Abstract—The performance of systems that exhibit repetitive disturbances can be significantly improved using repetitive control (RC). If the period-time of the repetitive disturbance is exactly known and constant in time, perfect asymptotic disturbance rejection can be achieved. In this paper, we apply RC to a high-precision stage driven by a walking piezo actuator with four bimorph piezoelectric legs. The repetitive nature of the walking movement introduces repetitive disturbances in the system, which are periodic with respect to the angular orientation of the legs, but not with respect to time. Therefore, an adjusted RC scheme is introduced, which incorporates a time-varying delay that is dependent on the momentary orientation of the legs during the walking movement. Experiments show that the tracking error can be significantly reduced by the adjusted RC method compared to standard RC. Furthermore, the adjusted RC scheme can also suppress repetitive disturbances for varying setpoint velocities.

I. INTRODUCTION

The performance of systems that perform repetitive tasks or that are subject to repetitive disturbances can be improved significantly using repetitive control (RC) [1], [2]. Repetitive controllers employ the internal model principle to enable asymptotic rejection of the periodic disturbances. If the period of the repetitive disturbances is known and constant in time, asymptotic disturbance rejection is achieved. However, if the repetitive disturbances are periodic with respect to another variable than time, convergence of the repetitive controller is not guaranteed anymore.

In this paper, we apply RC to a walking piezo actuator, which is used to drive a high-precision stage. The walking piezo actuator employs four bimorph piezo legs to obtain a repetitive walking movement. This results in repetitive disturbances in the system, which are periodic with respect to the angular orientation of the piezo legs, but not with respect to time. We propose an adjusted RC scheme, which incorporates a time-varying delay that is dependent on the momentary leg orientation. The time-varying delay is adjusted at a fixed sampling time, which makes the method applicable for real-time control purposes that use a fixed sampling frequency of the controller.

Several solutions for the application of RC to systems that have a varying repetitive period with respect to time have been presented in literature. Adaptive RC methods continuously estimate the time-varying period of the repetitive disturbances and adjust the sampling frequency accordingly [3], [4], [5]. The adaptive repetitive controllers are however implemented at a varying sampling rate, which makes them hard to be used in combination with a feedback controller at a fixed sampling frequency. The adaptive RC scheme proposed in [6] adapts the repetitive delay based on a physical model of the time-varying character of the repetitive delay. The variation is assumed to be slowly in time, which is not valid in various applications including the walking piezo actuator considered in this paper.

Robust RC methods [7], [8], [9] use $\mathcal{H}_\infty$ control theory to design repetitive controllers that are robust against system variations. However, no robustness against time-variations in the repetitive period is considered.

Systems that exhibit spatial repetitive disturbances, e.g., disturbances that are periodic with respect to a rotation angle, can be transformed to the rotational-angle domain, which renders the repetitive delay constant in the new variable being the rotation angle [10], [11]. However, the design of the stabilizing feedback controller becomes very complicated since the transformed system becomes nonlinear.

High-order RC uses multiple memory loops to provide robustness against small variations in the period-time of repetitive disturbances [12], [13]. High-order RC makes a trade-off between robustness for changes in the period-time and the reduction of the error spectrum in-between the harmonic frequencies [14]. A systematic design approach for high-order RC yielding optimal performance trade-offs is developed in [15]. The performance indicators are incorporated in the RC design using linear matrix inequalities (LMIs). Although this approach is very interesting, it is not considered in this paper due to the resulting large size of the LMIs with matrices in the order of $250 \times 250$.

The contribution of this paper is the application of RC to a high-precision stage driven by a walking piezo actuator. For this purpose, an adjusted RC scheme is developed, referred to as delay-varying repetitive control (DVRC), which incorporates a time-varying delay that is dependent on the momentary leg orientation. With DVRC, the tracking performance of the high-precision stage driven by the walking piezo actuator is improved significantly. Experimental results show the improvement of DVRC compared to standard RC. Also a comparison to high-order RC is made.

This paper is organized as follows. The proposed delay-varying repetitive control (DVRC) method is presented in Section III. The walking piezo actuator and the experimental setup are treated in Section IV together with the design of the learning filters. The results of the experiments for constant velocity setpoints without RC, with standard RC, with high-order RC, and with DVRC are contained in Section V. Furthermore, DVRC is applied for setpoints with varying velocity in Section V. Finally, conclusions are drawn in
Section VI.

II. THE WALKING PIEZO ACTUATOR

The high-precision stage and the walking piezo motor are shown in Fig. 1. The piezo motor consists of four piezoelectric drive legs, which can be driven by electric waveforms through the connector [16]. The drive pads of the legs are pressed against the drive strip of the one degree-of-freedom (DOF) stage using a motor suspension [17] and preload springs such that the \((x_m, y_m, z_m)\)-axes of the motor coincide with the \((x, y, z)\)-axes of the stage. The position of the stage is measured using an optical incremental encoder with a resolution of 0.64 nm. The movement of the back of the motor housing in \(y_m\)-direction is measured using a capacitive sensor with a resolution of 0.44 nm.

The drive legs of the walking piezo motor employ a bimorph working principle through two electrically separated piezo stacks. The piezo legs are driven in pairs of two by four independent waveforms, i.e., the first pair \(p_1\) is driven by \(V_1(t)\) and \(V_2(t)\) and the second pair \(p_2\) is driven by \(V_3(t)\) and \(V_4(t)\). The tip displacements of the piezo legs can be described as

\[
\begin{align*}
    x_{m,p_i}(t) &= c_x(V_{2i-1}(t) - V_{2i}(t)), \\
    y_{m,p_i}(t) &= c_y(V_{2i-1}(t) + V_{2i}(t)),
\end{align*}
\]

where \(i \in \{1, 2\}\) and \(c_x\) (m/V) and \(c_y\) (m/V) are the constant bending and extension coefficients, respectively. In [17], asymmetric waveforms have been developed, which result in periodic tip trajectories with a take-over between the driving pair of legs at a non-zero velocity of the legs in \(x\)-direction. The asymmetric waveforms enable the stage to be driven continuously at velocities in the range of nanometers per second to millimeters per second. The asymmetric waveforms \(u_i(t)\) \((V)\), \(i \in \{1, 2, 3, 4\}\) are defined as

\[
V_i(t) = \frac{A}{A_0} + A_0 \sum_{k=1}^{4} \{a_k \cos[k\alpha(t) + k\psi_i(t)] + \}
\]

\[
b_k \sin[k\alpha(t) + k\psi_i(t)]\},
\]

with Fourier coefficients \(a_0 = 28.80, a_1 = -10.78, b_1 = 18.73, a_2 = 2.387, b_2 = 4.097, a_3 = 1.985, b_3 = -0.007792, a_4 = 0.2298,\) and \(b_4 = -0.3901\). The amplitudes \(A = 46\) V and \(A = 46\) V. The phases equal \([\psi_1, \psi_2, \psi_3, \psi_4] = [0, \pi/2, \pi, 3\pi/2]\) rad. In (2), \(\alpha(t)\) (rad) denotes a nominal angle of the legs on the tip trajectory. The angle \(\alpha(t)\) follows from the drive frequency \(f_\alpha(t)\) (Hz) as \(\alpha(t) = 2\pi \int_0^t f_\alpha(\tau) d\tau\).

III. DELAY-VARYING REPETITIVE CONTROL

The repetitive disturbances in the high-precision stage driven by the walking piezo actuator are repetitive with respect to the angular orientation \(\alpha\) of the piezo legs, but not with respect to time. Since the angular orientation \(\alpha\) of the piezo legs is known [17], the constant delay in the standard RC scheme is replaced by a time-varying delay \(z^{-N(\alpha(t))}\), which is dependent on the angular orientation \(\alpha(t)\) (rad).

A schematic representation of a feedback controlled system with the adjusted RC scheme is shown in Fig. 2, where \(H(z)\) denotes the linear system and \(C(z)\) the feedback controller. The input of the system is denoted by \(u(t)\), the output by \(y(t)\). The input \(d(t)\) represents both the repetitive disturbances caused by the repetitive movement of the piezo legs as well as all other disturbances acting on the system. The tracking error is defined as \(e(t) = y(t) - r(t)\), where \(r(t)\) is the reference setpoint. The delay-varying repetitive controller \(M_{zd}(z, \alpha)\) is depicted within the dashed block, in which \(L(z)\) is the learning filter with a phase delay of \(l\) samples and \(Q(z)\) the linear-phase robustness filter with a phase delay of \(q\) samples [13]. Finally, \(z^{-N(\alpha)}\) denotes the time-varying delay in discrete-time, where

\[
N(\alpha) = k - k^*(\alpha)
\]

and the sample index

\[
k^*(\alpha(kT_s)) = \arg \min_{l \in \mathbb{N}} (\alpha(lT_s) - \alpha(kT_s) + P_{\alpha})^2,
\]

in which \(T_s > 0\) is the sampling time, \(k\) is the current sample and \(P_{\alpha}\) is the repetitive period. The introduced phase delay of the filters \(L\) and \(Q\) is compensated for in the memory loop of \(N(\alpha)\) samples by redefining \(N(\alpha) := N(\alpha) - l - q\).

The transfer function of the repetitive controller \(M_{zd}(z)\), i.e., between the tracking error \(e\) and the output \(w\), equals

\[
M(z) = \frac{L(z)Q(z)z^{-N(\alpha)}}{1 - Q(z)z^{-N(\alpha)-l}}.
\]

The system of Fig. 1 has an inherent nonlinearity since the output \(x(t)\) contains for a constant drive frequency \(f_\alpha(t)\) repetitive components with other period-times than \(1/f_\alpha(s)\).
This nonlinearity is caused by the harmonic components in the waveform generation (2), resulting in a repetitive movement of the drive legs (see also [17]). The disturbances introduced by the walking movement are fully repetitive with respect to the angular orientation $\alpha$, which is chosen to be the repetitive variable. The repetitive period equals $P_{\alpha} = 2\pi$ rad, i.e., one complete cycle of the piezo legs. The delay $z^{-N(\alpha)}$ equals the elapsed time since the leg had the same angular orientation during the previous step, i.e., when it was at angle $\alpha^* = \alpha - 2\pi$. The delay $z^{-N(\alpha)}$ varies with changing leg velocity, i.e., with varying angular frequency $\frac{d}{dt}(\alpha) = 2\pi f_{\alpha}(t)$.

The system is considered to be composed of a linear part with an additive non-linearity that is dependent on the angular orientation $\alpha(t)$. The periodic repetitive disturbances are assumed to be caused by the non-linear part and contained in the disturbance $d(t)$ of Fig. 2 through a disturbance model that is dependent on the angular orientation $\alpha(t)$.

### A. Stability

The sensitivity function $S(z)$, relating the disturbances $d(t)$ to the tracking error $e(t)$ is given by

$$
S(z) = (1 + H(z)C(z)[1 + M_{sd}(z)])^{-1} = \tilde{S}(z)M_{s}(z),
$$

where $\tilde{S}(z) = 1/(1 + H(z)C(z))$ is the sensitivity without RC. The modifying sensitivity function $M_{s}(z)$ [12] is given by

$$
M_{s}(z) = \frac{1 - Q(z)z^{-N(\alpha)-t}}{1 - Q(z)z^{-N(\alpha)-t}(1 - T(z)L(z)z^{+t})}
$$

and $T(z) = H(z)C(z)/(1 + H(z)C(z))$ is the complementary sensitivity function. For a fixed value of the delay $N(\alpha)$ the system is asymptotically stable if the following two conditions are fulfilled:

1) the sensitivity $\tilde{S}(z)$ has all poles in the open left half of the complex plane,

2) the modifying sensitivity function $M_{s}(z)$ (6) has a $\mathcal{H}_\infty$ norm smaller than one, i.e.,

$$
|Q(z)z^{-N(\alpha)-t}(1 - T(z)L(z)z^{+t})| < 1, \quad \text{for all } z \text{ with } |z| = 1.
$$

The convergence criterion (7) only holds for fixed values of the repetitive delay $N(\alpha)$. However, for time-varying delays the frequency domain convergence criterion (7) cannot be used. Although the simulation and experimental results show that the DVRC scheme is stable, the formal stability proof remains for future research.

### B. Design L and Q filter

From (7) it follows that a straightforward choice for the learning filter is the inverse of the complementary sensitivity function, i.e., $L(z) = T^{-1}(z)$. If an exact inverse cannot be obtained, e.g., when $T(z)$ is non-minimum phase or non-proper, an approximation of the inverse is made. One generally used method to obtain a proper and stable inverse is using the zero-phase-error-tracking-control (ZPETC) method [18]. The filter $Q(z)$ is designed to account for mismatches between $L(z)$ and $T^{-1}(z)$ such that the convergence criterion (7) is fulfilled. The use of the $Q(z)$ filter also restricts the learning performance of RC in certain frequency bands since part of the frequency content in the tracking error is reduced [13].

### IV. FEEDBACK AND LEARNING CONTROLLERS

This section discusses the control configuration and the design of the repetitive controllers.

#### A. Control configuration

For feedback control, the angular frequency of the legs $f_{\alpha}$ (Hz) is chosen as the control input to the system, i.e., $u(t) = f_{\alpha}(t)$ in Fig. 2. The output of the system is the stage position, i.e., $y(t) = x_s(t)$. The measured frequency response function (FRF) from the angular frequency $f_{\alpha}(t)$ to the stage position $x_s(t)$ is shown in Fig. 3 with the solid black line. The measured FRF shows a decay of 20 dB/decade with a corresponding phase of -90 deg at low frequencies. At a frequency of 527 Hz the first resonance can be seen, directly followed by an anti-resonance and resonance at 624 Hz and 650 Hz respectively. Furthermore, the FRF shows a phase delay of three samples at a sample frequency of $f_s = 4$ kHz.

To design the feedback controller and the learning filters, a parametric linear model $\hat{H}(s)$ containing a pure integrator, two resonances and one anti-resonance is fitted to the measured FRF as

$$
\hat{H}(s) = \frac{2\pi f_{p1}}{s} \frac{c}{s^2 + 2\pi f_{p1}b_{p1}s + (2\pi f_{p1})^2} \left( \frac{1}{s^2 + 2\pi f_{b1}b_{s1}s + (2\pi f_{b1})^2} \right) \left( \frac{1}{s^2 + 2\pi f_{p2}b_{p2}s + (2\pi f_{p2})^2} \right),
$$

where $c = 14.3$, $f_{p1} = 527$ Hz, $b_{p1} = 0.033$, $f_{b1} = 624$ Hz, $b_{s1} = 0.02$, $f_{p2} = 650$ Hz and $b_{p2} = 0.175$. The phase delay of three samples is added to the model by multiplying
it after discretization with a discrete-time delay $z^{-3}$. The model $H(z)$ approximates the measured FRF well, as shown by the dashed grey line in Fig. 3.

The feedback PI controller $C(s)$ is designed using loop-shaping techniques as

$$C(s) = k \frac{s + 2\pi f_{zc}}{s},$$

where the gain $k = 2.8 \cdot 10^6$ and the frequency of the zero at which the integrating action stops equals $f_{zc} = 5$ Hz, resulting in a closed-loop bandwidth $f_{BW} = 5$ Hz. The controller is then discretized using a Tustin discretization at a sampling frequency of $f_s = 4$ kHz.

**B. Learning filter**

The learning filter $L(z)$ is derived as a proper stable approximation of a discrete-time model of the complementary sensitivity function $\tilde{T}(z) = H(z)C(z)/(1 + H(z)C(z))$ using the ZPETC method [18], i.e., $L(z)\tilde{T}(z) \approx 1$. In Fig. 4 can be seen that the phase of $L(z)$ is an exact inverse of the phase of the complementary sensitivity function $\tilde{T}(z)$. The magnitude of the learning filter deviates mainly at high frequencies from $\tilde{T}^{-1}$ to obtain a proper and stable filter $L(z)$. In order to guarantee stability of the repetitive controller, a low-pass $Q(z)$ FIR filter with 100 taps and a cut-off frequency of 220 Hz is used.

**V. Results**

In this section, the results of standard RC with a fixed delay, i.e., $N(\alpha) = N$ samples, and of DVR are discussed for both constant velocity and varying velocity reference setpoints.

For comparison, a high-order repetitive controller that incorporates two periods, i.e., with two memory loops [13], is designed. The high-order repetitive controller equals

$$M_{HO}(z) = \frac{L(Z)W(z)Q(z)z^{-p}}{1 - Q(z)Q(z)z^{-p}},$$

where $W(z) = \sum_{i=1}^{n_{HO}} w_i z^{-(i-1)p}$ is the high-order repetitive function and $n_{HO} = 2$ is the order. The optimal weighting filter for a second order repetitive controller is determined in [13] as $W_{opt} = (w_{opt,1}, w_{opt,2}) = (2, -1)$.

For the application of DVR to the walking piezo motor, the repetitive variable is the angular position of the piezo legs $\alpha$ (rad). In practice, the time-varying delay is implemented as a first-in-first-out (FIFO) buffer of sufficient length in which the tracking errors $e(t)$ and the corresponding angles $\alpha(t)$ are stored. The output of the buffer is the error $e_{out}(t)$ of $N(\alpha(t))$ samples ago, calculated using (3).

**A. Constant velocity**

The tracking errors of the experiments for $\dot{\alpha} = 10$ $\mu$m/s without RC, with RC, with high-order RC and with DVR are shown in Fig. 5. The tracking error without RC is shown in the top left figure of Fig. 5. The root-mean-square (rms) value of the tracking error without RC equals $\text{rms}(e) = 109$ nm. A clear repetitive structure is present in the error as shown in the zoom plot of Fig. 6 with the light grey line.

The application of RC reduces the tracking error by 83% to an error of $\text{rms}(e_{RC}) = 18.3$ nm, as shown in the top right figure of Fig. 5. Although the error is reduced significantly, a clear fluctuation in the magnitude of the tracking error is visible. This fluctuation is caused by the fact that the repetitive disturbances are not periodic with respect to time. The zoom plot of Fig. 6 shows that the remaining error with RC (black dashed line) still contains a significant repetitive part.

The high-order repetitive controller reduces the tracking error further to $\text{rms}(e_{HO}) = 13.7$ nm as shown in the bottom left figure of Fig. 5. The convergence of the error is clearly visible. However, the second order repetitive controller is not able to completely remove the fluctuation in the error, indicating that it is not able to cope with the amount of variation in the repetitive delay. Increasing the order of the repetitive controller is not possible since this would require
the cut-off filter of the $Q$ filter to be lower than the frequency content of the repetitive tracking error in order to guarantee stability.

Finally the results of the experiment with DVRC are shown in the bottom right figure. The remaining tracking error with DVRC equals $\text{rms}(e_{DVRC}) = 2.77 \text{ nm}$ and has a faster convergence rate. DVRC reduces the tracking error by 97% compared to the tracking error without RC, by 85% compared to standard RC and by 80% compared to the high-order repetitive controller. After convergence the tracking error shows a noisy character in which no deterministic part is visible anymore as can be seen by the solid black line in Fig. 6.

The cumulative power spectral densities (CPSDs) of the tracking errors in Fig. 6 clearly show the reduction of DVRC with respect to the other RC experiments. For frequencies $f \to \infty$, the cumulative PSDs converge to the squared rms values of the tracking errors.

**B. Varying velocity**

Since the repetitive delay is continuously adjusted in DVRC, it can be used for setpoints that have a varying velocity, i.e., which have an inherent variation in repetitive delay for the walking piezo actuator. Fig. 7 shows the results of an experiment with and without DVRC for a changing reference velocity $\dot{v}(t) = 2 \times 10^{-4} + 10^{-4} \sin \left( \frac{2\pi}{100} t \right)$. Standard RC and high-order RC are not applied since they cannot cope with such fast and large changes in the repetitive period.

The error reduction of DVRC compared to the experiment without RC can clearly be seen in the top left axis of Fig. 7. The sinusoidal shape of the tracking error with DVRC corresponds to the variation in velocity and is larger for higher reference velocities (larger drive frequencies $f_\alpha$) and smaller for low velocities (small $f_\alpha$). This corresponds to the magnitude of the sensitivity $\dot{S}$ on which the repetitive ‘notches’ are places, which has a smaller magnitude for low frequencies (low velocities) and vice versa. The rms values of the errors over the complete experiment equal $\text{rms}(e) = 173.3 \text{ nm}$ and $\text{rms}(e_{DVRC}) = 34.3 \text{ nm}$, which is a reduction of 80%.

The zoom plot in the left bottom axis of Fig. 7 shows the repetitive nature of the tracking error obtained without RC by the grey line. With DVRC hardly any repetitive structure is present anymore in the tracking error, as shown by the black line in the left bottom axis of Fig. 7.

The CPSDs of the error signals without and with DVRC (right axis of Fig. 7) clearly show the applicability of DVRC for reference signals with a varying velocity. The variation in the velocity directly results in a variation of the repetitive delay. The CPSDs show that the largest error reduction is obtained by DVRC at low frequencies $f < 5 \text{ Hz}$, i.e., in the frequency range where the base repetitive frequencies are contained. Furthermore, it can be seen that DVRC reduces the tracking error for all frequencies up to the cut-off frequency of the $Q$ filter, i.e., for $f < 200 \text{ Hz}$.

**C. Discussion**

The constant repetitive period $N$ of standard RC is rounded to a fixed number of samples. However, in practice the true repetitive period will almost never be an integer number of samples. The mismatch between the true and implemented repetitive period becomes smaller when a higher sampling frequency is used. A high sampling frequency is also beneficial for the case of DVRC since the errors contained in the buffer are discretized at a fixed sampling period and since it also reduces the discretization of the angular orientation of the legs $\alpha(t)$.

The experiments presented in this paper all have reference setpoints in which the velocity does not change sign, i.e., the angular orientation $\alpha$ of the piezo legs is monotonically increasing (or decreasing). One possible method to extend DVRC for velocity setpoints that change sign involves detecting the sign change and resetting the memory buffer.
In this paper, we applied DVRC for setpoint velocities that do not change in sign. For these setpoints the repetitive variable, i.e., the angular orientation of the piezo legs, is monotonically increasing in time. Future work involves extending the DVRC scheme to allow also non-monotonically increasing repetitive variables, i.e. references with a changing sign of the velocity.

VII. ACKNOWLEDGMENTS

This research is part of the Micro and Nano Motion project, which is supported by Point One.

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