Robust control to suppress clutch judder

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Judder is a well-known phenomenon occurring in dry clutches. Especially during drive-off, clutch judder induces undesired driveline oscillations. This paper presents the design and implementation of a robust control concept to actively damp judder in a clutch system. A DK-iteration procedure, combining \( H_\infty \) controller synthesis and \( \mu \)-analysis is adopted for robust controller design. Simulations are performed for validation of the control concept.

1. INTRODUCTION

Focus of this research lies on the powertrain of Heavy Duty Vehicles (HDVs) incorporating an Automated Manual Transmission (AMT). An AMT typically incorporates a gearbox in combination with a dry plate or lock-up clutch system. The clutch transmits torque from the engine to the rest of the driveline (see Fig. 1). The driveline in combination with the engine will be referred to as the powertrain.

![Fig. 1. Schematic representation of a typical HDV powertrain.](image)

Oscillations in the driveline specifically originating from the clutch, are referred to as clutch (engagement) judder [⁴]. Judder is a friction-induced vibration between masses with sliding contact and is a well-known phenomenon in dry clutches [⁶], [¹²]. The resulting oscillations inherit the first resonance frequency of the driveline. These oscillations introduce undesired dynamic loads, increase slip and wear effects in the clutch and reduce driver comfort [⁴].

Generally speaking, two main causes of clutch judder can be distinguished [¹²]: Firstly, variation in the friction characteristics of the clutch facings material and secondly, mechanical tolerances and misalignment in the driveline. The effect of judder may be solved by i) changing vehicle driveline properties by means of mechanical adjustments, e.g. increasing damping and stiffness of the shafts or improving the characteristics of the clutch facings material, or ii) application of control to actively damp the oscillations in the driveline. This research focuses on the design of a controller.

Much research regarding active damping of driveline oscillations focuses on control of the engine output, as the engine can be regarded as an easy-to-use actuator (e.g. [³]). Judder however, occurs in the slipping clutch phase, when the engine is decoupled from the driveline. Hence, this research focuses on the design of a controller utilizing the clutch actuator. This actuator provides the clamping force to close the clutch. For example [¹²], [¹¹] propose such control algorithms to counteract the judder phenomenon. These algorithms, however, are based on a learning feedforward filter. As soon as the judder phenomenon is detected, the filter is initialized and the oscillations are opposed by the resulting feedforward signal. Although some successful results are reported, the controller is unable to stabilize the driveline in all working conditions. This can be attributed to the lack of feedback control, which results in a non-robust system.

The contribution of this paper comprises the design of a robustly stable feedback controller to actively damp judder-induced driveline oscillations [¹]. The control concept is tested in simulations.
2. MODELING

2.1 Modeling of the clutch system

The clutch consists of several plates, which are clamped together. The plate facings are covered with a material with a high friction coefficient $\mu$. The clamping force $F_n$ is provided by the clutch actuator. Friction induced judder models typically comprise a combination of static $\mu_{st}$ and viscous $\mu_{kin}$ friction, \cite{12}.

$$\mu = \text{sign}(\omega_{sl}) \mu_{st} + \mu_{kin} \omega_{sl}$$  \hspace{1cm} (1)

where $\omega_{sl} = \omega_c - \omega_g$ (see Fig. 2). For simplicity, assume focusing on driving-off on a flat road, starting from standstill. This implies $\omega_c > \omega_g$ and hence

$$\mu = \mu_{st} + \mu_{kin} \omega_{sl}$$  \hspace{1cm} (2)

Assuming constant pressure across the surface of the clutch plates, the torque transmitted by the clutch is given by

$$T_{cl} = F_n R_m n_{cl} \mu$$  \hspace{1cm} (3)

with constants $R_m$ and $n_{cl}$ the mean clutch radius and the number of clutch plates respectively, as defined in (2) and $F_n \geq 0$ the clamping force.

Combining (2) and (3) shows that $F_n R_m n_{cl} \mu_{kin}$ can be regarded as a damping term. The values of $\mu_{st}$ and $\mu_{kin}$ may vary as a function of temperature, wear, moist, etc. \cite{12}. For $\mu_{kin} < 0$, this may thus induce instabilities which will be referred to as clutch judder.

Fig. 2. Mass-spring-damper model of the HDV powertrain. All inertias and rotations are lumped, taking into account the gearbox and differential ratios. The engine torque $T_e$ and the force $F_n$, with which the clutch plates are clamped together are considered as inputs. Indicated are the engine, gearbox and vehicle inertias, $J_e$, $J_g$, $J_v$ (the latter one includes all external loads) and their corresponding rotational speeds $\omega_e$, $\omega_g$ and $\omega_v$ respectively, the damping and stiffness of the drive shafts, $d_e$ and $k_e$ respectively, the engine damping $d_e$ and the friction coefficient of the clutch plates facings $\mu$. $T_{cl}$ indicates the torque transmitted by the clutch.

2.2 Modeling of the powertrain

In Fig. 2 a mass-spring-damper model of the powertrain including the main variables and parameters is shown. Correspondingly, a nonlinear, MIMO model description $\Sigma_{nl}$ is derived.

$$\Sigma_{nl} : \begin{cases} J_e \dot{\omega}_e &= T_e - d_e \dot{\omega}_e - T_{cl} \\ J_g \dot{\omega}_g &= T_{cl} + k_e \dot{\theta} + d_e \dot{\theta} \\ J_v \dot{\omega}_v &= -k_e \dot{\theta} - d_e \dot{\theta} \\ \dot{\theta} &= \omega_v - \omega_g \end{cases}$$  \hspace{1cm} (4)

where $\dot{\theta}$ represents the winding of the drive shafts, i.e. $\dot{\theta} = \omega_v - \omega_g$. Defining $\mu_{st}^* = R_m n_{cl} \mu_{st}$, $\mu_{kin}^* = R_m n_{cl} \mu_{kin}$ and combining (3) and (4) yields

$$\begin{cases} J_e \dot{\omega}_e &= T_e - d_e \dot{\omega}_e - F_n \mu_{st}^* + \ldots \\ & \quad \ldots + F_n \mu_{kin}^* (\omega_c - \omega_g) \\ J_g \dot{\omega}_g &= -d_e \dot{\omega}_g + k_e \dot{\theta} + d_e \dot{\theta} + \ldots \\ & \quad \ldots + F_n (\mu_{st}^* (\omega_c - \omega_g) + \mu_{kin}^*) \\ J_v \dot{\omega}_v &= -k_e \dot{\theta} - d_e \dot{\theta} (\omega_v - \omega_g) \\ \dot{\theta} &= \omega_v - \omega_g \end{cases}$$  \hspace{1cm} (5)

The goal is to damp oscillations in the driveline while driving-off. The gearbox rotational speed $\omega_g$ is directly linked to the clutch and provides a measurement in which the driveline oscillations due to clutch judder are well observable. As discussed in the introduction, $F_n$ will be used to control $\omega_g$. To assure the engine keeps running while driving-off, the engine output $T_e$ will be used to control the engine rotational speed $\omega_e$.

Define the state $x = (\omega_c, \omega_g, \omega_v, \dot{\theta})^T$, the input vector $u = (T_e, F_n)^T$ and the output vector $y = (\omega_c, \omega_g)^T$.

Consider linearisation of (5) around nominal working conditions $x = \bar{x}$, $u = \bar{u}$ and $y = \bar{y}$. Focusing on (small) perturbations around the nominal working conditions $\bar{x} = \delta x$, $\bar{u} = \delta u$ and $\bar{y} = \delta y$, this yields the linear model

$$\Sigma_{l} : \begin{cases} \dot{\bar{x}} &= A(\bar{x}) \bar{x} + B(\bar{x}_2) \bar{u} \\ \bar{y} &= C \bar{x} + D \bar{u} \end{cases}$$  \hspace{1cm} (6)

where

$$A(\xi_1) = \begin{bmatrix} -d_e - \xi_1 & \xi_1 & 0 & 0 \\ 0 & -d_e - \xi_1 & d_e & k_e \\ 0 & -1 & 1 & 0 \end{bmatrix}$$

$$B(\xi_2) = \begin{bmatrix} \frac{1}{T_e} & -\xi_2 - \mu_{st}^* \xi_1 \\ 0 & \frac{\xi_2 - \mu_{st}^* \xi_1}{T_e} \\ 0 & 0 \end{bmatrix}$$

$$C = \begin{bmatrix} \bar{E}^{2 \times 2}, \bar{G}^{2 \times 2} \end{bmatrix}, \quad D = \begin{bmatrix} [0^{2 \times 2}] \end{bmatrix}$$

and $\xi_1 = F_n \mu_{kin}^*$, $\xi_2 = \omega_{sl} \mu_{kin}^*$ with $F_n$ and $\omega_{sl}$ nominal values of $F_n$ and $\omega_{sl}$ respectively.

2.3 Sensor, actuator and communication dynamics

Compared to practice, the theoretical models $\Sigma_{nl}$ and $\Sigma_{l}$ lack actuator, sensor and communication dynamics. For the sake of simplicity, these additional dynamics are not taken into account in the calculations. The results as well as the final robust controller design however, do incorporate these dynamics. See e.g. Fig. 3 in which an additional time-delay is clearly visible.

Following Fig. 3 and utilizing measurement results, limited bandwidth of the actuators as well as limited resolution of the sensors is accounted for via corresponding low-pass filters. Furthermore, a time-delay is modeled using a Padé approximation.
2.4 Model validation

For a powertrain with closed clutch, $\omega_c = \omega_g$ holds, reducing the order of the model (4). The resulting model $\Sigma_p : T_e \mapsto \omega_c$ is linear.

$$
\begin{align*}
\Sigma_p : & \begin{cases}
(J_e + J_g)\dot{\omega}_c = T_e - d_e\omega_c + k_c\theta + d_s\dot{\theta} \\
J_g\dot{\omega}_g = -k_g\theta - d_s\dot{\theta} \\
\dot{\theta} = \omega_c - \omega_g
\end{cases}
\end{align*}
$$

(8)

The model described by (8) is compared to experimental measurement results, utilizing frequency response measurement techniques [7]. On the basis of these measurements, the actual parameter values of $\Sigma_p$ are estimated by fitting the model onto the measurement results, see Fig. 3. These values are then used in the models $\Sigma_{nl}$ and $\Sigma_{l}$, incorporating the powertrain including a slipping clutch.

In Fig. 4 the response of $\omega_g$ to a step in the clamping force $F_n$, for constant $T_e$ is compared to measurement results. The result indicates that the eigenfrequency of the driveline as well as the corresponding damping are modeled appropriately.

![Fig. 4. Step response of the model $\Sigma_{nl} : F_n \mapsto \omega_g$ for constant $T_e$ (dashed grey), compared to measurement results (solid black).](image)

In Fig. 5, the resulting model $\Sigma_{nl} : T_e \mapsto \omega_c$ is compared to measurement results (solid black). In the upper plot, the magnitude of the off-diagonal terms is shown. In the lower plot, the magnitude of the diagonal terms is shown. In the lower plot, the magnitude of the diagonal terms is shown. In the upper plot, the magnitude of the off-diagonal terms is shown.

![Fig. 5. Bode plot of the linearized model $\Sigma_{nl} \mapsto \omega_g$ for $\mu_{kin} > 0$ (black) and $\mu_{kin} < 0$ (grey).](image)

Fig. 5. Bode plot of the linearized model $\Sigma_{nl} \mapsto \omega_g$ for $\mu_{kin} > 0$ (black) and $\mu_{kin} < 0$ (grey).

A Relative Gain Array (RGA) analysis provides a measure for the interaction between the inputs and outputs in the model (9). In case the resulting RGA is close to identity, crosswise interactions are relatively small. In Fig. 6, the magnitudes of the diagonal and the off-diagonal RGA terms of $\Sigma_1$ (6) are shown. In the low as well as in the high frequency range, the RGA is close to identity, whereas for frequencies of the order $10^{-1}$, the RGA is worse. The desired closed-loop bandwidth of the system is in the order of $10^0$, at which the RGA is close to identity. The RGA analysis thus shows that the MIMO model (6) may be regarded as decoupled.

![Fig. 6. Result of the RGA analysis of $\Sigma_1$ (6). In the upper plot, the magnitude of the diagonal terms is shown. In the lower plot, the magnitude of the off-diagonal terms is shown.](image)

Fig. 6. Result of the RGA analysis of $\Sigma_1$ (6). In the upper plot, the magnitude of the diagonal terms is shown. In the lower plot, the magnitude of the off-diagonal terms is shown.

2.5 Model characteristics

Regarding the linear model $\Sigma_1$, (6), (7), the terms in the upper left $3 \times 3$ block of $A(\xi_1)$ can be regarded as damping terms. Assume for the moment $d_e = d_s = 0$. The sign and size of these damping terms then depend on $\xi_1$, i.e. on $\mu_{kin} = R_m n_{cl} \mu_{kin}$ and $\overline{F_n}$. As mentioned before, $\mu_{kin}$ may vary as a result of wear, moist or temperature. With $R_m > 0$ and $n_{cl} > 0$, this may yield a negatively valued $\mu_{kin}$. As $\overline{F_n} \geq 0$ holds, this may result in negatively valued damping terms. In Fig. 5, the frequency response functions of (6) for $T_e \mapsto \omega_g$ for different values of $\mu_{kin}$ are shown. For $\mu_{kin} < 0$, three unstable poles are present, whereas the system is stable for $\mu_{kin} > 0$. Hence, $\mu_{kin} < 0$ will lead to instability, which explains the term 'negatively damped' or 'self-induced' judder [4], [2], [13]. The size of the instability depends on the size of $\mu_{kin}$ and $\overline{F_n}$.

3. CONTROLLER SYNTHESIS

Utilizing sequential loop-closing techniques, the controller design is divided into two parts. The first part deals with stabilization of the driveline. The second part assures control of the engine rotational speed. Focusing on the former one, a perturbed plant and an uncertainty matrix are derived [10]. Correspondingly, a controller is...
designed using a DK-iteration scheme. This approach enables robust stability and performance analysis.

3.1 Sequential loop-closing

Based on the RGA analysis (Sect. 2.5), a decentralized diagonal controller \( K(s) = \text{diag}(K_c(s), K_g(s)) \) instead of a full MIMO controller is chosen. Sequential Loop-closing (SQL) techniques are adopted for the design [8]. In this way, off-diagonal interaction terms are accounted for.

Transforming the state-space representation \( \Sigma_t \) to a transfer function \( H(s) \) yields

\[
\begin{bmatrix}
\omega_c \\
\omega_g
\end{bmatrix} = \begin{bmatrix}
H_{11} & H_{12} \\
H_{21} & H_{22}
\end{bmatrix}
\begin{bmatrix}
T_e \\
F_n
\end{bmatrix}
\]  
\tag{9}

where \( H(s) \) is partitioned into 4 blocks \( H_{ij}(s), i, j \in \{1, 2\} \). In Fig. 7 a schematic representation of \( H(s) \) in combination with the diagonal controller \( K(s) \) is shown.

The Hankel Singular Value (HSV) ratio, i.e. the ratio of the largest and the smallest HSV, is an indicator for the observability and controllability of a system. For the system may be optimized by the design of the rest of the driveline when the clutch is slipping, this appropriately. As the engine is (partly) decoupled from the engine will not be able to attenuate driveline oscillations already designed \( K \), as account, as in this step of the controller design, as \( K_c(s) \) on the other hand, the already designed \( K_g(s) \) has to be accounted for as well. This yields an equivalent model \( H_{11}^*(s) \), which follows directly from the scheme in Fig. 7

\[
H_{11}^* = H_{11} - \frac{H_{12}K_gH_{21}}{1 + H_{22}K_g}
\]  
\tag{10}

As stabilization of the powertrain is the main topic of this paper, the remainder of the controller synthesis focuses on the design of \( K_g(s) \). The subsequent design of \( K_c(s) \) is relatively easy, as \( H_{11}^*(s) \) represents a stable system. This design will not be discussed further in this paper.

Hence, consider the model \( \Sigma_g \) with state vector \( x_g = \bar{x}_g \), input \( u_g = \bar{F}_n \) and output \( y_g = \bar{\omega}_g \) (see (6))

\[
\Sigma_g : \begin{cases}
\dot{x}_g = A_g(\xi_1)x_g + B_g(\xi_2)u_g \\
y_g = C_gx_g + D_gu_g
\end{cases}
\]  
\tag{11}

where

\[
A_g(\xi_1) = A(\xi_1), \\
B_g(\xi_2) = \begin{bmatrix}
-\frac{\zeta_2-s\omega_2}{\omega_2} & \frac{s\omega_2}{\omega_2}
\end{bmatrix}
\]  
\tag{12}

\[
C_g = [0, 1, 0, 0], \\
D_g = 0
\]

The corresponding transfer function of \( \Sigma_g \) is given by

\[
H_g(s) = H_{22}(s).
\]

3.2 Uncertainty modeling

Focusing on the design of a controller \( K_g(s) \) for the model \( \Sigma_g (11) \), robust stability with respect to the real parametric uncertainties \( \xi_1 \) and \( \xi_2 \) is required. Define upper and lower bounds \( \xi_i^- \leq \xi_i \leq \xi_i^+ \), nominal values \( \xi_i = \xi_i^\pm / 2 \), and scaling factors \( s_i = \frac{1}{2}(\xi_i^+ - \xi_i^-) \), \( i \in \{1, 2\} \). Given the normalized real-valued perturbations \( \delta_i \in [-1, 1], i \in \{1, 2\} \), the parametric uncertainties are thus decomposed into a nominal part and an uncertain part

\[
\xi_1 = \xi_{1,n} + \delta_1s_1, \\
\xi_2 = \xi_{2,n} + \delta_2s_2
\]  
\tag{13}

Substituting this into the state-space model \( \Sigma_g (11) \), yields the perturbed state-space model \( \Sigma_{g,p} \)

\[
\Sigma_{g,p} : \begin{cases}
\dot{x}_g = (A_n + \delta_1A_\delta)x_g + (B_n + \delta_2B_\delta)u_g \\
y_g = Cgx_g
\end{cases}
\]  
\tag{14}

where \( A_n = A_g(\xi_{1,n}), \delta_1A_\delta = A_g(\delta_1s_1) - A_g(\xi_{1,n}) \) and \( B_n = B_g(\xi_{2,n}), \delta_2B_\delta = B_g(\delta_2s_2) \).

3.3 Linear Fractional Transform

To enable robust control, a Linear Fractional Transformation (LFT) of the model \( \Sigma_{g,p} (14) \) is desirable (see Fig. 8). Define the uncertainty matrix \( \Delta_g = \text{diag}(\delta_1, \delta_2) \) and the corresponding input and output vectors \( y_\Delta = (y_{\Delta 1}, y_{\Delta 2})^T \) and \( u_\Delta = (u_{\Delta 1}, u_{\Delta 2})^T = \Delta_\Delta y_\Delta \) respectively.

Following [10], the corresponding augmented model \( \Sigma_{g,a} \), with input and output vectors \( u_{\Delta} = (u_{\Delta 1}, u_{\Delta 2})^T \) and \( y_{\Delta} = (y_\Delta, y_\Delta)^T \) respectively, is defined as

\[
\Sigma_{g,a} : \begin{cases}
\dot{x}_g = \begin{bmatrix}
A_n & B_n \\
C_g & 0
\end{bmatrix} \begin{bmatrix}
x_g \\
u_g
\end{bmatrix} + B_\Delta u_\Delta \\
y_\Delta = C_\Delta \begin{bmatrix}
x_g \\
u_g
\end{bmatrix}
\end{cases}
\]  
\tag{15}

Introducing the scalings

\[
\delta_1A_\delta = B_\delta, \delta_1C_\delta \\
\delta_2B_\delta = B_\delta, \delta_2C_\delta
\]  
\tag{16}

the matrices \( B_\Delta \) and \( C_\Delta \) follow from substituting \( u_\Delta = \Delta_\Delta y_\Delta \) in (15), and combining the result with the original perturbed model (14).
This yields
\[ \Sigma_{g,a} : \begin{cases} \dot{x}_g &= A_g x_g + B_g u_a \\ y_a &= C_g x_g + D_g u_a \end{cases} \] (17)
where
\[ B_g = \begin{bmatrix} B_{\delta 1}, B_{\delta 2}, B_n \end{bmatrix} \]
\[ C_g = \begin{bmatrix} C_{\delta 1}, 0_{1\times4}, C_g \end{bmatrix}^T \]
\[ D_g = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & C_{\delta 2} \\ 0 & 0 & 0 \end{bmatrix} \] (18)
Transforming the state-space representation \( \Sigma_{g,a} \) to a transfer function \( H_{g,a}(s) \) (see Fig. 8), yields
\[ H_{g,a} = C_g (sI - A_n)^{-1} B_g + D_g \] (19)
\[ = \begin{bmatrix} H_{g,a,11} & H_{g,a,12} \\ H_{g,a,21} & H_{g,a,22} \end{bmatrix} \]
3.4 Performance demands
Performance demands are incorporated in the controller design via a second-order low-pass output weighting filter \( W_p(s) \). The closed-loop sensitivity \( S(s) \) represents the transfer from a reference signal \( r \) to the error signal \( y_e = r - y_g \) and is an indicator of closed-loop performance. Hence, the performance requirement is defined as
\[ |S(s)| < \frac{1}{|W_p(s)|} \] (20)
Incorporating this in the LFT representation of the system, yields the model \( H_{g,a}(s) \) with inputs \( u = [u_\Delta, r, u_g]^T \) and outputs \( y = [y_\Delta, z, y_K]^T \), where \( z = W_p(s)y_K \) (see Fig. 8).
\[ H_{g,a}^* = \begin{bmatrix} H_{g,a,11} & 0 & H_{g,a,12} \\ -W_p H_{g,a,21} & W_p & -W_p H_{g,a,22} \\ 0 & 0 & -H_{g,a,22} \end{bmatrix} \] (21)
3.5 Robust performance and stability analysis
The matrix \( \Delta_\delta \) represents a structured uncertainty incorporating real perturbations only. Define \( D \) the set of block-diagonal matrices whose structure is compatible to the structure of \( \Delta_\delta \). Utilizing scalings \( D \in D \), a \( \mu \)-analysis provides the least conservative robust performance and stability conditions [9]. The scalings reduce the conservativeness of the conditions by exploiting the fact that \( \Delta_\delta \) incorporates real perturbations only.

Given a robust controller \( K_g(s) \), define the lower LFT of \( H_{g,a}^* \) and \( K_g \) by
\[ N = Fl(H_{g,a}^*, K_g) \]
\[ = H_{g,a,11} + H_{g,a,12} K_g (I - H_{g,a,22} K_g)^{-1} H_{g,a,21} \]
If nominal stability of the model \( N(K_g) \) is guaranteed, robust performance is achieved for
\[ \mu_p(N(j\omega, K_g(j\omega))) < 1, \quad \forall \omega \] (23)
where
\[ \mu_p(N(K_g)) \leq \min_{D_p \in D} \sigma(D_p N(K_g) D_p^{-1}) \] (24)
defines the performance of the system via the upper bound on the scaled singular value of \( N(K_g) \).

Robust stability is achieved if nominal stability is guaranteed, and
\[ \mu_s(M(j\omega)) < 1, \quad \forall \omega \] (25)
where \( M(j\omega) = N_{11} \) the transfer functions from the output to the input of the perturbation matrix \( \Delta_\delta \), and \( \mu_s \) is defined by
\[ \mu_s(M) \leq \inf_{D_p \in D} \sigma(D_p M D_p^{-1}) \] (26)
a generalization of the upper bound on the scaled structured uncertainty \( \Delta_\delta \), with scalings \( D_s \in D \).
3.6 DK-iteration
A robust controller is designed utilizing a DK-iteration scheme, which combines \( H_\infty \)-synthesis and the \( \mu \)-analysis discussed in the previous section. The DK-iteration scheme is given by [9]

1) For a given (initial) controller \( K_g(s) \), compute the scaling \( D_p(j\omega) \in D \) minimizing \( \mu_p(N(j\omega, K_g(j\omega))) \) for all \( \omega \).
2) Fit the magnitude of each element of \( D_p(j\omega) \) to a stable, minimum-phase transfer function \( D_p^*(s) \).
3) For fixed \( D_p^*(s) \), synthesize a \( H_\infty \)-controller \( K_g(s) \) for the scaled problem
\[ \min_{K_g} \left( \|D_p^* N(K_g) D_p^{-1}\|_\infty \right) \]
4. RESULTS
As mentioned in Section 2.3 the additional actuator, sensor and communication dynamics are taken into account in the final controller synthesis and evaluation discussed in this section.

In Fig. 9 the values of \( \mu_p(j\omega) \) and \( \mu_s(j\omega) \) are shown. As nominal stability of \( N \) is guaranteed and \( \mu_s(j\omega) < 0 \) dB \( \forall \omega \) holds, robust stability of the resulting closed-loop system is achieved. The desired performance however, may not be met, as \( \mu_p(j\omega) > 0 \) dB.

The resulting sensitivity \( |S(s)| \) for varying \( \xi_1 \) and \( \xi_2 \) is shown in Fig. 10. In general, the performance demand (20) is achieved sufficiently. In worst case situations however, the desired performance will not be achieved.
5. CONCLUSIONS AND FUTURE WORK

A model of the powertrain, incorporating an uncertainty model for unmodeled friction dynamics, is validated using experiments. Based on this model, a robustly stable feedback controller is designed. Simulations show the appropriate functioning of the controller in suppressing the judder-induced driveline oscillations. Future work includes further research to improve the closed-loop performance as well as implementation on a real vehicle.

REFERENCES


NOMENCLATURE

Roman letters

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<th>Symbol</th>
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<td>$\delta/\Delta$</td>
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Greek letters

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Subscripts

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<td>$s$</td>
<td>drive shafts</td>
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<td>$e$</td>
<td>engine</td>
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<td>$g$</td>
<td>gearbox</td>
</tr>
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<td>$v$</td>
<td>vehicle</td>
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<tr>
<td>$(n)l$</td>
<td>(non)linear</td>
</tr>
<tr>
<td>$\Delta$</td>
<td>scaling</td>
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