



## Velocity and acceleration estimation for optical incremental encoders<sup>☆</sup>

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### ARTICLE INFO

#### Keywords:

Encoders  
Data acquisition  
Sensors  
Angular acceleration  
Angular velocity  
Angular position  
Quantization

### ABSTRACT

Optical incremental encoders are extensively used for position measurements in motion systems. The position measurements suffer from quantization errors. Velocity and acceleration estimations obtained by numerical differentiation largely amplify the quantization errors. In this paper, the time stamping concept is used to obtain more accurate position, velocity and acceleration estimations. Time stamping makes use of stored events, consisting of the encoder counts and their time instants, captured at a high resolution clock. Encoder imperfections and the limited resolution of the capturing rate of the encoder events result in errors in the estimations. In this paper, we propose a method to extend the observation interval of the stored encoder events using a skip operation. Experiments on a motion system show that the velocity estimation is improved by 54% and the acceleration estimation by 92%.

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### 1. Introduction

Optical incremental encoders are widely used to apply feedback control on motion systems where the position is measured at a fixed sampling frequency. They are available in both rotational and linear form. The position accuracy is limited by the quantized position measurement of the encoder, i.e. it is limited by the number of slits on the encoder disk.

The quantization errors can be reduced using more expensive encoders with more increments at the expense of increased cost price. Velocity and acceleration information is often obtained by numerical differentiation of the quantized position signal. Direct differentiation mostly leads to signals that are not useful [6]. The quantization errors limit the performance in high accuracy control applications. Smart signal processing techniques can be used in combination with cheap low resolution encoders to obtain position estimations with the same accuracy as expensive high resolution encoders.

In literature, several methods have been proposed to improve the accuracy of velocity and acceleration estimations for optical incremental encoders. The methods can be divided into two kinds; fixed-time (clock-driven) methods and fixed-position (encoder-driven) methods.

For real-time control purposes, a fixed-time method is desired since the controller is generally evaluated at fixed-time intervals. Fixed-time velocity and acceleration estimations can be obtained

using three different approaches; predictive postfiltering techniques, linear state observers and indirect measurement techniques.

Predictive postfiltering techniques perform a filtering on differentiated position signals. Euler based methods [10] and polynomial delayless predictive differentiators [14] both disregard the variable rate of occurrence of the encoder events to estimate the velocity or acceleration. The transition based logic algorithm of [9] estimates only the velocity under the assumption that the sampling frequency is much larger than the rate of the encoder events.

Linear state observer techniques use the encoder position measurements, without the need for differentiation. Dual-sampling rate observers [7] and Kalman filters [2] require accurate system models to be available. The non-model based observer of [13] switches between two estimation filters based on an estimation error, which is generally not available. Data based observers using neural networks [4] or fuzzy logic [15] estimate the velocity using only the position information, thus disregarding the non-constant time occurrence of the encoder events.

Indirect measurement techniques are based on analog or digital postprocessing of available position and/or velocity signals. In [12], the velocity is estimated by a polynomial fitting through a number of encoder counts. No time information of the encoder events is taken into account and no acceleration information is obtained. Both encoder counts and their time instants are used in [3] to estimate the velocity. This is called the time stamping concept. However, simulations only are performed and at a practically unrealistic sampling frequency of 1 MHz. Furthermore, all encoder events are taken into account, which is not practically applicable.

Since for control purposes fixed-time methods are desired and since the encoder events have a fixed-position nature, occurring

<sup>☆</sup> This research is part of the Micro and Nano Motion project, which is supported by SenterNovem/Point One.

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at a varying rate in time, a combination of the two approaches would be favorable. Therefore, in this paper, we propose a method to accurately estimate the velocity as well as the acceleration using the time stamping method of [3]. The time stamping concept was used for position estimation in combination with the calibration and compensation of encoder errors in [11]. The time stamping concept involves the capturing of encoder events, i.e. the encoder transitions and their time instants, at a high clock frequency added to the encoder in the data-acquisition hardware. The encoder events are stored in a register and transferred to the controller at a lower fixed sampling rate. Polynomial interpolation through the encoder events and extrapolation of the obtained polynomial make it possible to estimate position, velocity and acceleration at the time of interest.

In this paper, we propose a skip option for the event selection as an extension of the time stamping concept of [11]. The need for the skip option is originated by two factors. Firstly, the storage register for encoder events is of finite length. The skip option is a selection method for the storage of the encoder events captured at the high resolution clock in the data-acquisition hardware and enables a more flexible use of the available register space. Secondly, the velocity and acceleration estimation using time stamping are distorted by the presence of encoder imperfections. For this use, the skip option can perform a spatial low-pass filter on the encoder events to reduce the effect of the encoder imperfections. Since the most recent event is always included in the register, the skip option effectively does not influence the resolution or the quadrature of the encoder. It only influences the spatial data history in the register used for the polynomial interpolation.

The proposed method is an indirect measurement approach which is fully data-based, so no model of the system is required. Note that the required model for estimation of the position, velocity and acceleration is different from the model required for feedback controller design, e.g. the model required for estimation purposes should include friction, which is difficult to model accurately.

Experiments on a motion system show the applicability of the proposed method to obtain more accurate velocity and acceleration signals. Since the skip option and the capturing of the encoder events are implemented in hardware, the estimation of the signals is of the fixed-time kind. Therefore, the proposed method is applicable in real-time closed-loop experiments.

This paper is organized as follows. The time stamping concept is explained in more detail in Section 2. The position reconstruction method is briefly addressed in Section 3. In Section 4, the principle of skip in time stamping is introduced and its effects on the position reconstruction are described. The influence of skip is shown in Section 5 by means of experiments on a motion system. Finally, conclusions are drawn in Section 6.

## 2. Time stamping concept

In most motion control applications the position is measured by reading out the quadrature encoder counter value at the fixed sampling rate  $T_c$  of the controller. This introduces even for ideal encoders a quantization error in the position measurement of maximally half an encoder count. The quantized signal contains the encoder counter value at the sample times of the controller  $t_c$ , as can be seen in Fig. 1.

A possibility for increasing the accuracy of the position information with the same resolution encoders is using the concept of time stamping [3]. The time stamping concept stores the time instants  $t_e$  of a number of encoder transitions together with their position  $x_e$ . The pair  $(t_e, x_e)$  is called an encoder event. The encoder events are captured by a high resolution clock with a sampling-period  $T_e \ll T_c$ .

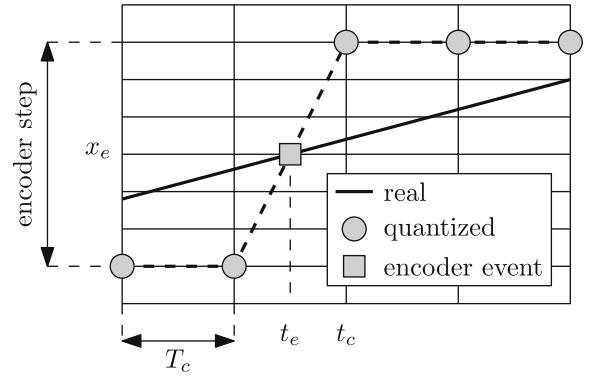


Fig. 1. Time stamping concept; the encoder transition  $x_e$  and time  $t_e$  are captured and stored as an event.

## 3. Position reconstruction

The use of encoder events for feedback control is not trivial since the encoder events are obtained at a variable rate proportional to the instantaneous velocity of the system during the measurement. To obtain a position estimation at the equidistant sampling times of the controller, a polynomial is fitted through a number of past encoder events. This polynomial is extrapolated to the desired time instant of the controller. Velocity and acceleration estimations are obtained by differentiation of the fitted polynomial with respect to time and extrapolation to the fixed sampling times of the controller.

For the position, velocity and acceleration estimations, a low order polynomial is fitted through a number of encoder events by the least squares method. Let  $n$  be the number of encoder events used in the fit,  $m$  the order of the fit, and  $k$  the index number of the events. Furthermore, let  $p_0, \dots, p_m$  be the polynomial coefficients to be estimated,  $t_1, \dots, t_n$  the time information of the encoder events, and  $x_1, \dots, x_n$  the position information of the encoder events. Now define the matrices  $A \in \mathbb{R}^{n \times m+1}$ ,  $P \in \mathbb{R}^{m+1}$ , and  $B \in \mathbb{R}^n$  as follows

$$A = \begin{bmatrix} t_{k-n+1}^m & t_{k-n+1}^{m-1} & \cdots & 1 \\ \vdots & \vdots & & \vdots \\ t_{k-1}^m & t_{k-1}^{m-1} & \cdots & 1 \\ t_k^m & t_k^{m-1} & \cdots & 1 \end{bmatrix}, \quad (1)$$

$$P = [p_m \quad p_{m-1} \quad \cdots \quad p_0]^T, \quad (2)$$

$$B = [x_{k-n+1} \quad \cdots \quad x_{k-1} \quad x_k]^T. \quad (3)$$

To prevent numerical problems with the higher order terms in (1), the time variable  $t$  can be redefined to be zero every controller sample time  $t_c$ , i.e.  $t := t - t_c$ .

If  $n = m$ , an exact fit is made through the events. For the least squares method  $n > m$ . The over-determined system of linear equations to be solved for the polynomial fit equals

$$AP = B.$$

The polynomial coefficients can be calculated using the least squares method as

$$P = (A^T A)^{-1} A^T B. \quad (4)$$

The polynomial coefficients  $P$  of (4) have to be calculated in real-time. For this purpose, LU-factorization without pivoting is used [8]. The inverse matrix will not be calculated in a recursive manner since the events contained in the register are likely to vary every sampling time of the controller and thus the information contained in the  $A$  matrix is different every time. Instead, (4) will be solved at each controller sample.

If the time instants between the events in the register are very small, the  $A$  matrix can become ill-conditioned. To avoid numerical problems, the time span  $\Delta t_K$  in-between the time instants of the first and last event is scaled to 1 for the polynomial fit. In this way, the time instants of the events are of order 1 and the  $A$  matrix is well conditioned.

As an example of the time scaling, let the true time instants of the last five encoder events equal  $t_K = [1 \ 2 \ 3 \ 4 \ 5] \times 10^{-5}$  s. The time range of the events equals  $\Delta t_K = 4 \times 10^{-5}$  s. The time instants are scaled as  $t_k^* = \frac{t_k}{\Delta t_K} = [0.25 \ 0.5 \ 0.75 \ 1 \ 1.25]$  s. The scaled time instants are of order 1 and the scaled time range equals exactly 1. For a second order fit with five events, i.e.  $m = 2$  and  $n = 5$ , the condition number of the  $A$  matrix (1) with the scaled time instants equals  $\text{cond}(A(t_k^*)) = 26.96$  whereas the condition number of the  $A$  matrix with the original time instants equals  $\text{cond}(A(t_k)) = 5.976 \times 10^9$ . The time scaling improves the conditioning properties of the  $A$  matrix for the estimation of the polynomial coefficients.

Since the position, the velocity and the acceleration estimations are required at the sampling times of the controller, the polynomial with the fitted coefficients  $P$  is extrapolated to the desired time instant  $t_c$ . The extrapolation of the polynomial to the time instant  $t_c$  results in an estimated position  $\hat{x}$ , estimated velocity  $\hat{\dot{x}}$ , and estimated acceleration  $\hat{\ddot{x}}$  as

$$\hat{x}(t)|_{t=t_c} = p_m(t_c \Delta t_K)^m + p_{m-1}(t_c \Delta t_K)^{m-1} + \dots + p_0,$$

$$\hat{\dot{x}}(t) = \dot{\hat{x}}(t),$$

$$\hat{\ddot{x}}(t) = \ddot{\hat{x}}(t).$$

The time scaling factor used for the estimation of the polynomial coefficients is accounted for in the extrapolation to obtain the correct position, velocity and acceleration estimations.

If the estimation exceeds the quantized position measurement by more than one count, the estimation is replaced by the quantized measurement. This results in an used estimation signal  $\hat{x}^*(t)$  as

$$\hat{x}^*(t) = \begin{cases} \hat{x}(t), & \text{if } |\hat{x}(t) - \bar{x}(t)| \leq 1, \\ \bar{x}(t), & \text{else,} \end{cases}$$

where  $\bar{x}(t)$  (counts) denotes the quantized position measurement. This can for example occur when no new events are detected over a longer time interval.

The easiest motion profile for the time stamping concept would be a constant velocity reference since this would result in an equally distributed series of events in time. Oscillating signals that change sign in the velocity are more difficult to be handled since these signals have a lower event rate at the turnaround points. In this paper we consider sinusoidal setpoint profiles since these have a constant changing event rate in time and also contain the turnaround points.

#### 4. Skip option

The encoder events  $(t_e, x_e)$  suffer from errors due to encoder imperfections, such as a non-uniform slit distribution, misplacement of the sensor photodiodes, eccentricity of the encoder disc, etc. The encoder imperfections introduce an error between the real and observed encoder event. The time instant  $t_e$  of the encoder event can have an error of maximum  $T_e$  due to the limited resolution  $T_e$  of the high resolution clock.

For real-time experiments,  $n$  events are used in the polynomial fit. The errors in the encoder events act as disturbances on the position information. These disturbances are amplified in the velocity estimation and even more in the acceleration estimation. A possible solution would be to increase the number of events. However,

in most hardware, the number of available events is limited. In this section, a skip option is proposed to extend the time span covered by the  $n$  events in the fit without the need for more events.

##### 4.1. Skip

The skip option makes it possible to skip a fixed number of events in between two stored events. In Fig. 2, the skip option is shown graphically for a skip number of  $\sigma = 2$  counts. The real signal and the quantized signal are shown by the solid and dashed line respectively. The arrows show the events to be discarded since the last stored event. The stored events are shown by the dark grey circles. The light grey circles are the discarded events. In between two dark circles always  $\sigma = 2$  events are skipped.

The skip option performs a low-pass filtering on the encoder events with a spatial cut-off frequency that is dependent on the momentary velocity.

For a skip factor of  $\sigma$  (counts), the index numbers of the events to be stored and used for the polynomial fit can be calculated using

$$k_\sigma(i) = 1 + i(\sigma + 1), i \in \mathbb{Z}^+, \quad (5)$$

where  $\mathbb{Z}^+$  is the set of nonnegative natural numbers including zero.

##### 4.2. Position reconstruction

The events to be used for the position reconstruction with skip are determined by (5). In most control applications, the controller is sampled at a fixed sample interval. With skip it can occur that the last event before a controller interrupt is discarded. However, the last encoder event before a controller interrupt is the most recent measurement. Therefore, the last encoder event is always stored in the set of adjusted events. If the most recent encoder event (with index  $k$ ) is to be stored and used for the polynomial fit, i.e.  $k = k_\sigma(N)$ ,  $N = \max(i)$ , then the last  $n$  events satisfying 5 are stored in the register. If the most recent event before a controller interrupt was to be skipped, i.e.  $k \neq k_\sigma$ , the event is nevertheless stored in the register together with the last  $n - 1$  events satisfying (5). This results in the set  $K \in \mathbb{R}^n$  with the index numbers of the events to be stored in the register when using a skip option of  $\sigma$  (counts) of

$$K = \begin{cases} [k \ k_\sigma(N) \ k_\sigma(N-1) \ \dots \ k_\sigma(N-n+2)]^T, & \text{if } k \neq k_\sigma(N), \\ [k_\sigma(N) \ k_\sigma(N-1) \ \dots \ k_\sigma(N-n+1)]^T, & \text{if } k = k_\sigma(N), \end{cases}$$

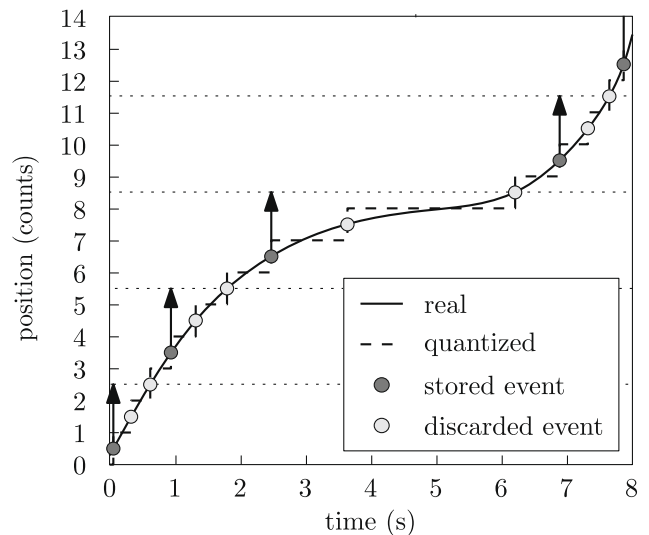


Fig. 2. Visualization of the skip option in time stamping for  $\sigma = 2$  counts.

where  $N = \max(i)$  is the last element of the vector with stored time stamps with skip. For the polynomial fit of (4), the matrices  $A$  and  $B$  equal with skip

$$A_\sigma = [t_K^m \quad t_K^{m-1} \quad \dots \quad 1], \quad B_\sigma = [x_K].$$

#### 4.3. Discussion

One might think that the obtained position information when using the skip option is equal to using a lower resolution encoder or to cancelation of the quadrature signal in the case that  $\sigma = 3$  counts.

However, the skip option only affects the information that is stored in the register and which occurred in history. Since the most recent event, i.e. the encoder event just before a controller interrupt, is always included in the register (see Section 4.2) the resolution or quadrature of the encoder is not affected by the skip option.

As an illustrative example, the position information for a signal when using the skip option and with a lower resolution encoder are compared in Fig. 3. To illustrate both the effects of a lower resolution encoder and cancelation of the quadrature, a skip option of  $\sigma = 3$  counts is chosen. The true signal is shown in Fig. 3 with the solid black line. The encoder events captured at a high resolution clock are shown by the dots. The light grey dots are skipped events and the dark grey events are stored in the register to be used for the polynomial fit. Note that for the fit at  $t = 20$  s, the last event before this time is also stored. The position information used for the polynomial fit with  $\sigma = 3$  counts is clearly different from the information of an encoder with a four times lower resolution as indicated by the black dashed line in Fig. 3. A fit through the stored events in Fig. 3 can yield a more accurate position estimation than the low resolution quantized position.

### 5. Experimental results

In this section, the results of the application of the time stamping concept to a motion system are discussed. Experiments are performed for different skip values  $\sigma$  and for sinusoidal reference profiles.

The experimental setup consists of the mechanical setup, an amplifier, a TUEdACs Microgiant data acquisition device [5] and a computer, as shown in the block diagram of Fig. 4.

The mechanical setup, shown in Fig. 5, consists of a DC motor, which is connected to a rotating mass. On the DC motor, a HEDS-5540 encoder [1] with a resolution of 100 slits/revolution ( $1.5707 \times 10^{-2}$  rad/slit) is mounted. On the opposite site, a

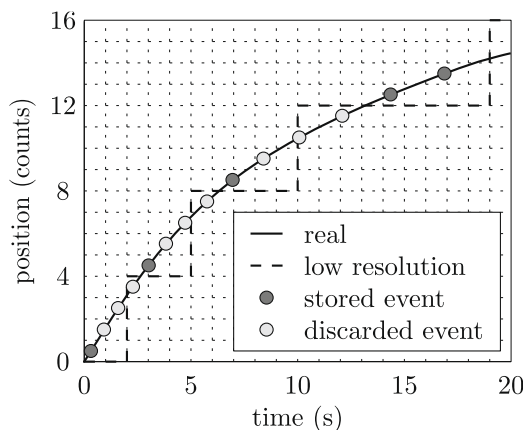


Fig. 3. Position information for using the skip option with  $\sigma = 3$  counts and a 4 times lower resolution encoder.

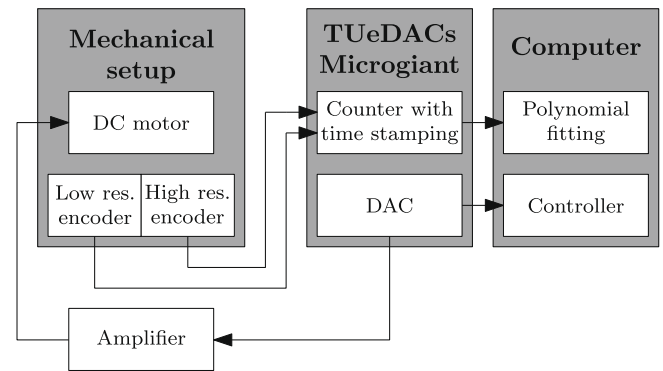


Fig. 4. Block diagram of the experimental setup.

Heidenhain ROD-426 encoder with 5000 slits/revolution ( $3.1416 \times 10^{-4}$  rad/slit) is connected to the rotating mass. The output of the Heidenhain encoder will be used as a reference to determine the improvement of the time stamping concept.

The data acquisition device, shown in Fig. 6, is equipped with 32 bit quadrature counter inputs with a clock frequency of 20 MHz and can thus capture encoder events with a time resolution of 50 ns [5]. Up to five encoder events can be stored in a register. The data of the register is transferred to the controller through USB at a fixed sampling rate of 1 kHz. Furthermore, the Microgiant is equipped with a DAC output, which is used to drive the mechanical setup. The selection and storage of the encoder events when using skip is also performed in the Microgiant.

The real-time application is hosted by a fully preemptive Linux kernel. The computer reads the time stamping registers of the Microgiant for the polynomial fitting and generates the control signal to the system in order to track a reference profile.

To compare the results for different skip values  $\sigma$ , the system must follow a known reference profile. Therefore, the system is feedback controlled. The time stamping concept is applied with a second order polynomial fit ( $m = 2$ ) though five encoder events ( $n = 5$ ). Experiments are performed without skip and with skip values  $\sigma \in \{1, 2, 3, 4, 5, 10, 20\}$ . The calculation time of the polynomial fit and the extrapolation is in the order of microseconds, which is much smaller than the controller sample time  $t_c = 1$  ms and can thus be performed in real-time.

In order to evaluate the estimation accuracy, reference signals are made by off-line anti-causal filtering of the high resolution position measurement  $x_{enc,hr}$  by a fifth order low-pass filter  $L(s)$  with cut-off frequency at 50 Hz as

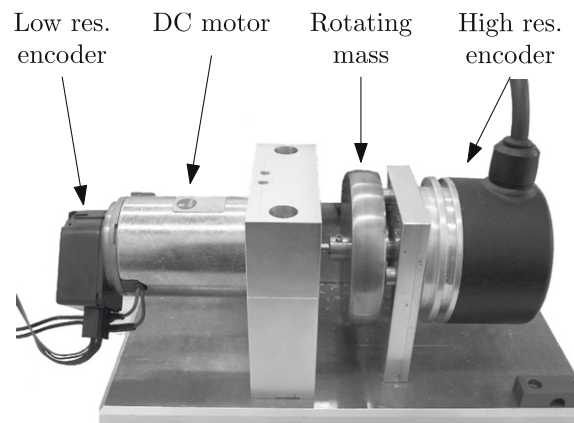


Fig. 5. The mechanical setup.

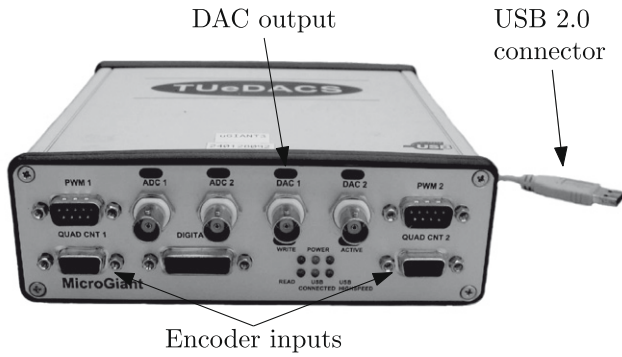


Fig. 6. The TUeDACs Microgiant data acquisition device.

$$X_{ref}(s) = L(s)X_{enc,hr}(s),$$

$$V_{ref}(s) = sX_{ref}(s),$$

$$A_{ref}(s) = sV_{ref}(s)$$

and

$$x_{ref} = \mathcal{L}^{-1}(X_{ref}(s)),$$

$$\dot{x}_{ref} = \mathcal{L}^{-1}(V_{ref}(s)),$$

$$\ddot{x}_{ref} = \mathcal{L}^{-1}(A_{ref}(s)),$$

where  $\mathcal{L}$  denotes the Laplace operator. The bandwidth of the low-pass filter  $L(s)$  is chosen sufficiently above the frequency of the reference profile and with a sufficiently high order to suppress the quantization effects present in the high resolution reference encoder. The estimation errors are defined as

$$e_x = x_{ref} - \hat{x}, \quad (6)$$

$$e_v = \dot{x}_{ref} - \hat{\dot{x}}, \quad (7)$$

$$e_a = \ddot{x}_{ref} - \hat{\ddot{x}}. \quad (8)$$

The skip option performs a time-independent spatial filtering on the encoder events. For constant velocity setpoints, the smallest errors are obtained for maximum skip values. This results in an observation window with the largest position history and thus maximally reduces the effect of the generally high frequent event errors. In this paper, sinusoidal setpoint profiles are used since these contain varying event rates in time and turnaround points.

Experiments are performed with sinusoidal reference signals  $r(t) = A \sin(2\pi ft)$ . The influence of varying amplitudes on the estimations is investigated. A changing frequency does not affect the optimal skip option since a changing frequency only affects the time properties of the signal and not the amplitude. The position estimations of time stamping with  $\sigma = 0$  and  $\sigma = 3$  are shown in Fig. 7 for  $f = 1$  Hz and  $A = \pi/2$  rad. For the sake of clarity, the position curves are offset from each other by 0.2 rad. The largest errors occur at the maxima of the reference signal, where the velocity equals zero and the time in between events is large.

The measured and the estimated velocities without skip and with  $\sigma = 3$  are depicted in Fig. 8. For clarity, the overlapping curves are offset from each other by 2 rad/s. The velocity obtained by differentiation of the quantized position (grey dotted) is clearly not useful for control purposes. With  $\sigma = 3$  counts, the estimated velocity (black) results in a lower error than with  $\sigma = 0$  counts (dark grey). The momentary oscillations caused by the event errors are filtered out by the skip option. The estimation with  $\sigma = 3$  is 54% more accurate than without skip ( $\sigma = 0$ ).

The measured and estimated acceleration signals are shown in Fig. 9, for clarity, the overlapping curves are offset from each other by 30 rad/s<sup>2</sup>. The quantized acceleration obtained by two times differentiation of the quantized position is completely useless.

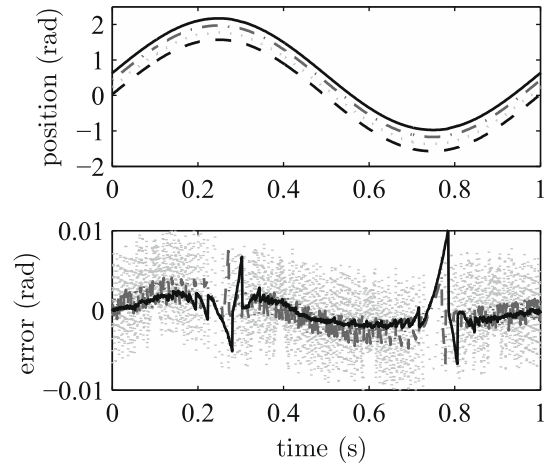


Fig. 7. Position signals, reference (dashed, black), quantized measurement (dotted, light grey, added offset = 0.2 rad), estimated with  $\sigma = 0$  counts (dash-dotted, dark grey, added offset = 0.4 rad) and with  $\sigma = 3$  counts (solid, black, added offset = 0.6 rad).

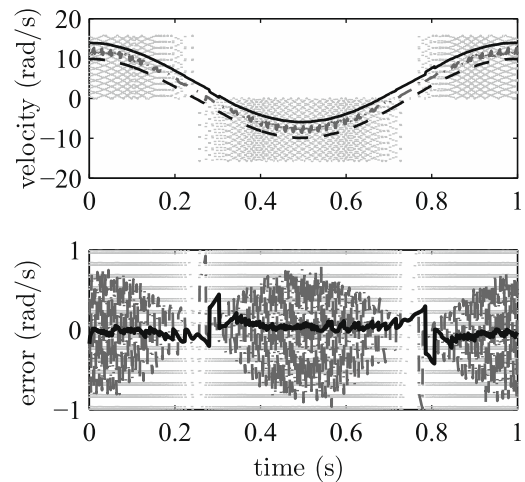


Fig. 8. Velocity signals, reference (dashed, black), quantized measurement (dotted, light grey), estimated with  $\sigma = 0$  counts (dash-dotted, dark grey, added offset = 2 rad/s) and with  $\sigma = 3$  counts (solid, black, added offset = 4 rad/s).

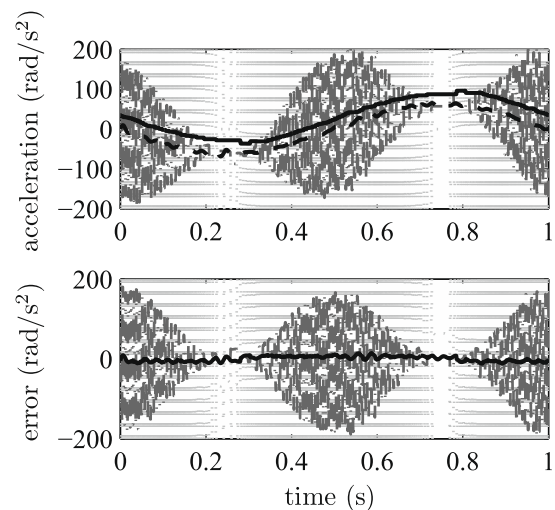
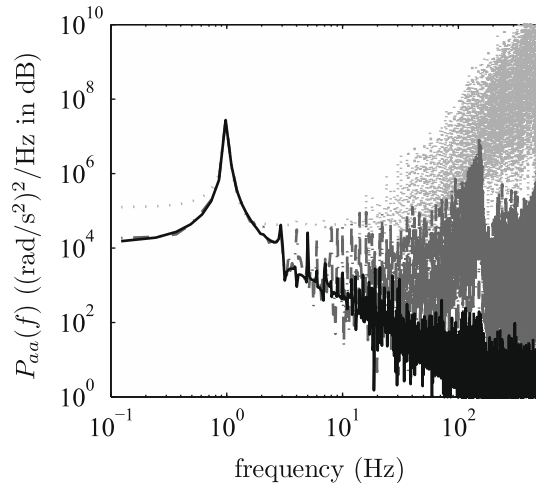
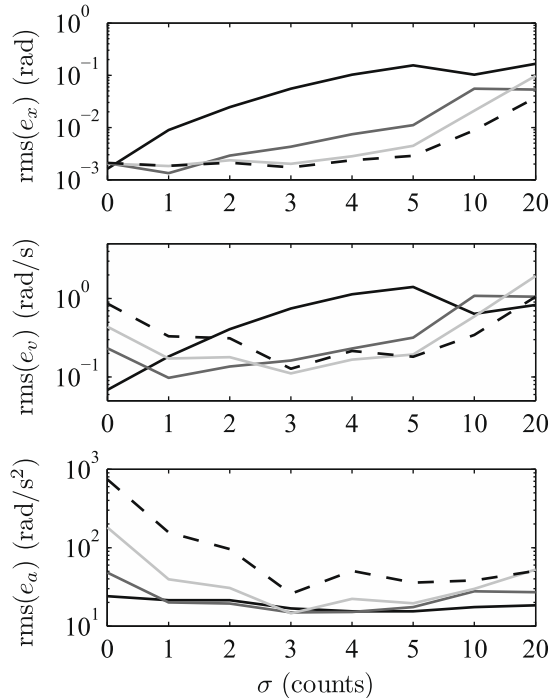


Fig. 9. Acceleration signals, reference (dashed, black), quantized measurement (dotted, light grey), estimated with  $\sigma = 0$  counts (dash-dotted, dark grey) and with  $\sigma = 3$  counts (solid, black, added offset = 30 rad/s<sup>2</sup>).



**Fig. 10.** PSDs of the measured and estimated accelerations, quantized (dotted, light grey), estimated with  $\sigma = 0$  counts (dash-dotted, dark grey) and with  $\sigma = 3$  counts (solid, black).



**Fig. 11.** Estimation errors with various skip values  $\sigma$  for sinusoidal reference with varying amplitude  $A$ ,  $\pi/16$  rad (black),  $\pi/4$  rad (dark grey),  $\pi/2$  rad (light grey) and  $\pi$  rad (dashed).

The acceleration obtained with time stamping and  $\sigma = 0$  counts shows large bursts and is not useful for control purposes. With a skip of  $\sigma = 3$  counts, an acceleration signal that is useful for control purposes is estimated. The acceleration estimation with  $\sigma = 3$  is 92% more accurate than with  $\sigma = 0$  counts.

The power spectral densities (PSDs) of the measured and estimated acceleration signals, depicted in Fig. 10, show the influence of time stamping and the skip option. The PSD of the quantized acceleration is for all frequencies located above the PSDs of the estimated accelerations. The PSD of the estimation with  $\sigma = 3$  counts clearly shows the reduction of the high frequencies in the estimation.

The estimation errors for a frequency  $f = 1$  Hz and a varying amplitude  $A \in \{\pi/16, \pi/4, \pi/2, \pi\}$  are shown in Fig. 11. For

varying amplitude, the optimal skip value varies. A change in the amplitude changes the amount of encoder counts over one period. Since the skip values perform a spatial low-pass filtering based on the encoder counts, a change in the amplitude changes the cut-off frequency for constant skip value. The optimal skip setting, i.e. the skip setting to obtain the smallest estimation error, is dependent on the amplitude. For increasing amplitude the optimal skip value also increases.

In case of a skip of  $\sigma = 3$  counts, every fourth subsequent count is stored. For a movement in one direction this corresponds to storing the same event of the four quadrature signals of one encoder slit. As an additional advantage, a skip value of  $\sigma = 3$  counts therefore eliminates all event errors that are caused by phase errors between the quadrature signals. The phase errors have the largest contributions at twice the period time of the TTL signals. For sinusoidal setpoints, the velocity changes in time and with this the period time of the TTL signals. The amount of the phase error reduction by the time stamping concept with  $\sigma = 3$  counts cannot be quantified easily in either the time or the frequency domain due to the change in period time and the existence of other encoder errors [11].

## 6. Conclusions

The position measurements of optical incremental encoders suffer from quantization errors. Velocity and acceleration estimations obtained by numerical differentiation of the quantized position measurements show large spikes. The time stamping concept uses encoder events, consisting of the counter value and the corresponding time instant. Through the stored events a polynomial is fitted and extrapolated to the desired time instant. Differentiation of the fitted polynomial and extrapolation lead to velocity and acceleration estimations that are applicable for control purposes.

As more and more general-purpose embedded processors have time stamping capabilities, the opportunities for real-time implementation are highly feasible. This paper shows that the feature of encoder time stamping is really useful in control system design.

Encoder imperfections and the limited resolution with which the encoder events are captured lead to errors on the encoder events. These errors result in oscillations in the velocity and acceleration estimations. Increasing the time span covered by the stored encoder events reduces the oscillations in the estimations. However, the amount of events to be stored is limited. Therefore, a skip option is proposed to discard a fixed number of events in between stored events. The skip option increases the covered time span without the need for additional events. The skip option performs a spatial low-pass filtering on the encoder counts.

Experiments show the improvement of the velocity and acceleration estimations with a skip of three events in comparison to differentiation of the quantized position measurement and in comparison to the time stamping concept without skip. Compared to time stamping without skip, the velocity estimation is improved by 54% and the acceleration estimation by 92%.

The optimal skip value is independent on the velocity of rotation or the frequency of oscillation. However, it is dependent on the amplitude of oscillation since this changes the amount of events per period.

Future research includes the derivation of explicit conditions for the optimal number of events, the optimal fit order and the optimal skip value for various operating conditions.

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