Wave Variables for Haptic Purposes

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Master's thesis

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Preface

This report is written after a traineeship of 3.5 months at the University of Victoria (UVIC, BC, Canada) under supervision of dr. Daniela Constantinescu. The aim of the traineeship was to implement a new control strategy to connect a physical pantograph setup to a virtual environment. Unfortunately this didn’t work out as planned. Only at the end of my stay I accomplished the initial goal of the traineeship. The knowledge and experience I have obtained during the traineeship is, however, valuable. Therefore Prof. M. Steinbuch, as my supervisor in the Netherlands, suggested to make a small report which would summarize my findings and provide additional theoretical background. Eventually the size of the report increased due to additional presented background information.

I would like to thank Daniela and Maarten for providing the opportunity to carry out a traineeship abroad. Daniela’s support during the project is still greatly appreciated. Never before did I encounter such a dedication towards solving encountered obstacles.

Jesse Scholtes

June 17, 2009
Abstract

In the physical world, humans rely on their sense of touch to do everyday tasks such as driving a car, using a cell phone, playing a musical instrument, drinking a glass of water. The touching or sensing cues we receive while carrying out such tasks are related to haptics. In general the word haptic is used as a gather name for subjects related to the sense of touch.

In the context of mechanical engineering, haptics is referred to as haptic technology. This is the means or knowledge to develop, design and build systems that are able to provide or present information to the user by the sense of touch. Haptic technology provides force information to a user by means of a device(master), which is connected to another similar device(slave). A movement of the master results in a similar movement of the slave and if the slave touches something in its environment it is felt at the master. One class of problems which is encountered within the research of haptic technology is the instability problem caused by communication delays (due to large distances) between the master and the slave device. The wave variable algorithm, developed by Günter Niemeyer and Jean-Jacques E. Slotine, is capable of providing a passive communication channel solving this problem.

Motivated by the interesting research by Günter Niemeyer and Jean-Jacques E. Slotine, the assignment for the traineeship was to implement this wave variable algorithm on the experimental setup. Where, two other subgoals were recognized: a literature study towards the wave variable algorithm and as a result of the implementation demonstrate its characteristics. The traineeship was carried out at the University of Victoria (UVIC, BC, Canada) under supervision of dr. Daniela Constantinescu. The experimental setup consist of a physical master device and a virtual slave device.

The wave variable algorithm originates from transmission line theory. Here, use is made of a special class called ‘lossless transmission line. Combined with the representation of a transmission line as a two-port network realizes a notion of the transmission line where signals traveling through it, are independent of time. These signals are called wave variables and are a combination of a force and a velocity signal. The adjustable characteristic impedance parameter trades of the velocity against the forces and allow the user so suit its needs or preference. By means of passivity theory the (i.e. net energy flow) the stability of the transmission line is proven.

During the experiments position drift was observed. It could not be established whether the amount of position drift was a result of numerical integration, discrete sampling, data loss or the controller used to feedback the force at the slave device to the master device. However, an improved controller, with a higher bandwidth, resulted in less position drift. Also, wave reflections were observed when lowering the value for the characteristic impedance.

The wave variable algorithm provides a passive communication line between master and slave device and solves the instability problem caused by transmission delays. However, it can also lead to a conservative system in the sense of reduced performance with respect to presenting the user with relevant information. Also, position drift is observed. Solution for these problems are also stated in literature and its use should be shown through implementation in the experimental setup. Only in this case the full potential of the wave variable algorithm can be judged.
## Symbols and Units

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Unit</th>
<th>Description</th>
</tr>
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<tbody>
<tr>
<td>α</td>
<td>[1/m]</td>
<td>attenuation factor</td>
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<tr>
<td>β</td>
<td>[rad/s]</td>
<td>phase factor</td>
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<tr>
<td>Δx</td>
<td>[m]</td>
<td>length of an infinitesimal circuit element.</td>
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<tr>
<td>ω</td>
<td>[rad/s]</td>
<td>frequency</td>
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<td>γ</td>
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<td></td>
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<td>ρT</td>
<td>[-]</td>
<td>reflection coefficient</td>
</tr>
<tr>
<td>b</td>
<td>[Ns/m]</td>
<td>characteristic impedance</td>
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<tr>
<td>C</td>
<td>[F/m]</td>
<td>shunt capacity of the transmission line per unit length.</td>
</tr>
<tr>
<td>F</td>
<td>[N]</td>
<td>force</td>
</tr>
<tr>
<td>G</td>
<td>[Ω/m]</td>
<td>shunt conductance of the transmission line per unit length.</td>
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<tr>
<td>i</td>
<td>[A/m]</td>
<td>current per unit length of line.</td>
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<tr>
<td>I</td>
<td>[A]</td>
<td>Current</td>
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<tr>
<td>I⁺</td>
<td>unit</td>
<td>incident current wave (Here, used in transmission line theory).</td>
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<tr>
<td>I⁻</td>
<td>unit</td>
<td>reflected current wave (Here, used in transmission line theory).</td>
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<tr>
<td>I₁</td>
<td>unit</td>
<td>incident current wave (Here, used in two-port network theory).</td>
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<tr>
<td>I₂</td>
<td>unit</td>
<td>reflected current wave (Here, used in two-port network theory).</td>
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<tr>
<td>K</td>
<td>[m/N]</td>
<td>mechanical compliance</td>
</tr>
<tr>
<td>L</td>
<td>[H/m]</td>
<td>total series inductance of the transmission line per unit length.</td>
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<tr>
<td>M</td>
<td>[Ns²/m]</td>
<td>mass of a moving body</td>
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<td>T</td>
<td>[s]</td>
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<td>u</td>
<td>[√Watt]</td>
<td>incident wave variable</td>
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<tr>
<td>v</td>
<td>[√Watt]</td>
<td>reflected wave variable</td>
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<tr>
<td>v</td>
<td>[V/m]</td>
<td>voltage per unit length of line.</td>
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<td>V</td>
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<td>[m/s]</td>
<td>velocity</td>
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<td>[Ω]</td>
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<td>Zₘ</td>
<td>[Ω]</td>
<td>impedance of the master device</td>
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<tr>
<td>Zₛ</td>
<td>[Ω]</td>
<td>impedance of the slave device</td>
</tr>
<tr>
<td>Zₜ</td>
<td>[Ω]</td>
<td>terminal load impedance</td>
</tr>
<tr>
<td>Zₛ</td>
<td>[Ω]</td>
<td>source load impedance</td>
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Chapter 1

Introduction

This chapter gives a brief introduction to the contents of this report followed by an explanation of the layout and the works of the experimental setup. The chapter ends with an explanation of the assignment for this traineeship.

The research for this traineeship is related to the class of control systems used in the field of haptic technology. Haptics refers to: "pertaining to the sense of touch"; haptic technology refers to the technology which achieves this (e.g. interfaces, control methods). Providing information about haptic interfaces and technologies is not the objective of this report and would be too extensive. For more information the reader is referred to the report "Haptic Control for Dummies: An introduction and analysis" [7], previous research of Dr. D. Constantinescu [5] and previous work of TU/e student M. A. Beenakkers [9].

The contents of the report are as follows:

**Chapter 1: Introduction** The first chapter is the introduction of this report. It provides information about the experimental setup used during the traineeship and the contents of the assignment.

**Chapter 2: Transmission Line Theory** Parts of transmission line theory are explained, to provide an understanding and relevant background knowledge towards the contents of the assignment. Treated subjects are: reflections, impedance, two-port networks and a special class of transmission lines which are referred to as lossless.

**Chapter 3: Wave Variables** The third chapter describes the concept of wave variables. Its background is explained together with a mathematical derivation. Furthermore, several results of experiments are presented.

**Conclusion & recommendations** The last section provides conclusive remarks regarding the wave variable algorithm and presents recommendations towards future work.
CHAPTER 1. INTRODUCTION

1.1 The Pantograph Setup

The purpose of the experimental setup, also called the pantograph setup is to allow arbitrary body interaction within planar virtual environments (VEs). The haptic interface comprises two five-bar mechanisms mounted in parallel as depicted in figure 1.1. The design allows the device handle to translate in the horizontal plane and to rotate around a vertical axis. The three degrees of freedom (DOF) of the haptic interface (two translations and one rotation) are actuated by four electric motors mounted on the base joints. Position encoders are attached to the motor shafts. The position of the user’s hand, i.e., the position of the device handle, is known from the encoder readings and the kinematics of the haptic interface. VORTEX, a physics-based simulation engine from CM Labs Inc., is used to simulate and display the VE. The experimental setup uses two computers: one computer (host PC) runs Windows 2000 and simulates the VE at visually interactive rates of 10 to 60 [Hz]; a second computer (remote target) runs VxWORKS and computes the haptic control and rendering algorithms at 512 [Hz]. For more information on this setup, see [10].

Figure 1.1: 3-DOF pantograph experimental setup

1.2 Using the Pantograph Setup

Extra information on the experimental setup is presented in this subsection. In this case the configuration used in this particular traineeship is discussed. The location where the VE is computed can differ from configuration to configuration. In figure 1.2 a schematic overview of the setup is given.

In this diagram the three main components of the experimental setup are: the Pantograph, the remote target PC and the host PC. The latter runs various software packages enabling visualizations, controller development and enabling communication to the remote target PC. To get acquainted with the different components of the pantograph setup, a short description of the different software tools are given:

FTP Server The FTP Server program establishes a UDP-link (User Datagram Protocol; used in
applications for fast data transfer where a quick response time is more important then data integrity; opposed to e.g. TCP) between the host PC and the Remote PC;

**Hyper Terminal** The remote target PC hasn't got its own monitor screen, Hyper Terminal is used to monitor its activities and important errors during debugging.

**VxWorks** VxWORKS is a Unix-like real-time operating system (RTOS) made and sold by Wind River Systems. The RTOS system provides an operating environment in which data acquisition of the master is handled and interactions in the VE are computed at a fixed rate of 512 [Hz].

**Tornado** TORNADO provides a set of core and optional cross-development tools that enables developers to create device software running on VxWORKS faster. TORNADO consist of the VxWORKS run time system, which is a high performance RTOS that executes on the target PC (in our case the CPU on the Remote PC).

**MATLAB / Simulink / Real-Time Workshop** MATLAB is a numerical computing environment and programming language. MATLAB allows easy matrix manipulation, plotting of functions and data, implementation of algorithms, creation of user interfaces, and interfacing with programs in other languages. SIMULINK is a tool for modeling, simulating and analyzing dynamical systems. Real-Time Workshop (RTW) generates stand-alone C-code for developing and testing algorithms modeled in SIMULINK. The resulting code can be used for real-time and non-real time applications. It enables users to run and interact with the code outside the MATLAB and SIMULINK environment.

**Microsoft Visual Basic** VISUAL BASIC is a programming language for developing sophisticated
professional applications for Microsoft Windows. It makes use of Graphical User Interface (GUI) for creating robust and powerful applications. The GUI, as the name suggests, uses illustrations for text, which enable users to interact with an application. This feature makes it easier to comprehend processes in a quick and easy way. In this case Visual Basic is used to create the interacting VE.

Furthermore, towards providing additional information, appendix A provides a step-by-step guide for running an experiment or demo on the experimental setup and appendix E provides practical information with respect to the setup. Appendix C provides for useful explanations of terms stated in this report.

1.3 Assignment

In 1991 Günter Niemeyer and Jean-Jacques E. Slotine introduced the concept of wave variable transformations in haptic control [11]: a transformation of velocity- and force signals into a so called wave variables which are transmitted, back and forth, through a communication line between master and slave device. The presented theory leads to a passive communication channel despite the existence of (communication) delays. Since the introduction of this theory, several papers have been written and now seems as an interesting technique in the field of haptic control. As a result the assignment for the traineeship was stated as follows:

Implement the wave variable algorithm on the experimental setup.

By implementing the algorithm it directly provides information about its handling and feeling outside the scope of computer simulations and information given in papers. Within the assignment the following subgoals were recognized:

• Perform a literature study;
• Investigate stability, robustness and performance of the algorithm.
Chapter 2

Transmission Line Theory

Wave variable theory originates from the field of electrical engineering, in particular transmission line theory. To provide a very useful extension in understanding wave variables, this chapter treats some parts of transmission line- and network theory. Both provide information to derive the wave variable algorithm. Terms as impedance, two-port networks, distributed vs. lumped models, wave reflections, impedance and lossless transmission lines will be treated throughout this chapter. Those familiar with the former terms may skip the remainder of this chapter and continue with the next chapter.

This chapter is based on the book ‘Transmission lines’ written by Robert A. Chipman \cite{chipman}. Based on the relevance with obtaining the wave variable equations, a selection of theory is presented in this chapter.

This chapter starts of with a subsection about transmission line theory \secref{2.1} and clarifies terms as impedance and wave reflections. The second subsection describes two-port networks \secref{2.2} whereas subsection \secref{2.3} describes the class of transmission lines which are lossless. Subsection \secref{2.4} ends this chapter with a summary.

\section{2.1 Transmission Line Theory}

Transmission lines are mathematically described as a distributed system consisting of electrical circuit elements. Theory associated with this model was initiated by William Thomson in 1855 and finished by Oliver Heaviside in 1885. Transmission lines are assumed to be uniform, which implies that the material, dimensions and cross-sectional geometry of the line and its surrounding medium remains constant throughout its length.

A basic transmission line scheme is presented in figure \ref{fig:transmission_line_scheme}. The depicted transmission line consists of a signal source and a terminal load.

A distributed model/system consisting of electronic (or mechanic) elements assumes that each circuit element is infinitesimal. The behavior of an infinitesimal section circuit element in a transmission line is assumed to be described by four distributed electric circuit coefficients, which are, per unit of line, constant every where on the line.

- $\Delta x$ [m], length of an infinitesimal circuit element
CHAPTER 2. TRANSMISSION LINE THEORY

- R [Ω/m], total series resistance of the transmission line per unit length.
- L [H/m], total series inductance of the transmission line per unit length.
- C [F/m], shunt capacity of the transmission line per unit length.
- G [Ω/m], shunt conductance of the transmission line per unit length.

It is essential to know that these coefficients are, at a given frequency, determined by only the materials and dimensions of the line conductors and the surrounding medium. They do not vary in time or due to current/voltage. Therefore the line is a linear passive network.

The differential equations for a uniform transmission line are found by focussing on an infinitesimal section of line with length ∆x [m]. The block diagram of an infinitesimally short length of the transmission line along x is depicted in figure 2-2, the unit for time t is in [s].

Applying Kirchhoff’s laws for electric circuits gives the following expressions for an infinitesimal section of the transmission line.

\[
\frac{\partial v(x,t)}{\partial x} = -Ri(x,t) - L \frac{\partial i(x,t)}{\partial t} \tag{2-1}
\]

\[
\frac{\partial i(x,t)}{\partial x} = -Gv(x,t) - C \frac{\partial v(x,t)}{\partial t} \tag{2-2}
\]

The solution of equation (2-1) and (2-2) can, in the frequency domain (harmonic steady-state), be represented as:

\[
V(x) = V^+e^{-\gamma x} + V^-e^{\gamma x} \tag{2-3}
\]

\[
I(x) = I^+e^{-\gamma x} + I^-e^{\gamma x} \tag{2-4}
\]

1 more information for the word shunt is given in appendix C
2 more information on Kirchhoff’s laws is given in appendix C
3 the derivation to obtain equations (2-3), (2-4) and (2-5) are given in [5] pages 20-26
2.1. TRANSMISSION LINE THEORY

with,

\[ \gamma = \alpha + j\beta = \sqrt{(R + j\omega L)(G + j\omega C)} \]  (2-5)

if \( R, L, G \) and \( C \) are considered constant and the voltage \( v(x, t) \) and current \( i(x, t) \) are considered to be time harmonic variations at angular frequency \( \omega \) [rad/s],

- \( \alpha \) [1/m] is the attenuation factor, a quantity (here voltage/current) is attenuated when a physical quantity monotonically diminished as function of an increasing independent variable (here \( x \)). The attenuation of voltage and current waves on a transmission line are described by the term \( e^{-\alpha x} \) which decreases (exponentially) with the increase per length of line. \( \alpha \) refers to the attenuation per length of line.

- \( \beta \), is the phase factor. The term \( e^{-j\beta x} \) states that harmonic voltage and current waves will travel in the increasing direction of \( x \). The phase angles decrease uniformly and at a constant rate of \( \beta \) [rad/m].

In the next subsections the occurrence of reflections and the significance of impedances will be explained.

2.1.1 Reflections

From equation (2-3) it can be stated that the term \( V^+ e^{-\gamma x} \) represents the forward traveling (incident) wave, i.e. the wave traveling in the direction of increasing \( x \). Similarly \( V^- e^{\gamma x} \) represents the backward traveling (reflected) wave, i.e traveling in the direction of decreasing \( x \). Note that \( V^+ \) is the complex value of the first wave as it leaves the point \( x = 0 \). Where as \( V^- \) is the complex value of the second wave as it arrives at \( x = 0 \). The resulting and actual voltage (measurable voltage) \( V \) at \( x = 0 \) equals \( V^+ + V^- \). A similar reasoning hold for the harmonic current waves \( I^+ \) and \( I^- \).

![Figure 2-3: Schematic view of a basic transmission line, with impedance load \( Z_T \)](image-url)

From the previous statement it can also be concluded that it is possible that voltage and current waves can travel in both directions on the transmission line when there is only one signal source. This is where one of the major characteristics of waves in general comes forward, namely reflection. Whenever traveling waves (i.e. light waves, sound waves, water waves) and in this case voltage and current waves, encounter a discontinuous change in the medium in which they are traveling, they are partially or totally reflected.

Suppose that current and voltage waves are traveling along the line in the direction of increasing \( x \). Then, with reaching the end of the line at the termination, denoted as \( Z_T \), reflections come into existence. The terminal load \( Z_T \) dictates a certain magnitude and phase relation between the voltage and current and the terminal load. Therefore the complex value of the reflected waves will be such
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that when they are combined with the complex values of the incident waves, the boundary condition imposed by $Z_T$ will be satisfied. The reflected voltage and current waves will travel back along the line to the point $x = 0$, and in general will (due to the same reasoning) be partial reflected by the source termination $Z_S$.

2.1.2 Impedance

The word *impedance* originates from the verb ‘(to) impede’. The dictionary provide the following definition:

*Impede: to retard in movement or progress by means of obstacles or hindrances; obstruct; hinder.*

The electrical impedance, or simply impedance, relates current to voltage at a certain port. It describes a measure of opposition to a sinusoidal alternating current, describing not only the relative amplitudes of the voltage and current, but also the relative phases. Impedance is a complex quantity often denoted with the symbol $Z$ and is usually represented as $Z = R + jX$, where $R$ is the *ohmic resistance* and $X$ is the *reactance*.

Transmission line theory is no exception in using impedances for describing the behavior and effects of waves propagating through a transmission line. A terminal load has impedance $Z_T$ and is described as:

$$ Z_T = R_T + jX_T $$ (2-6)

The internal impedance of a generator connected at the signal source end of the line had the symbol $Z_S$, with:

$$ Z_S = R_S + jX_S $$ (2-7)

A unique quantity appearing in transmission line theory, is the characteristic impedance denoted as $Z_0$. $Z_0$ is characteristic for the line itself and is only dependent on the frequency $\omega$ [rad/s]. It is a function of the distributed coefficients the characteristic impedance and is described as:

$$ Z_0 = R_0 + jX_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}} $$ (2-8)

If a uniform transmission line is not terminated with its characteristic impedance, i.e. $Z_0 \neq Z_T$ then reflected waves will be present on the line. Where the impedance at every point of the line differs from the characteristic impedance $Z_0$.

The relation between impedance and the reflection of voltage and current waves becomes more apparent if one takes a closer look at the terminal load. Here, the ratio of the the complex voltage expression (equation (2-3)) to the complex current expression (equation (2-4)) is constrained to be equal to the connected terminal load impedance $Z_T$. At the end of the transmission line ($x = l$) the following expression yields for the terminal load impedance.

---

sometimes denoted by the term ‘complex resistance’
2.2. TWO-PORT NETWORKS

\[ Z_T = \frac{V(x = l)}{I(x = l)} = Z_0 \left( \frac{V^+ e^{-\gamma l} + V^- e^{\gamma l}}{V^+ e^{-\gamma l} - V^- e^{\gamma l}} \right) \]  

(2-9)

With \( I \) expressed as:

\[ I(x = l) = \frac{1}{Z_0}(V^+ e^{-\gamma l} - V^- e^{\gamma l}) \]  

(2-10)

\( V^+ e^{-\gamma x} \) is the wave traveling in the direction towards the termination and \( V^- e^{\gamma x} \) the wave traveling away from the termination in the direction of decreasing \( x \). In the basic transmission line circuit (figure 2-3) the signal source initiates a harmonic voltage wave in the direction of the termination (incident wave). Note that \( V^- e^{\gamma x} \) (reflected wave) only comes into existence through the physical process of reflectance.

The distinction between the incident and reflected waves, both consisting of pairs of voltage and current waves, is in the sign as can be seen in equation (2-11),

\[ I^+ = \frac{V^+}{Z_0}, \quad I^- = -\frac{V^-}{Z_0} \]  

(2-11)

The ratio between the incident and the reflected wave is called the reflection coefficient, which is the ratio of two complex quantities, and is described as follows:

\[ \rho_T = V^- e^{\gamma l} \left( \frac{V^-}{V^+} \right) e^{2\gamma l} \]  

(2-12)

The magnitude of the complex number reflection coefficient \( \rho_T \) is the ratio of the magnitude of the reflected wave to the magnitude of the incident wave at the point of reflection. When the magnitude of the reflection coefficient is unity, \( |\rho_T| = 1 \), then an incident wave will completely be reflected at the terminal (i.e. no energy is absorbed by the load). In this particular case, the terminal impedance has no resistive component and does not absorb power from an incident wave, hence the wave is totally reflected. Resistive terminations having a reflection coefficient less than unity dissipate a finite fraction of the power of the incident wave and reflect the balance. Figure 2-4 represents the space of the reflection coefficient. Short circuit refers to an electrical circuit with an abnormal low-resistance connection between two nodes of an electrical circuit that are meant to be at different voltages. An open circuit is its opposite, meaning an infinite resistance between two terminations.

Whenever two waves of identical frequency travel in opposite directions on a transmission line the fundamental phenomena of interference or standing waves\(^5\) may occur in steady-state situations. The magnitude of each complex wave variable exhibits periodic maxima and minima along the system, at intervals determined by the wavelength of individual waves.

2.2 Two-port Networks

In order to model networks consisting of transmission lines the theory of two-port networks is used. A port within an electrical network is defined as any pair of physical coincident terminals at which two complementary variables (here, voltage and current) exist. Any section of an uniform transmission

\(^5\) more information on standing waves is given in appendix C
line can be seen as a two-port. Here the purpose of the two-port network theory is to develop formulas useful in expressing the nature of networks consisting of uniform transmission lines. The behavior of the network is characterized by the results of specified measurements made at the network terminals. A typical example of a two-port network consists of a concatenation of different two-ports as depicted in figure 2-5.

Summarized, two-port network theory is designed to model and simulate the behavior of larger networks consisting of many electrical elements like transmission lines. Note that, previously each transmission line was described as a distributed model. Where as in the framework of two-port network it is viewed as a lumped element with characteristic impedance $Z_0$.

As can be seen, figure 2-6 uses another set of notations. Previously superscripts $^+$ and $^-$ were used to indicate the incident and reflected current ($I^+$ and $I^-$) and voltage waves ($V^+$ and $V^-$) to describe the distributed circuit elements. In the two-port framework the subscripts $^+$ and $^-$ are respectively
interchanged by subscripts 1 and 2. The analysis of transmission lines within two-port networks is carried out in the frequency domain with frequency \( \omega \) in [rad/s]. The conventional choice of sign in two-port network analysis for the current on the right is opposite to that used for a terminal load current in transmission line theory.

Referring to equation (2-11), the set of equations relating the four complex currents and voltages can be rewritten into equation (2-13). It is commonly written as a 2-by-2 matrix referred to as the *impedance matrix* describing the two-port

\[
\begin{bmatrix}
V^+ \\
V^-
\end{bmatrix}
= Z
\begin{bmatrix}
I^+ \\
I^-
\end{bmatrix}
\quad \Leftrightarrow \quad
\begin{bmatrix}
V_1 \\
V_2
\end{bmatrix}
= Z
\begin{bmatrix}
I_1 \\
-I_2
\end{bmatrix}
\]  (2-13)

with

\[
Z = Z_0 \begin{bmatrix}
\coth(\gamma l) & \text{cosech}(\gamma l) \\
\text{cosech}(\gamma l) & \coth(\gamma l)
\end{bmatrix}
\]  (2-14)

representing the open circuit impedance matrix of a length \( l \) of a uniform transmission line, having characteristic impedance \( Z_0 \), attenuation factor \( \alpha \) and phase factor \( \beta \).

The input impedance from either side of the two-port (i.e. diagonal terms in \( Z \) are the same). As the transmission line is assumed to be symmetric.

For future purpose equation (2-13) is rewritten into equation (2-15).

\[
\begin{bmatrix}
V_1 \\
I_1
\end{bmatrix}
= H
\begin{bmatrix}
V_2 \\
-I_2
\end{bmatrix}
\]  (2-15)

with,

\[
H = \begin{bmatrix}
\cosh(\gamma l) & Z_0\sinh(\gamma l) \\
Z_0\sinh(\gamma l) & \cosh(\gamma l)
\end{bmatrix}
\]  (2-16)

It can be seen that voltage and current corresponding to the left port is at the left hand side and those of the right port is at the right hand side. This form is referred to as the *hybrid matrix* representation.

### 2.3 Lossless Transmission Line

With regard to the wave variable algorithm some more details are given on the class of lossless transmission lines. A lossless transmission line does not dissipate energy. Hence, no attenuation of the voltage and current waves occurs. Therefore, the attenuation factor \( \alpha = 0 \). This implies that the distributed electric circuit coefficients \( R = 0 \) and \( G = 0 \). The transmission line equations for voltage and current stated in equations (2-1) and (2-2) are now described as:

\[
\frac{\partial V(x, t)}{\partial x} = -L \frac{\partial I(x, t)}{\partial t}
\]  (2-17)

\[
\frac{\partial I(x, t)}{\partial x} = -C \frac{\partial V(x, t)}{\partial t}
\]  (2-18)

The lossless line is now represented as a cascade of inductors in series and capacitors in parallel as depicted in figure 2-7.

---

\( \text{The derivation of impedance Z is given in appendix [1] } \)
CHAPTER 2. TRANSMISSION LINE THEORY

An attenuation factor of zero also implies that the hybrid matrix as stated in equation (2-15) changes to:

\[
\begin{bmatrix}
V_1 \\
I_1
\end{bmatrix} = H_{\text{lossless}} \begin{bmatrix}
V_2 \\
-I_2
\end{bmatrix}
\]  

(2-19)

with,

\[
H_{\text{lossless}} = \begin{bmatrix}
\cosh(j\beta l) & Z_0 \sinh(j\beta l) \\
Z_0 \sinh(j\beta l) & \cosh(j\beta l)
\end{bmatrix}
\]  

(2-20)

2.4 Summary and Observations

In this chapter parts of the transmission line theory were treated and explained. A summary of the most important points with regard to wave variables are stated once more.

**Theory** It has been established that all possible frequency time harmonic voltage distributions on a uniform transmission line are described by:

\[
V(x) = V^+e^{-\gamma x} + V^-e^{\gamma x}, \quad \gamma = \alpha + j\beta
\]  

(2-21)

With \(V^+\) and \(V^-\) arbitrary complex voltage quantities which are determined by the boundaries / terminations at the end of the line and \(\alpha\) the attenuation factor and \(\beta\) the phase factor. Note that the time harmonic variation of the voltage is represented by an implicit multiplying factor \(e^{j\omega t}\).

The corresponding equation for the harmonic current distribution is equivalently represented by:

\[
I(x) = \frac{1}{Z_0}(V^+e^{-\gamma x} - V^-e^{\gamma x})
\]  

(2-22)

with \(Z_0\) the characteristic impedance of the transmission line. The current waves have the same frequency, phase velocity, wavelength\(^7\) and attenuation as the voltage waves. The first term of equation (2-21) and (2-22) (\(V^+\) and \(I^+\)) describe a wave traveling from the source towards the load. The second terms (\(V^-\) and \(I^-\)) describe a wave traveling from the load to the source. The first wave (\(V^+\) and \(I^+\)) are referred to as the incident wave whereas the second wave (\(V^-\) and \(I^-\)) is referred to as the reflected wave.

**Impedance** The electrical impedance, or simply impedance, relates current to voltage at a certain port. It describes a measure of opposition to a sinusoidal alternating current, describing not only the relative amplitudes of the voltage and current, but also the relative phases. Impedance

\[^7\] The phase velocity of the voltage wave is defined as \(v_p = \frac{\omega}{\beta}\). Its wave wavelength \(\lambda = \frac{2\pi}{\beta}\).
2.4. SUMMARY AND OBSERVATIONS

is a complex quantity often denoted with the symbol \( Z \) and is usually represented as \( Z = R + jX \), where \( R \) is the ohmic resistance and \( X \) is the reactance.

A terminal load has impedance \( Z_T \) and is described as:

\[
Z_T = R_T + jX_T \tag{2-23}
\]

The internal impedance of a generator connected at the signal source end of the line has the symbol \( Z_S \), with:

\[
Z_S = R_S + jX_S \tag{2-24}
\]

The characteristic impedance is denoted as \( Z_0 \). \( Z_0 \) is characteristic for the line itself and is only dependent on the frequency \( \omega \) [rad/s]. As a function of the distributed coefficients the characteristic impedance is described as:

\[
Z_0 = R_0 + jX_0 = \sqrt{R + j\omega L \div G + j\omega C} \tag{2-25}
\]

Reflectance It is stated that reflected waves occur on a transmission line whenever a terminal load impedance is not equal to the characteristic impedance \( Z_0 \) of the line. A complex number stating the ratio of reflection is referred to as \( \rho_T \) and can be defined as

\[
\rho_T = \frac{V^- e^{\gamma l}}{V^+ e^{-\gamma l}} = \frac{Z_T - Z_0}{Z_T + Z_0} \tag{2-26}
\]

The reflection coefficient for harmonic current waves is found to be \( -\rho_T \). If a wave is entirely reflected, then the magnitude of the reflection coefficient is \(|\rho_T| = 1\).

Two-port networks A port of a network is defined as any pair of physical coincident terminals at which the instantaneous current into one of the terminals is equal to the instantaneous current out of the other terminal. Any section of an uniform transmission line is a two-port network. The purpose of two-port network theory is to develop formulas useful in expressing the nature of networks consisting of uniform transmission lines. The nature of the network is defined by the results of specified measurements made at the network terminals.

The open circuit impedance matrix of a length \( l \) of a uniform transmission line, having characteristic impedance \( Z_0 \), attenuation factor \( \alpha \) and phase factor \( \beta \) yields:

\[
\begin{bmatrix}
V^+ \\
V^-
\end{bmatrix} = \mathbf{Z} \begin{bmatrix}
I^+ \\
I^-
\end{bmatrix} \iff \begin{bmatrix}
V_1 \\
V_2
\end{bmatrix} = \mathbf{Z} \begin{bmatrix}
I_1 \\
-I_2
\end{bmatrix} \tag{2-27}
\]

with

\[
\mathbf{Z} = Z_0 \begin{bmatrix}
\coth(\gamma l) & \coth(\beta l) \\
\text{cosech}(\gamma l) & \text{cosech}(\beta l)
\end{bmatrix} \tag{2-28}
\]

Lossless transmission line A lossless transmission line does not dissipate energy. Hence, no attenuation of the voltage and current waves occurs. Therefore, the attenuation factor \( \alpha = 0 \). This implies that the distributed electric circuit coefficients \( R = 0 \) and \( G = 0 \). The transmission line equations for voltage and current stated in equations (2-7) and (2-8) are now described as:
\[
\frac{\partial V(x, t)}{\partial x} = -L \frac{\partial I(x, t)}{\partial t} \quad (2-29)
\]
\[
\frac{\partial I(x, t)}{\partial x} = -C \frac{\partial V(x, t)}{\partial t} \quad (2-30)
\]

An attenuation factor of zero also implies that the hybrid matrix as stated in equation (2-15) changes into:
\[
\begin{bmatrix}
V_1 \\
I_1
\end{bmatrix} = H_{\text{lossless}} \begin{bmatrix}
V_2 \\
-I_2
\end{bmatrix} \quad (2-31)
\]

with
\[
H_{\text{lossless}} = \begin{bmatrix}
\cosh(j\beta l) & Z_0 \sinh(j\beta l) \\
Z_0 \sinh(j\beta l) & \cosh(j\beta l)
\end{bmatrix} \quad (2-32)
\]
Chapter 3

Wave Variables

This chapter is dedicated to the introduction and explanation of the wave variable algorithm. The chapter starts of with information about the algorithms background. The second subsection presents a derivation of the wave variable algorithm along with block diagrams which can be useful for implementation. The third subsection provides a stability analysis based on passivity theory. The fourth, fifth, and sixth subsection are dedicate to explanation of the algorithms characteristics and behavior.

3.1 Introduction and Background

The wave variable algorithm is designed to solve the instability problem caused by communication delays in teleoperation systems. It is a highly robust algorithm that guarantees stability of the teleoperator system under arbitrary time delays. However, at the cost of a natural feeling (haptic experience) in general due to poorly damped reflections. Anderson and Spong[1][2] were the first to use the concept of wave variables in a context of haptic control. They took the passivity approach to deal with time delays in long range communications. For example intercontinental master / slave operations. Their approach is inspired on methods from electrical engineering used for controlling high power transmission lines which cover great distances. The paper attempts to model a teleoperation transmission line as an ideal (lossless) transmission line as discussed in chapter 2. The transmission line is assumed to be represented as a two-port. Stability was proven by means of the passivity and scattering theory.

The idea proposed by Spong and Anderson was used by Niemeyer and Slotine who continued their research in the direction and actually introduced the term wave variables within the field of haptic control. The wave variable is defined as an intermediate variable that is transmitted between the master and the slave device. After Niemeyer and Slotine, other researchers proceeded to improve on the wave variable algorithm while still taking advantage of its robustness. One of the active areas of research is a method for smart/active impedance matching. It’s used to reduce the wave reflections by artificially matching the characteristic impedance of the communication line to either the impedance of the master or slave.

more information on scattering theory is given in appendix
Niemeyer and Tanner have continued research related to the wave variable algorithm and have been researching the application of wave variables in sensing textures via high-frequency force feedback \[15\]. They propose injecting environmental force directly into the returning data after passing through a high-pass filter. They also proposed a solution to the above mentioned reflections and position drift. A self tuning algorithm that adjusted the wave impedance online depending on the slave impedance has been developed \[15\].

### 3.2 Wave Variable Theory

In the previous chapter, transmission line theory, which lays at the foundation of the wave variable algorithm was explained. Concepts as reflections, impedance, two-port networks and lossless transmission lines have been discussed. This subsection will derive the wave variable algorithm. For this, the concepts of impedance and hybrid matrix will be used again. This is also the point from which a mechanical notation will be used instead of an electrical notation. The electrical current \( I \) [A] and voltage \( V \) [V] are replaced by the mechanical equivalents velocity \( \dot{x} \) [m/s] and force \( F \) [N]. The mechanical hybrid matrix for a lossless transmission line in the Laplace domain can now be stated as,

\[
\begin{bmatrix}
F_1(s) \\
\dot{x}_1(s)
\end{bmatrix} = \begin{bmatrix}
\cosh(sT) & b \sinh(sT) \\
b \sinh(sT) & \cosh(sT)
\end{bmatrix} \begin{bmatrix}
F_2(s) \\
\dot{x}_2(s)
\end{bmatrix}
\]

(3-1)

The characteristic impedance is now interchanged for the symbol \( b \). And \( T \) denotes the delay in the transmission line. It is now possible to isolate the delay term by means of diagonalization of the hybrid matrix and yields:

\[
\begin{bmatrix}
\cosh(sT) & b \sinh(sT) \\
b \sinh(sT) & \cosh(sT)
\end{bmatrix} = \begin{bmatrix}
\sqrt{\frac{b^2}{2}} & -\sqrt{\frac{b^2}{2}} \\
\sqrt{\frac{b^2}{2}} & \sqrt{\frac{b^2}{2}}
\end{bmatrix} \begin{bmatrix}
e^{sT} & 0 \\
0 & e^{-sT}
\end{bmatrix} \begin{bmatrix}
\sqrt{\frac{b^2}{2}} & -\sqrt{\frac{b^2}{2}} \\
\sqrt{\frac{b^2}{2}} & \sqrt{\frac{b^2}{2}}
\end{bmatrix}^{-1}
\]

(3-2)

As can be seen, the transformation matrices in equation (3-2) are independent of the time delay. The matrices can be used to transform velocity and force into the incident and reflected wave variables \( u \) and \( v \):

\[
\begin{bmatrix}
F_1(s) \\
\dot{x}_1(s)
\end{bmatrix} = \begin{bmatrix}
\sqrt{\frac{b^2}{2}} & -\sqrt{\frac{b^2}{2}} \\
\sqrt{\frac{b^2}{2}} & \sqrt{\frac{b^2}{2}}
\end{bmatrix} \begin{bmatrix}
u_1(s) \\
v_1(s)
\end{bmatrix}
\]

(3-3)

\[
\begin{bmatrix}
u_1(s) \\
v_1(s)
\end{bmatrix} = \begin{bmatrix}
e^{sT} & 0 \\
0 & e^{-sT}
\end{bmatrix} \begin{bmatrix}
u_2(s) \\
v_2(s)
\end{bmatrix}
\]

(3-4)

\[
\begin{bmatrix}
F_2(s) \\
\dot{x}_2(s)
\end{bmatrix} = \begin{bmatrix}
\sqrt{\frac{b^2}{2}} & -\sqrt{\frac{b^2}{2}} \\
\sqrt{\frac{b^2}{2}} & \sqrt{\frac{b^2}{2}}
\end{bmatrix}^{-1} \begin{bmatrix}
u_2(s) \\
v_2(s)
\end{bmatrix}
\]

(3-5)

\[\text{note that the term } (j\beta l) \text{ is interchanged by } (sT). \text{ Remember that } \beta = \frac{\omega}{v_p} \text{ (subscript } p \text{ denoting the phase velocity), and that } T = \frac{l}{v_p} \text{ expresses the pure time delay of the transmission line.}\]

\[\text{note that: } \sinh(sT) = \frac{e^{sT} - e^{-sT}}{2} \text{ and } \cosh(sT) = \frac{e^{sT} + e^{-sT}}{2}\]
It can now be seen that only the wave variables undergo the time delay. However, note that the relation described in equation (3-4) is non causal. To obtain a causal expression, one should interchange the models inputs and outputs and obtain the wave transform matrices:

\[
\begin{bmatrix}
u_1(s) \\ F_1(s)
\end{bmatrix} = \begin{bmatrix}
\sqrt{2b} & -1 \\
b & -\sqrt{2b}
\end{bmatrix} \begin{bmatrix}
\dot{x}_1(s) \\ V_1(s)
\end{bmatrix}, \quad \begin{bmatrix}
\dot{x}_2(s) \\ v_2(s)
\end{bmatrix} = \begin{bmatrix}
\frac{\sqrt{2b}}{1} & -\frac{1}{2} \\
-\frac{b}{\sqrt{2b}} & \frac{b}{\sqrt{2b}}
\end{bmatrix} \begin{bmatrix}
u_2(s) \\ F_2(s)
\end{bmatrix} \tag{3-6}
\]

Note that $T$ represents the physical delay in the communication line, which will assumed to be fixed. However, the characteristic impedance $b$ is an adjustable parameter. Its use and possibilities will be discussed later on. Furthermore, the wave variables $u$ and $v$ travel from left-to-right (master-to-slave) and right-to-left (slave-to-master) respectively. From matrices (3-3), (3-4) and (3-5) the direct relations for the wave variables can be found,

\[
u_1 = \frac{F_1 + bx_1}{\sqrt{2b}}, \quad \nu_2 = \frac{F_2 + bx_2}{\sqrt{2b}} \tag{3-7}
\]

\[
V_1 = -\frac{F_1 + bx_1}{\sqrt{2b}}, \quad V_2 = -\frac{F_2 + bx_2}{\sqrt{2b}} \tag{3-8}
\]

By using the equations before mentioned, a transmission line can now be represented by the block diagram depicted in figure [3-1].

---

**Figure 3-1: Block diagram representation of the transmission line using the wave variable transformation**

The wave variable algorithm applied in a master/slave takes the form as shown in figure [3-2].

### 3.3 Stability and Passivity

One of the other advantages of using the wave variable theory is that it provides good insight in the energyflow between the inputs and outputs of the system as the product of force and velocity represents a notion of power. This subsection contains information regarding the stability of the wave variable algorithm. The discussion is based on the work of Günter Niemeyer and Jean-Jacques E. Slotine [12] and Ching Ho [4].

\[\text{As mentioned in the first chapter, information about teleoperation master slave systems is required. Appendix C provides for definitions and explanations for terms used, for instance terms in 3.2.}\]
The power input for the transmission line yields:

\[ P_{in} = \dot{x}^T F = \frac{1}{2} u^T u - \frac{1}{2} v^T v \]  \hspace{1cm} (3-9)

The condition for passivity requires that, over time, the output energy is equal or lower than the input energy:

\[ \int_0^t P_{in} \, d\tau = \int_0^t (\dot{\dot{x}}^T F) \, d\tau \geq -E_{store}(0), \quad \forall t \geq 0 \]  \hspace{1cm} (3-10)

where \( E_{store}(0) \) denoted the initial stored energy.

Rewritten into the wave domain, equation (3-10) yields:

\[ \int_0^t \frac{1}{2} v^T v \, d\tau \leq \int_0^t \frac{1}{2} u^T u \, d\tau + E_{store}(0), \quad \forall t \geq 0 \]  \hspace{1cm} (3-11)

As each wave 'contains' its own power, then passivity only requires that the output or return wave should be limited by the input or command wave regardless of phase. Then, the delayed wave signal does not alter passivity. Instead, it stores the energy in the wave for the delay time and releases it thereafter. In the case of a constant delay, the total power input is (i.e. power absorbed by the element):

\[ P_{in} = \dot{x}_1^T F_1 - \dot{x}_2^T F_2 = \frac{1}{2} (u_1^T u_1 - v_1^T v_1) - \frac{1}{2} (u_2^T u_2 - v_2^T v_2) \]  \hspace{1cm} (3-12)

With the introduction of constant delay \( T \), \( u_2 \) and \( v_1 \) can be expressed as,

\[ u_2(t) = u_1(t-T) \]  \hspace{1cm} (3-13)

\[ V_1(t) = V_1(t-T) \]  \hspace{1cm} (3-14)

Substitution in (3-12) gives,

\[ P_{in} = \frac{1}{2} \int_0^t [u_1^T u_1(\tau) - v_2^T v_2(\tau - T) - u_1^T u_1(\tau - T) + v_2^T v_2(\tau)] \, d\tau \]  \hspace{1cm} (3-15)

\[ = \frac{1}{2} \int_0^t [u_1^T u_1(\tau) - u_1^T u_1(\tau - T)] \, d\tau + \frac{1}{2} \int_0^t [v_2^T v_2(\tau) - v_2^T v_2(\tau - T)] \, d\tau \]  \hspace{1cm} (3-16)

and the next step is to combine the limits of the integral,

\[ \int_0^t P_{in} \, d\tau = E_{store}(t) = \frac{1}{2} \int_{t-T}^t u_1^T u_1(\tau) \, d\tau + \frac{1}{2} \int_{t-T}^t v_2^T v_2(\tau) \, d\tau \geq 0 \]  \hspace{1cm} (3-18)
assuming zero initial conditions.

With equation (3-18) it can be seen that the left term is always greater than or equal to zero because the integral terms are always positive regardless of the signs of the wave variables $u$ and $v$. Since equation (3-18) describes the net energy flow, it can be concluded that the network is now always a passive element. However, this proves is not conclusive because it considers that the delays in both directions are identical. Therefore, the delay $T$ is now split up in a delay term from port 1 to port 2 $T_L$ and visa versa the delay term $T_R$. Equation (3-18) can now be written as,

$$E_{store}(t) = \frac{1}{2} \int_{t-T_L}^{t} u_1^T u_1(\tau) d\tau + \frac{1}{2} \int_{t-T_R}^{t} v_2^T v_2(\tau) d\tau \geq 0 \quad (3-19)$$

The use of delays $T_L$ and $T_R$ does not change the net energy flow, which indicates that the asymmetric transmission delay has no effect on passivity. Furthermore, the unit of wave variables is the square root of power. It can be seen as power flow across the transmission line, and the pure transmission delay become temporary energy storage elements. The term temporary is used here because the power is only integrated over the duration of the transmission delay. If the wave variable, i.e. power flow, becomes zero for longer than the duration of the transmission delay, all the energy that is stored in the transmission delay is delivered to the output. Which again shows that transmission line is a lossless passive element.

### 3.4 Reflections and Impedance cont’

From figure 3-1 and the intermediate variables (equation (3-6)) it can be noted that the incident wave $u$ is a function of the reflected wave $v$ and in addition to either the force or velocity (i.e. parts of the incident wave are reflected back or visa versa). From the transmission line theory explained in this report, it is known that power transmitted along a line is reflected (at the ports at either sides) when the characteristic impedance of the line does not equal the impedance at the port.

The characteristic impedance, represented by $b$, is a tuning parameter which can trade off the speed of motion and the level of forces. Increasing $b$ will place a larger weight on the velocity compared to the force. Hence, a large $b$ will make the ‘transmission line’ act as a stiff and heavy rod. A small $b$ will make the ‘transmission line’ act as a light and compliant rod. This may be seen from equation (2-8) with $R = 0$ and $G = 0$.

**Intermezzo: the unit of characteristic impedance $b$**

As extra information regarding the characteristic impedance it could be useful to derive its unit. By taking the unit of the mechanical analog for $C$ (capacitance - mechanical compliance such as a spring) and $L$ (inductance - mass of a moving body).

$$L \ [\text{H/m}] \iff M \ [\text{Ns}^2/\text{m}]$$
$$C \ [\text{F/m}] \iff 1/K \ [\text{m/N}]$$
For a lossless line the mechanical characteristic impedance is described as

\[ Z_0|_{\text{lossless}} = \sqrt{\frac{L}{C}} \iff b \sqrt{\frac{M}{K}} \]

and its unit:

\[ b = \sqrt{\frac{M}{K}} = \sqrt{MK} = \sqrt{\frac{[\text{Ns}^2]}{[\text{m}]}} \cdot \frac{[\text{N}]}{[\text{m}]} = \frac{[\text{Ns}]}{[\text{m}]} \]

which corresponds with the unit of viscous damping.

Summarizing, the wave variable algorithm inherits effects as reflections which are also present in transmission line theory. The amount of reflection is dependent on the impedance of the transmission line (the characteristic impedance) and the impedance at its terminations (here impedance at the slave and master side). The difference between these impedances is a measure for the amount of reflection. However, the wave algorithm provides the opportunity to alter the characteristic impedance according to the preference or the needs of the user/operator. The amount of reflections in the system can be reduced by matching the characteristic impedance with the impedances of the master and/or the slave.

The choice of \( b \), however, sets a problem as a master/slave teleoperation system doesn’t always move in free motion or is not always in contact with a rigid environment. In other words: the characteristic impedance would have to be adjusted (on the fly) according to the (contact) state of the slave.

To illustrate the effects of \( b \) towards the level of reflections in master-slave system, two cases are considered. The explained cases are based on situations encountered whilst implementing the wave variable algorithm. They could provide a helpful insight on the choice of characteristic impedance parameter \( b \).

- Case 1: master in free motion\(^5\), slave in contact with a rigid environment\(^6\) and a large characteristic impedance (\( Z_m << b \approx Z_s \));

- Case 2: master in free motion, slave in contact with a rigid object and a small characteristic impedance (\( Z_m \approx b << Z_s \));

- Extra case: This is virtually an extension of case 2 where instead of a constant force, an impact force acting on the slave is considered. In this case the characteristic impedance is also considered to be small (\( Z_m \approx b << Z_s \)).

First, figure 3-1 is used and generalized to visualize the flow of signals through the system as depicted in figure 3-3. The only information given with this generalized view is the direction of flow and the interaction. The main point of this diagram is to emphasize and visualize the flow of signals through the wave variable diagram.

\(^5\)Here, ‘in free motion’ refers to a state in which the master or the slave experience no force due a lack of interaction

\(^6\)Here, ‘in contact’ refers to a state in which a constant force is applied and measured at either the master or the slave
3.4. Reflexions and Impedance Cont’

Case 1:

First, the implication of a large value for the characteristic impedance parameter $b$ is discussed. Secondly the implications for a signal traveling from the master device to the slave device through a transmission line is discussed.

A large value for parameter $b$ implies the following: for a large $b$ the transmission line can be seen as a component with a high resistivity (the stiff and heavy rod). In this particular case it matches with the highly resistive rigid object encountered at the slave device. Thus, in this case, the amount of reflections at the master side is large compared to the reflections occurring at the slave side. This is due to the fact that the characteristic impedance of the transmission line complies with the impedance of the rigid object and therefore absorbers most of its power.

The same conclusion can be obtained from another approach, using an observation of the flow of a signal through the wave variable diagram. A large value for the characteristic impedance parameter $b$ implies that the term $1/b$ (as seen in figure 3-3) becomes very small. As a result, the incoming force $F_2 [N]$ has less influence on the outgoing speed $\dot{x}_2 [m/s]$ as $b$ becomes larger. To emphasis this, the arrow has been removed as depicted in 3-4. At the left hand side of figure 3-4 an opposite situation holds. There, the speed $\dot{x}_1 [m/s]$ is directly fed back to force $F_1 [N]$. It can be seen that the larger $b$ is, the more dominant $\dot{x}_1 [m/s]$ is in the outgoing force signal $F_1 [N]$. This notion is stressed by a double arrow in figure 3-4.

It can be concluded that the larger parameter $b$ is, the more dominant velocity is with respect to the force. Besides this, a force measured at the slave will flow via $F_2 [N]$ towards the master to $F_1 [N]$. Due to the fact that $b$ is large, the force signal $F_1 [N]$ only contains a small part of the force $F_2 [N]$. In practice the operator experiences the slave (through the master) as sluggish or highly ‘damped’.

No realistic representation of the slave environment can be felt or experienced at the master-device. However, a larger value for $b$ does not accumulate reflection in the system.

\[\text{With the word ‘damped’ the difference in characteristics between a force and velocity signal is meant. In the sense that if a force is taken and divided by its corresponding mass and then differentiated to obtain velocity, it has lost capricious information enclosed in the force signal. Which in obtaining viable performance or sensing capabilities is crucial.}\]
CHAPTER 3. WAVE VARIABLES

Figure 3-4: Case 1: Generalization of the wave variable diagram to represent the flow of signals in an extreme case where the $b$ is very high

Case 2:

Similar to the first case, first the influence of the choice of parameter $b$ is discussed and secondly a signal traveling through a master-slave system using the wave variable algorithm is discussed.

A small value for parameter $b$ implies the following: the transmission line can be seen as a component with little resistivity (the light and compliant rod). In this particular case it matches with the small resistivity of the master device in free motion, but not with the large resistivity at the slave device. Thus, in this case, the amount of reflection at the master side is small compared to the reflections occurring at the slave side. This is because $b$ does not match with the impedance of the rigid environment.

The same conclusion can be obtained from the other approach. As in case 1, the flow signals through the system is observed. In contrast to case 1 the force signals are now more dominant over the velocity signals. In an extrem case (in which $b$ is very small) the feedback signals (fig.3-3) from $\dot{x}_1 [m/s]$ to $F_1 [N]$ can be neglected because of its minor influence, as represented in figure 3-5.

Figure 3-5: Case 1: Generalization of the wave variable diagram to represent the flow of signals in an extreme case where parameter $b$ is very low

Extra case:

Up till now the degree of force measured at the slave side is considered to be constant. In practice more 'extreme' situations have to be considered, such as an impact force. Here, the slave hits an object in its environment space (or an object hits the slave), which delivers a large force spike into the system (assuming that the system is controlled and capable of measuring with a sufficient small sampling time to capture the spike). In this example, only one impact is considered and no other movements.

In the case that $Z_m \approx b << Z_s$, $\dot{x}_1 [m/s]$ has still some influence. The spike will partially reflect back to the slave and a loop (containing the spike) traveling back and forth via the master and the slave is created. In time the spike will disappear as it is partially absorbed at each terminal. However, if a case is considered in which the characteristic impedance is set to a very low value ($b <<$) the dominance
of the signals traveling from the slave will increase over the velocity signal $\dot{x}_1 \text{[m/s]}$. In this way no damping (or very little damping) of the spike takes place. In practise, this leaves a system which is not suitable for practical use.

### 3.5 The Wave Domain

In order to get a better understanding of the behavior of basic mechanical components in the wave domain, step responses of a mass, spring and damper are considered (figure 3-6).

In figure 3-6 the step response in wave domain is depicted for each of the mechanical elements. To grasp the physical idea of wave transformation, one needs to take into account that a free motion or rigid wall (doesn’t dissipate nor stores any energy) results in a straight line $v(t) = \pm 1$ in the wave domain. As the reflection coefficient $\rho_T = 1$ for free motion and $\rho_T = -1$ for rigid wall.

---

**Figure 3-7: Step response of individual elements in wave domain, impedance variable $b = 1$**

Description of the response of each element

---

*The figure and explanation are inspired on a section of the paper named: Using wave variables for system analysis and robot control by Günter Niemeyer and Jean-Jacques E.Slotine 1997

*Appendix B contains step responses in the wave domain for each element for different value for $b$)*
Inertia At $t = 0$ [s] the velocity of the mass is zero, and reacts as a rigid wall. As time $t$ [s] increases kinetic energy is build up. Velocity increases as force decreases, as the kinetic energy stored in the mass exponentially converges to $v(t) = 1$ in steady-state.

Spring Initially, the force acting on the spring is very low, the spring compresses easily and starts building up potential energy. As the potential energy builds up, the increase of forces slows down further motion and the response becomes negative. In steady state the spring feels like a rigid wall reflecting the applied force without any movement. In contrast to the inertia the spring doesn’t dissipate energy, but stores its energy as potential energy.

Damper In contrast to the spring and the inertia the damper dissipated its energy. From $B \cdot \dot{x} = F$, it shows that the damper acts as restriction on the input velocity and thereby dissipates energy.

### 3.6 Experiment

The wave variable algorithm was implemented in the experimental setup described in the first chapter. One type of experiment was carried out. The general idea of the experiment was to gain insight in the behavior of the wave variable algorithm, three aspects are considered:

- Demonstrate the position drift between master and slave device.
- Demonstrate the influence of changing the characteristic impedance parameter $b$.
- Demonstrate the influence of implementing an improved controller at the slave side. For comparison reasons time-response plots are shown of a teleoperation system using the wave variable algorithm and a teleoperation system using a so called 4-channel controller by Lawrence [8].

Figure 3-8 depicts how a connection between a master and a slave device is made with the use of the wave variable algorithm. Note that there is no controller present at the master side. The force $F_s$ is the controller output and due to the fact that velocity is controlled in contrast to position, a PI controller is applied. Also notice that the output of the master and the slave device is a velocity, both integrated from the position. For convenience and clarity some subscripts have been changed. The subscript 1 has been interchanged for the letter ‘m’ referring to the master. And Subscript 2 has been interchanged for the letter ‘s’ referring to the slave. Furthermore the subscript $sc$ stands for a velocity signal used as reference to control the slave device.

![Figure 3-8: Simplified block diagram of the experimental setup as used in the experiment](image)

The experiment is described as follows:
3.6. EXPERIMENT

The knob/grip of the pantograph device is actuated in the \( x \)-direction for a period of 5 seconds with a constant force of 0.5 [N]. Within that time, the slave contacts a virtual wall. The time-response of this of the force transmitted to the master is then analyzed.

To ensure that there is a minimal amount of interaction of the other two DOF’s, they are fixed by two separate controllers as illustrated in figure 3-9.

This experiment takes care for the same condition for each experiment, in this way it is possible to compare experiments with different settings.

![Virtual representation of the fixed DOF’s](image)

**Figure 3-9: Schematic representation of the experiment**

3.6.1 Position drift

In papers published by several authors, under which Niemeyer and Slotine and H.Ching, a position drift between master and slave device in time is mentioned. In the first trials the effect of position drift indeed appeared. Graphs (a) and (b) in figure 3-10 both demonstrate the position drift between the master- and slave device. The time 35 [s] to 40 [s] is the time in which the constant force of 0.5 [N] is applied. The experiment starts after 35 [s] due to several initialization steps that are carried out in advance of the experiment. Figure 3-10(a) represents the case where the normal controller is used and (b) the case in which a higher bandwidth controller is applied. Between the 35th and the 36th second, the mass in the VE moves towards the virtual wall. The decreasing slope is exponential and corresponds to the applied constant force. The position error between master and slave device is minimal. During the final tenths of this second the virtual mass hits the virtual wall. The virtual mass bounces 3 to 4 times against the wall until it settles and comes to a standstill. As can be seen the relative displacement of the virtual mass \( x_s \) after settlement is zero. However, when looked at the position of the pantograph device, whose physical boundaries are larger than the boundaries of the VE, after impact shows a constant decreasing displacement. The difference between figures (a) and (b) is apparent in the sense that the angle of the slope differ. The angle of the slope of \( x_m \) in (b) is smaller compared to the slope of \( x_m \) in (a).

If a closer look is taken at \( x_m \), in figure 3-10(b), a small but important difference can be seen compared with \( x_m \) in figure 3-10(a) is seen in the settling time. It settles faster, resulting in a smaller angle of the
slope. The reason why the position drift takes place is because of the numerical errors occurring when differentiating to obtain velocity signals, especially at very low speeds. These differentiating actions take place at both the master and the slave. The problem of position drift and a solution is addressed and proposed in a paper [14] by Günter Niemeyer and Jean-Jacques Slotine.

3.6.2 Varying the characteristic impedance parameter

In this subsection the time response of the force $F_m$ as transmitted through the wave variable diagram is analyzed for three different values for the characteristic impedance $b$.

The time response plots of the force $F_m$ are depicted in figure 3-11. Viewed from top to bottom the characteristic impedance $b$ is respectively $b = 10$, $b = 125$ and $b = 200$. As explained in subsection 3.4, the higher $b$ is, the more emphasis is put on the velocity transmitted from master to slave. For a smaller $b$, emphasis is put on the force transmitted from slave to master. Notice that the forces for a decreasing $b$ become less damped and less transient. The first observable transient is observed at time $t = 35 \ [s]$. This is the point were the constant force is applied and the virtual slave is moved towards the virtual wall. At $t \approx 35.7 \ [s]$ the virtual mass hits the wall and an impact force is transmitted to the master device. Directly after the impact the virtual slave bounces of the wall. Due to the constant force which is still applied, the virtual mass will hit the wall again at $t = 36 \ [s]$. Eventually, the virtual mass settles at the wall. Notice that after each hit or abrupt change in force, transient behavior occurs.

If one wants to experience the environment space at the slave side through a teleoperation system, one has to feel the exact same forces which occur at the slave side. It can be concluded that the amount of damping which is introduced using a large value of $b$ is not desired. Viewing figure 3-11 again from top to bottom, notice that at the top graph the transmitted force $F_m$ at impact is the largest. Also note that the transient behavior can be decreased significantly by decreasing $b$. 
3.6.3 Improvements and comparison

The last result which is presented in this report, is a comparison of three graphs as presented in figure 3-12. All three graphs represent the time response of the force $F_m$ as transmitted to the master according to the experiment as explained in 3.6. From top to bottom, the first graph represents response of a teleoperation system using the wave variable algorithm with a characteristic impedance parameter $b = 125$. The second graph described the same system but with the difference of an improved controller is used. Virtually meaning a PI controller with a higher bandwidth. In order to set the wave variable algorithm in perspective, one experiment is performed using a 4-channel teleoperator by Lawrence [8]. The bottom graphs represent the result using this control method.

It can be seen that the difference in the amount of force transmitted through the 4-channel teleoperator to the master is large. Although the amount of force transmitted through the 4-channel teleoperator is very large, the total amount of force is not felt at the pantograph setup due to limitations of the applicable torque of the DC-motors and due to saturations in the system with respect to safety (for the operator and for the setup itself). However, there is a big difference in experiencing the virtual environment with a 4-channel by Lawrence system compared with the wave variable algorithm. With the 4-channel operator the virtual wall actually feels like a rigid wall. With the implementation of an improved controller the sense of touching a rigid wall was more realistic but not as good as the 4-channel teleoperator by Lawrence.

It can be concluded that there was a big difference in experiencing the slave environment by either the wave variable algorithm or the 4-channel teleoperator. But, it can also be said that there is room for a lot of improvements of the wave variable algorithm as it is implemented at this point. Many of those improvements are described extensively by Günter Niemeyer and Jean-Jacques Slotine in [14] [15].
Figure 3-11: Time response plot of the force as transmitted back to the master for different values of the characteristic impedance; from top to bottom: $b = 10$, $b = 125$ and $b = 200$
Figure 3-12: Time response plot of the force as felt/experienced at the master with the use of a 4-channel teleoperator; from top to bottom: $b = 125$, $b = 125$ improved controller and 4-channel Lawrence teleoperator.
Chapter 4

Conclusions & Recommendations

4.1 Conclusions

This report is a result of a 3.5 month traineeship at the University of Victoria in Canada, accomplished under direct supervision of dr. D. Constantinescu. The traineeship involved the implementation of a so called wave variable algorithm in an experimental setup. The experimental setup consist of a physical master device and a virtual slave device. It is used as a platform to perform research in a field described as haptics, more specific: haptic technology. The latter involves the development of systems that are capable of presenting information to a user by the sense of touch i.e. forces. The wave variable algorithm, (as presented in this report) developed by Günter Niemeyer and Jean-Jacques E. Slotine, treats a specific part of this research: the instability problem caused by communication delays (due to large distances) between the master and the slave devices. The wave variable algorithm is capable of providing a passive communication channel solving this problem. However, it can also lead to a conservative system in the sense of reduced performance with respect to presenting the user with ‘realistic’ information. Motivated by the interesting research by Günter Niemeyer and Jean-Jacques E. Slotine, the assignment for the traineeship was stated as follows:

*Implement the wave variable algorithm on the experimental setup.*

By implementing the algorithm it directly provides information about its handling and feeling outside the scope of computer simulations and information given in papers. Within the assignment the following subgoals were recognized:

- Perform a literature study.
- Investigate stability, robustness and performance of the algorithm.

The literature study resulted in the following conclusions:

- The algorithm originates from transmission line theory. Here, use is made of a special class called ‘lossless transmission line’. Combined with the representation of a transmission line as a two-port network realizes a notion of the transmission line where signals traveling through it, are independent of time. These signals are called wave variables and are a combination of
a force and a velocity signal. The adjustable characteristic impedance parameter trades the velocity against the forces and allow the user so suit its needs or preference.

- The wave variable algorithm provides robustness to arbitrary delays. For small delays, the system remains stable and local controllers can be designed without change. For larger delays impedance matching or wave filtering can be used.

- Stability of the transmission line can be proven by means of passivity theory i.e. net energyflow in the transmission line. However, the practical stability of the system is not guaranteed because certain values for the characteristic impedance parameter (typical low values) introduce reflections.

- Cascading of passive elements within the communication line can be added without endangering the total passiveness of the transmission line.

- In practise, a position drift between master and slave is observed. Tracking between the master and the slave relies on the numerical integration of the desired slave velocity into a desired position. Which, in practise, leads to a tracking error between the master and slave device. Other causes are discrete sampling and/or data loss in the communication line.

With respect to the item of stability, robustness and performance; no appropriate validation has been carried out. However, an experiment was carried out in which the characteristics of the wave variable algorithm were demonstrated. As a result, the following observations can be made:

- Position drift was measured and experienced. It could not be established whether the amount of position drift was a result of numerical integration, discrete sampling, data loss or the controller used to feedback the force at the slave device to the master device. However, an improved controller, with a higher bandwidth, resulted in less position drift.

- Influence of the characteristic impedance parameter was demonstrated. As the value for the characteristic impedance is decreased, less damping and transient behavior was observed. However, it has to be noted that there exists a lower boundary for the characteristic impedance. Under this boundary, reflections make it impossible to interact with the slaves environment.

### 4.2 Recommendations

Recommendations for future research:

- Through experiments, perform an analysis of the stability, robustness and performance under constant and varying delays. It would especially be interesting to see how the characteristic impedance influences these results.

- Implement a solution against parameter drift, as provided in [13].

- An advantage towards using force and velocity signals is that the powerflow between the master and slave can be measured. This information could be used to make a supervisory controller, capable of intervening in cases where the energyflow reach certain (danger) levels.
4.2. RECOMMENDATIONS

- Implement an asymmetric controller providing high frequency force signals as transmitted to the user via the slave [15];
- Implement the wave variable algorithm for the other two DOF’s.
Appendix A

Haptics demo

Haptics demo

1. Start-up Client Computer (left);
   (a) User name: Administrator;
   (b) Password: DancingRobot;
   (c) Log on to: M348-PC04;
   (note: Do not touch Haptic Device after beginning step 2)

2. Start-up Haptics Demo;
   (a) Open desktop shortcut FTP server;
   (b) Open desktop shortcut Hyperterminal (VxWorksCOM1(3));
   (c) Start-up target computer (without monitor);
   (d) Open Simulink model:
      i. Open Matlab 7.0.4.;
      ii. Set current directory to C:\workSpaceConstraintsMatlab7\simulinkModel;
      iii. Run initModel.m;
      iv. Open simulinkmodel wsc7.mdl;
      v. Open Tools
         - External Mode Control Panel (do not minimize this window);
         - Use ‘Signal & Triggering’ to select which data signals you want to record (max.3);
         - Use ‘Data Archiving’ to enable/disable archiving;
   (e) Open Graphics:
      i. Open Microsoft Visual C++;
      ii. file -> recent Workspace;
      iii. Open workspace C:\workSpaceConstraintsMatlab7\graphics\grahpics.dsw;
   (f) Open Tornado:
      i. Open Tornado Registry, an icon appears in right bottom screen;
ii. Open Tornado (neglect error message, press OK);

iii. Open workspace C:\workSpaceConstraintsMatlab7\simulinkModel\Workspace0.wsp;

iv. Start-up target server (Tools -> Target Server -> pantcpu0). An icon appears in right bottom screen, check message;

v. Select pantcpu0@m348-pc04 in drop-down menu in Tornado;

vi. Launch shell (click -> i);

vii. Make sure the current directory is C:\workSpaceConstraintsMatlab7\simulinkModel
    • Type: pwd in shell to check the current directory;
    • Type: cd "C:\workSpaceConstraintsMatlab7\simulinkModel" (with parentheses);

3. Run Haptics Demo;

   (a) In Tornado Shell, type <start (uploads real time code to target, remote PC);
   (b) In Matlab Simulink, In the External Mode Control Panel, click connect;
   (c) In Microsoft Visual C++, press CTRL+F5. Two windows appear. (wait a bit);
   (d) In the External Mode Control Panel of the Simulink model, click Start Real-Time Code(this starts the haptics server);
   (e) Turn on the power source of the haptic device;
   (f) In the VE, when the link held by the user turns red, turn power to motors on by depressing the red kill-switch;
   (g) play time!
   (h) To stop the demo:
      i. Press the Kill Switch;
      ii. Click Stop Real-Time Code in External Mode Control Panel in the Simulink model;
      iii. Stop the graphics client by closing both windows opened by Microsoft Visual C++;
      iv. Turn off power source of haptic device, close all opened windows and shut-down Tornado Registry and Hyperterminal.

4. Restart Haptics Demo (when it crashes);

   (a) Restart target computer;
   (b) In Tornado, close the shell;
   (c) Click Stop Real-Time Code in External Mode Control Panel in the Simulink model. If this does not work, close Matlab and execute step 2.d;
   (d) When the target computer rebooted (check message in Hyperterminal), start a new shell in Tornado (steps 2.f.iv and 2.f.v)
   (e) Run Haptics Demo (step 3)
Appendix B

Step responses in the wave domain
Figure B-1: step response in the wave domain of a damper, with varying characteristic impedance parameter $b$

Figure B-2: step response in the wave domain of a inertia, with varying characteristic impedance parameter $b$
Figure B-3: step response in the wave domain of a spring, with varying characteristic impedance parameter $b$

Figure B-4: step response in the wave domain of a 2nd order system, with varying characteristic impedance parameter $b$
APPENDIX B. STEP RESPONSES IN THE WAVE DOMAIN
Appendix C

Explanatory

Admittance  In electrical engineering, the admittance (Y) is the inverse or reciprocal of the impedance (Z), (also see impedance)

Distributed element model  Otherwise referred to as a class of models. For example the transmission line model of electronic circuits. It assumes that each circuit element is infinitesimal, and that the connecting elements are not perfect conductors, i.e. they have impedance. It assumes non-uniform current along each branch and non-uniform voltage along each node. This model is more accurate but more complex than its opponent, the lumped element model. Where the theory of distributed circuits is the extension of lumped element circuit theory by the addition of a space(meters) variable.

Environment  That which environs or surrounds; surrounding conditions, influences, or forces, by which living forms are influenced and modified.

Haptic  Pertaining to the sense of touch, in general the word haptic is used as a gather name for subjects related to the sense of touch.

Haptic feedback  The process of communicating information back to the user using force signals. Feedback in the sense of human-human or machine-human communication and/or interaction by means force.

Haptic technology  The means or knowledge to develop, design and build systems that are able to provide or present information to the user by the sense of touch.

Immitance  Collective term for impedance and admittance

Immitance matrices  A mapping between flows and efforts from the two-port network theory.

Impedance, characteristic (in transmission line theory)  The characteristic impedance is denoted as \( Z_0 \). \( Z_0 \) is characteristic for the line itself and is only dependent on the frequency \( \omega \) [rad/s]. As a function of the distributed coefficients the characteristic impedance is described as:

\[
Z_0 = R_0 + jX_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}}
\]  \hspace{1cm} (C-1)

Impedance, electrical  In electrical engineering, impedance is a measure for the manner and degree a component resists the flow of electrical current if a given voltage is applied. It is denoted by the symbol Z and is measured in Ohms. Impedance differs from a simple resistance in that it takes into account possible phase offset. Impedance can be seen as complex resistance.

Kirchhoff laws  Kirchhoff’s circuit laws are a pair of laws that deal with the conservation of charge and energy in electrical circuits, and were first described in 1845 by Gustav Kirchhoff.
law 1: The principle of conservation of electric charge implies that:

At any point in an electrical circuit that does not represent a capacitor plate, the sum of currents flowing towards that point is equal to the sum of currents flowing away from that point.

\[ \sum_{k=1}^{n} I_k = 0 \]  
\[ (C-2) \]

\( n \) is the total number of currents flowing towards and away from a point, the formula is also considered to be complex.

law 2: The directed sum of the electrical potential differences around any closed circuit must be zero.

\[ \sum_{k=1}^{n} V_k = 0 \]  
\[ (C-3) \]

\( n \) is the total number of currents flowing towards and away from a point, the formula is also considered to be complex.

**Lossless transmission line** A lossless transmission line does not dissipate energy. Hence, no attenuation of the voltage and current waves occurs. Therefore the attenuation factor \( \alpha = 0 \). This implies that the distributed electric circuit coefficients \( R = 0 \) and \( G = 0 \).

**Lumped element model (of electronic circuits)** It makes the simplifying assumption that each element is an infinitesimal point in space, and that the wires connecting elements are perfect conductors. This is reasonable for many actual circuits, but breaks down when actual circuit impedances are very low, or when the length of the wire approaches/exceeds the wavelength of the circuit’s operating frequency. In these cases the distributed element model is used.

**Master** A device or mechanism intended for controlling another device operating in a similar way.

**Operator** One who, or that which, operates or produces an effect (by means of a mechanical device).

**Reflectance** It is stated that reflected waves occur on a transmission line whenever a terminal load impedance is not equal to the characteristic impedance \( Z_0 \) of the line. A complex number stating the ratio of reflection is referred to as \( \rho_T \).

\[ \rho_T = \frac{V - e^{-\gamma l}}{V + e^{-\gamma l}} = \frac{Z_T - Z_0}{Z_T + Z_0} \]  
\[ (C-4) \]

The reflection coefficient for harmonic current waves is found to be \(-\rho_T\). If a wave which is entirely reflected the magnitude of the reflection coefficient is \(|\rho_T| = 1\).

**Scattering theory** It is a framework for studying and understanding the scattering of waves and particles. Scattering theory consists of the study of how solutions of partial differential equations, propagating freely ‘in the distant past’, come together and interact with one another or with a boundary condition, and then propagate away ‘to the distant future’.

**Shunt** to divert (a part of a current) by connecting a circuit element in parallel with another.

**Slave** A device or mechanism under control of and repeating the actions of a similar mechanism.

**Traveling and standing wave phenomena** [http://www.glenbrook.k12.il.us/GBSSCI/PHYS/Class/waves/u10l4a.htm](http://www.glenbrook.k12.il.us/GBSSCI/PHYS/Class/waves/u10l4a.htm)

**Transient response (in mechanical engineering)** the response of a system to a change from equilibrium. It can be seen as the portion of the response that approaches zero after a sufficiently long time (i.e., as \( t \) approaches infinity).
**Teleoperation system** A system which is able to manipulate distant environments.

**Two-port networks** A port of a network is defined as any pair of physical coincident terminals at which the instantaneous current into one of the terminals is equal to the instantaneous current out of the other terminal. Any section of an uniform transmission line is a two-port network. The purpose of two-port network theory is to develop formulas useful in expressing the nature of networks consisting of uniform transmission lines. The nature of the network is defined by the results of specified measurements made at the network terminals.

The open circuit impedance matrix of a length $l$ of a uniform transmission line, having characteristic impedance $Z_0$, attenuation factor $\alpha$ and phase factor $\beta$ yields:

$$
\begin{bmatrix}
V^+ \\
V^-
\end{bmatrix}
= 
Z
\begin{bmatrix}
I^+ \\
I^-
\end{bmatrix}
\iff
\begin{bmatrix}
V_1 \\
V_2
\end{bmatrix}
= 
Z
\begin{bmatrix}
I_1 \\
-I_2
\end{bmatrix}
$$

(C-5)

with,

$$Z = Z_0
\begin{bmatrix}
\coth(\gamma)l & \cosech(\gamma)l \\
\cosech(\gamma)l & \coth(\gamma)l
\end{bmatrix}
$$

(C-6)

**Waves** A wave is a disturbance that propagates through space and time, usually with transference of energy.
Appendix D

Derivation of the open circuit impedance matrix

![Figure D-1: Schematic view of a basic transmission line, with impedance load $Z_T$](image)

**D.1 Derivation $z_{11}$ and $z_{22}$**

The derivation of the term

$$Z_{inp} = Z_{out} = V_{11} = V_{22} = Z_0 \coth(\gamma l)$$  \hspace{1cm} (D-1)

yields as follows. The impedance at any point on a uniform transmission line, with impedance $Z_T$, may be obtained using (2-3) and (2-4),

$$Z(x) = \frac{V(x)}{I(x)} = Z_0 \left( \frac{V^+ e^{-\gamma x} + V^- e^{\gamma x}}{V^+ e^{-\gamma x} - V^- e^{\gamma x}} \right) \iff \frac{Z(x)}{Z_0} = \frac{e^{-\gamma x} + \rho_T e^{-2\gamma l} e^{\gamma x}}{e^{-\gamma x} - \rho_T e^{-2\gamma l} e^{\gamma x}}$$  \hspace{1cm} (D-2)

Dividing the numerator and denominator on the right side of (D-2) by $V^+$ and substituting $\frac{V^-}{V^+} = \rho_T e^{-2\gamma d}$ from equation (2-12) yields,

$$\frac{Z(x)}{Z_0} = \frac{e^{-\gamma x} + \rho_T e^{-2\gamma l} e^{\gamma x}}{e^{-\gamma x} - \rho_T e^{-2\gamma l} e^{\gamma x}}$$  \hspace{1cm} (D-3)

Multiplying all terms on the right of equation (D-3) by $e^{\gamma l}$, and substituting $l - x$ for $d$,

$$\frac{Z(d)}{Z_0} = \frac{e^{\gamma d} + \rho_T e^{-\gamma d}}{e^{\gamma d} - \rho_T e^{-\gamma d}}$$  \hspace{1cm} (D-4)
As $Z(d = 0) = Z_T$,
\[
\rho_T = \frac{Z_T}{Z_0} = \frac{1 + \rho_T}{1 - \rho_T} \iff \frac{Z_T - Z_0}{Z_T + Z_0} = \frac{Z_T - 1}{Z_T + 1}
\] (D-5)

The term for $\rho_T$ can now be substituted in (D-4),
\[
\frac{Z(d)}{Z_0} = \frac{e^{\gamma d} (\frac{Z_T}{Z_0} + 1) + e^{-\gamma d} (\frac{Z_T}{Z_0} - 1)}{e^{\gamma d} (\frac{Z_T}{Z_0} + 1) - e^{-\gamma d} (\frac{Z_T}{Z_0} - 1)}
\] (D-6)

Which is equivalent to
\[
\frac{Z(d)}{Z_0} = \frac{(\frac{Z_T}{Z_0} + \tanh(\alpha + j\beta)d)}{1 + (\frac{Z_T}{Z_0}) \tanh(\alpha + j\beta)d}
\] (D-7)

with $\gamma = \alpha + j\beta$.

Returning to the first statement (D-3), a term is needed for the input impedance of a length of line of a transmission line. And virtually means the input impedance of the part of the line on the terminal load side. Previously stated as the impedance at a specific point on a line. From the previous equation (D-7), $Z(d)$ can be interchanged for the input impedance $Z_{inp}$.
\[
\frac{Z_{inp}}{Z_0} = \frac{(\frac{Z_T}{Z_0} + \tanh(\alpha + j\beta)l)}{1 + (\frac{Z_T}{Z_0}) \tanh(\alpha + j\beta)l}
\] (D-8)

For any general uniform transmission line terminated in an open circuit (i.e. $Z_T = 0$), its input impedance $Z_{inp}$ is determined completely by the propagation factors $\alpha$ and $\beta$, its characteristic impedance $Z_0$ and $l$.

Equation (D-8) can therefore be written as:
\[
Z_{inp} = Z_0 \coth(\gamma)l
\] (D-9)

### D.2 Derivation of $z_{12}$ and $z_{21}$

\[
V_{lr} = V_{rl} = z_{lr} = z_{rl} = Z_0 \coth(\gamma)l
\] (D-10)

The coefficient $z_{lr}$ is the value of the ratio $\frac{V_r}{I_l}$ when $I_r = 0$, and is determined with equations (2-3), (2-10) and (2-12).

\[
V_r = V(x = l) = V^+ e^{-\gamma l} + V^- e^{\gamma l}
\]
\[
I_l = I(x = 0) = \frac{V_l}{Z_0} - \frac{V_r}{Z_0}
\] (D-11)

and $z_{rl}$ now yields,
\[
z_{rl} = \frac{V_r}{I_r} = Z_0 \frac{V_l \{e^{-\gamma l} + (\frac{V_r}{V_l}) e^{\gamma l}\}}{V_l (1 - \frac{V_r}{V_l})}
\] (D-12)

---

1 Note that: $\tanh(\gamma) = \sinh(\gamma) \cosh(\gamma)$, $\coth(\gamma) = \frac{1}{\tanh(\gamma)}$, $\sinh(\gamma) = \frac{e^\gamma - e^{-\gamma}}{2}$, $\cosh(\gamma) = \frac{e^\gamma + e^{-\gamma}}{2}$
D.2. DERIVATION OF $Z_{12}$ AND $Z_{21}$

Where $\frac{V_r}{V_l} = \rho T e^{-2\gamma l}$, and that for an open circuit termination $1 + j0$, and $\cosech(\gamma) = \frac{1}{\sinh(\gamma)}$,

$z_{rl} = z_{lr}$ can be written as.

$$z_{rl} = z_{lr} = Z_0 \cosech(\gamma l)$$  \hspace{1cm} (D-13)
APPENDIX D. DERIVATION OF THE OPEN CIRCUIT IMPEDANCE MATRIX
Appendix E

Implementation Issues

The difficulties with the implementation of the wave variable algorithm virtually comes down to choosing an appropriate value for the characteristic impedance parameter $b$. In papers, it is seen that values for $b$ are commonly low, between $1 - 10 \ [\text{Ns/m}]$. At the end of the traineeship, after searching for months for errors in other parts of the software, it became clear that the $b$ parameter which was used in this setup was chosen to small. So what I saw as instability were actually undamped reflections within the system. With respect to this problem, my tip is to always begin by choosing a large value for $b$ (such as $b = 1000 \ [\text{Ns/m}]$). This has no other implications other than a highly damped system. If that works, then gradually decrease the value for $b$ to find the lower boundary.

Furthermore, the following obstacles can be encountered:

- Algebraic loops: this is a loop where the input of a block requires the output of the same block. At the first discrete step of a simulation it tries to process that block, but it is not capable of doing so because the output is not available. It’s only available after the second discrete time step. I solved this problem by introducing a 1 step time delay.

- Towards the wave variable control approach by Gunter Niemeyer and Jean Jacques Slotine, the control scheme which is presented in the papers has a $F_m = F_s$ relation, which didn’t correspond to the given wave variable transformation in which is stated that the only difference between the traveling waves is the sign of the force. In haptics literature this is a straightforward assumption because of the action = reaction relation.
APPENDIX E. IMPLEMENTATION ISSUES
Bibliography


