \beta_2 \) should be adapted depending on the disturbance characteristic, in this paper we only consider the adaptation of the center frequency \( f_n \). Several factors play a role in the determination of the constant reduction factor and width of the notch filter. If only steady-state disturbance rejection around the center frequency \( f_n \) is considered, a wide notch with a large reduction factor is desirable. Indeed, a wide notch will not only reduce disturbances at \( f_n \), but has also good disturbance rejection properties for frequencies near \( f_n \), while a larger reduction factor results in a higher reduction. Unfortunately, there are also downsides to using such a notch filter. Widening the filter reduces the disturbances in a larger area around the center frequency, but at the same time results in amplification of disturbances in another, larger, frequency region. This is illustrated in Figure 7(a), where the frequency response of the original actively controlled
plant (4) is compared to that of the same controlled plant including a notch filter of different widths. The same holds if the reduction factor of the notch is increased. Another issue related to the width and reduction factor of a notch is its transient response time. A short transient response time of the notch filter is desirable since it allows a fast adaptation of the controller to the current disturbance frequency. A wide notch filter has a shorter transient response time than a narrow one, just as a filter with a smaller reduction factor has a shorter transient response time than one with a higher reduction factor. The effect on the transient response caused by widening the notch filter is illustrated in Figure 7(b), where the response of the controlled plant to a sinusoidal disturbance of 10 Hz is depicted with the same notch filters of different width.

The transient response time of the notch filter depends on the center frequency similarly as the frequency identification time of the real-time frequency analyzer. Therefore, in a good overall controller design, the transient response time of the notch filter for a certain frequency, and the time it takes to identify that frequency should be matched. We will use a notch that reduces the amplitude of a sinusoidal disturbance with a frequency corresponding to the center frequency \( f_n \) to approximately half of the final reduction in 3 periods. Furthermore, we choose a reduction factor of 30. The above discussion results in \( \beta_1 = 0.6 \) and \( \beta_2 = 0.02 \).

IV. STATE-SPACE REALIZATION AND STABILITY

To arrive at a closed-loop state-space system from disturbance input \( f_d \) to measured output \( v \), we start by representing the uncontrolled AVIS (the passive vibration isolation system) (3) in state-space form as

\[
\dot{x} = Ax + Bf_d \\
v = Cx,
\]

where \( x \in \mathbb{R}^4 \) is the state vector, and \( A, B, C \) are matrices of appropriate dimensions. The state-space description of the original controller (2) is given by

\[
\dot{x}_o = A_o x_o + B_o v \\
u = C_o x_o,
\]

where \( x_o \in \mathbb{R}^4 \) is the original controller state vector, and \( A_o, B_o, C_o \) are matrices of appropriate dimensions. This controller will be augmented with a notch filter that adapts itself to the dominating disturbance. The LTI notch filter (5) can be changed to an LPV notch by using the center frequency as the scheduling variable, i.e., \( \delta := 2\pi f_n \). This LPV notch can be represented by various state-space realizations. It is well-known that the complexity and conservatism of the stability and performance analysis of LPV systems depends on the type of parameter dependence of that system [?] For instance, systems that are affinely parameterized allow a quadratic stability analysis without a relaxation gap, and hence we aim for an affinely parameterized closed-loop system. Since the state-space description of the controlled AVIS is affine in the controller state-space matrices, an affinely parameterized state-space description of the controller results in an affinely parameterized state-space description of the controlled AVIS.

The LPV notch will be described by the modal canonical state-space representation

\[
\begin{bmatrix}
\dot{x}_n \\
u
\end{bmatrix} = \begin{bmatrix}
A_n(\delta) & B_n(\delta) \\
C_n(\delta) & D_n(\delta)
\end{bmatrix} \begin{bmatrix}
x_n \\
v
\end{bmatrix},
\]

where \( x_n \in \mathbb{R}^2 \) is the notch state vector and where

\[
\begin{bmatrix}
A_n(\delta) & B_n(\delta) \\
C_n(\delta) & D_n(\delta)
\end{bmatrix} =
\begin{bmatrix}
\delta & \delta \\
\frac{-\delta^2 + 1}{\beta_2^2 - 1} & \frac{-\delta^2 + 1}{\beta_2^2 - 1}
\end{bmatrix}
\]

(9)

which is indeed affine in \( \delta \). The series connection of (8) and the original controller (7) results in the affine parameter dependent controller

\[
\begin{bmatrix}
\dot{x}_c \\
u
\end{bmatrix} = \begin{bmatrix}
A_c(\delta) & B_c(\delta) \\
C_c(\delta) & D_c(\delta)
\end{bmatrix} \begin{bmatrix}
x_c \\
v
\end{bmatrix},
\]

(10)
where $x_c^T = [x_d^T \ x_o^T]^T$ and
\[
\begin{bmatrix}
A_c(\delta) & B_c(\delta) \\
C_c(\delta) & D_c(\delta)
\end{bmatrix} =
\begin{bmatrix}
A_{c}(\delta) & 0 & B_{c}(\delta) \\
0 & A_{o} & B_{o}D_{o}(\delta)
\end{bmatrix}.
\]

The closed-loop state-space system from disturbance input $f_d$ to measured output $v$ is then given by
\[
\begin{bmatrix}
\dot{x}_d \\
v
\end{bmatrix} =
\begin{bmatrix}
A(\delta) & B(\delta) \\
C(\delta) & D(\delta)
\end{bmatrix}
\begin{bmatrix}
x_d \\
f_d
\end{bmatrix},
\]
where $x_{d}^T = [x^T \ x_o^T]^T$ and
\[
\begin{bmatrix}
A(\delta) & B(\delta) \\
C(\delta) & D(\delta)
\end{bmatrix} =
\begin{bmatrix}
A & -BC_c(\delta) & B \\
B_c(\delta)C & A_c(\delta) & 0 \\
0 & 0 & 0
\end{bmatrix}.
\]

Note that (12) is affine in the scheduling parameter $\delta$. Stability of affine parameter dependent systems such as (12) can be assessed by solving a set of LMI s. The complexity of this set depends on the allowed rate of variation of the scheduling parameter. In our application, the operating condition of the mounted machine may be changed in an arbitrary fashion and hence the scheduling parameter may vary arbitrarily fast. The following sufficient condition for the stability of affine parameter dependent systems with an arbitrary fast rate of variation is well-known.

**Theorem IV.1 (7)** Affinely parameterized system (12) with an arbitrary fast varying parameter $\delta \in [\delta, \bar{\delta}]$ is quadratically stable if and only if there exists $K = K^T > 0$ such that
\[
A(\delta)^T K + K A(\delta) \prec 0 \quad \text{for all } \delta \in [\delta, \bar{\delta}].
\]

Our application aims at reducing sinusoidal disturbances within the frequency range 4 to 10 Hz and therefore the scheduling parameter will be restricted to lie within the interval $\delta \in [2\pi 4, 2\pi 10]$. The LMI solver SeDuMi [7] is successfully used to find a positive definite matrix $K \in \mathbb{R}^{10 \times 10}$, which satisfies (14). Based on Theorem IV.1, we now have that system (12) is stable for arbitrarily fast parameter variations in the range $\delta \in [2\pi 4, 2\pi 10]$.

V. INTERCONNECTION OF THE SIGNAL ANALYZER AND THE CONTROLLER

Since both the real-time spectrum analyzer and the LPV controller have been designed, they can be interconnected to obtain the complete control setup as was proposed in Figure 2. Several measured signals could be used to identify the dominating disturbance frequency. A logical first choice would be to use the vertical velocity $v$ of the AVIS since the goal of the controller is to minimize this velocity. However, a direct consequence of this goal is that the amplitude of the disturbing frequency is largely reduced, and therefore it is difficult to recognize this frequency with the spectrum analyzer. A better alternative is to use the controller output $u$ as the input to the spectrum analyzer. In contrast to the vertical velocity of the AVIS, the amplitude of the disturbance frequency will not decrease due to the suppression of the controller in this signal. As long as the disturbance is acting on the AVIS, its frequency is present in the controller output. A schematic overview of the complete control system as it will be implemented is shown in Figure 8.

![Figure 8]( Diagram of the implementation of the controller and the spectrum analyzer. )

VI. EXPERIMENTAL RESULTS

In this section, experiments will be performed on the experimental setup as depicted in Figure 6. The rotating imbalance is used to generate disturbance forces $f_d$ that directly act on the payload. The imbalance is represented by a mass $m_d$ located at a distance $e_d$ from the center of rotation. For constant angular velocities $\omega_d$, the induced disturbance force on the payload equals
\[
f_d = m_d e_d \omega_d^2 \sin(\omega_d t).
\]

These disturbance forces are sinusoids with frequency $\omega_d/2\pi$ Hz and an amplitude that is proportional to $\omega_d^2$. The rotational velocity of the mass can vary in time, but is restricted between 4 and 10 revolutions per second. As mentioned before, environmental disturbances are present as well, but disturbances from the rotating mass dominate the disturbance spectrum.

During this experiment, the rotational velocity $\omega_d$ of the imbalance is controlled to vary continuously between 4 and 10 revolutions per second. The variation is obtained by using a sinusoidal reference velocity profile for the imbalance with a frequency of 0.02 Hz. The first half of the experiment, up to 50 seconds, the original LTI controller is applied to be able to quantify the performance improvement of the LPV controller. After 50 seconds the LPV controller is used together with the real-time spectrum analyzer. In Figure 7 the results from this experiment can be seen. Figure 9 shows the estimated frequency of the dominating disturbance by the real-time spectrum analyzer. The measured vertical velocity $v$ of the AVIS and the off-line computed time-frequency spectrum are depicted in Figure 10. Several observations can be made from this figure. The overall error is decreased between a factor 3 to 5, depending on the disturbance frequency. When the LPV controller is scheduled to increase the performance around 10 Hz an increase in the error level in the frequency range 10 to 15 Hz can be observed. This is in agreement with Figure 7, where it was shown that the application of a notch around 10 Hz results in amplification.
of the environmental disturbances in that range. As a final observation we note that there is a frequency component around 130 Hz in the vertical velocity of the payload. Although this cannot be explained from the first principles model of the isolator [1], frequency response measurements show that indeed a resonance frequency is present in this range. It is clear from this experiment that the LPV controller can offer a major increase in performance.

![Fig. 9. Estimated frequency.](image)

![Fig. 10. Measured signal v and its spectrum. Dark red indicates no frequency content while blue indicates maximum frequency content.](image)

VII. CONCLUSIONS

In this paper we proposed a nonlinear controller setup for the active control of a vibration isolation system. This setup consists of two parts: (i) a real-time multiresolution spectrum analyzer that is able to identify the currently dominating disturbance, and (ii) an LPV controller that adapts itself to the available disturbance information. This resulted in a control system that is able to adapt itself to the current operating condition resulting in a closed-loop system with an overall increased performance when compared to an LTI controller.

The part of the controller that was scheduled according to the identified disturbance frequency was a notch filter. Such a notch filter has a certain width and reduction factor. These design parameters should be chosen with the application and expected disturbance variation in mind. A wide notch with a large reduction factor results in good disturbance reduction properties around the center frequency of the notch. However, at the same time environmental disturbances in other frequency regions will be amplified by such a notch. A too narrow notch with a small reduction factor on the other hand may not be adequate to reduce the dominating disturbance at the center frequency.

The proposed control system was validated by experiments that were performed on an active vibration isolation system. Compared to the originally designed LTI controller, the LPV control system was able to decrease the error by a factor 3 to 5 in case the disturbance spectrum is dominated by a single sinusoid with varying frequency. If the disturbance frequency varies very fast, the LPV control system is not able to adapt itself to the disturbance. This is both caused by the real-time spectrum analyzer as well as by the controller itself. The desired accuracy of the analyzer and the width and amplification factor of the notch may be altered to match the changed disturbance properties.

Although ideally the center frequency, the width, and the reduction factor of the notch filter should all three be adapted conform to the disturbance, in this paper we only considered the adaptation of the center frequency. The real-time spectrum analyzer not only offers the frequency of the dominating disturbance, but also its amplitude. This estimated amplitude can be used in conjunction with the given performance specifications to additionally schedule the reduction factor of the notch filter. The spectral analysis of past data can be used to estimate the rate of change of the disturbance spectrum and this information can also be used to schedule the width and amplification factor of the notch filter. All this will be subject of future research.

REFERENCES


