Active vibration isolator design for ambulance patients

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Master's thesis

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Preface

This report describes the results of my graduation project at the Dynamics and Control group of the Eindhoven University of Technology. The project is carried out for Janssen Precision Engineering B.V., located at Maastricht-Airport. During my research I received a lot of help and support by many people, to whom I would like express my gratitude.

First of all I would like to thank prof. dr. Henk Nijmeijer for his supervision, input and critical comments. I would like to thank ir. Huub Janssen for providing me with an interesting graduation project, a workplace and the support and input for this project. Secondly, I would like to thank my coaches; dr. ir. Roell van Druten, dr. ir. Igo Besselink and ir. Bart van Bree for all their input, support, criticism, supervision and all the interesting discussions we had.

Furthermore, I would like to thank all my colleagues at Janssen Precision Engineering for their support, the discussions we had but most of all for the pleasant atmosphere during my stay.

Finally, I would like to thank all those in my personal environment for their support during my graduation project and my studies altogether. Especially my parents, Joan and Agnes who have given me the opportunity and support to finish my studies at the Eindhoven University of Technology.

A.J.M. (Fons) Raemaekers
Abstract

The level of vibrations that a patient is subjected to in an ambulance is often too high. Measures have to be taken by the ambulance personnel to reduce these vibrations. This is done by either reducing the velocity of the ambulance or by deviating from the quickest or shortest route to avoid speed bumps and other obstacles that cause high vibration peaks. These vibrations are most severe in the vertical direction. The human body is most sensitive to vibrations in the range from 0.1 to 80 Hz, and it is within this range that the dominant vertical vibrations of an ambulance stretcher occur.

These vibrations can be reduced by placing an active suspension system between the ambulance body and the stretcher carrying the patient. This active suspension system is able to exert two forces. A pneumatic force is used to compensate the mass of the stretcher, the patient and medical equipment if present. An electromagnetic force is used to actively isolate the patient from the ambulance body vibrations. The requirements for the Active Vibration Isolator (AVI) are derived using a quarter ambulance model.

By introducing a reduction between the ambulance body, the AVI and the stretcher, the design requirements of the AVI system become more advantageous. A reduction of 1:3 is found to give the best trade off between the required electromagnetic force, the required mechanical power, the required stroke and the dimensions of the overall design.

On the basis of these requirements, the Active Vibration Isolator is designed. The key features of the design are a pneumatic system with low stiffness, a low friction force of the internal guiding system, low energy consumption when the system is in steady state and low cost. It potentially has low wear because the design comprises very few moving parts.

A control system is designed to regulate the AVI movements. This controller consists of a skyhook term to reduce the vibrations and a leveling term that reduces the required working space of the AVI and brings the stretcher to its neutral position to reduce the energy consumption of the system. These two actions oppose each other and a trade off must be made between the amount of vibration reduction and the required working space. For this control system, the position (stroke) of the AVI and the absolute velocity of the stretcher and patient mass must be measured.

To verify the AVI performance by experiment, a test rig is designed. With this test rig it is possible to measure the static and dynamic properties of the AVI. It
is also possible to introduce disturbances to the system and measure the vibration isolating capacities of the AVI. It is attempted to match the dynamics of the test setup to those of the ambulance dynamic model.

The design and control of the AVI is based on a quarter ambulance model which only addresses the vertical vibrations. The get insight in the real world performance of the system, a full, 6-dof ambulance model is created. The performance of the ambulance with the active suspension system is compared to the performance of a normal ambulance, where the stretcher is rigidly attached to the ambulance body, and to a system where the stretcher has passive suspension.

Simulations show that the active system is able to significantly (factor 30 on average) reduce the vibrations of the patient and stretcher mass for both discrete events such as speed bumps and for random road irregularities. It is estimated that with the active suspension system an ambulance can reduce the time to reach the hospital from a trauma scene up to 60 - 75 percent compared to a conventional ambulance, while providing good comfort to the patient. The system with passive suspension provides reasonable vibration isolation (up to a factor 5) compared to a normal ambulance.
Samenvatting

De trillingen waaraan een patiënt wordt blootgesteld in een ambulance zijn vaak te heftig. Maatregelen moeten genomen worden door het ambulance personeel om deze trillingen te reduceren. Dit wordt gedaan door ofwel de snelheid van de ambulance aan te passen of uit te wijken van de snelste/kortste route zodat drempels en andere obstakels die grote schokken veroorzaken ontwijken kunnen worden. Deze trillingen zijn het heftigst in de verticale richting. Het menselijk lichaam is het gevoeligst voor trillingen in de range van 0.1 tot 80 Hz, en het is binnen deze range dat de dominantie verticale trillingen van de stretcher van een ambulance plaatsvinden.

Deze trillingen kunnen worden gereduceerd door een actief veer systeem te plaatsen tussen de carrosserie van de ambulance en de stretcher waarop de patiënt ligt. Dit actieve veer systeem kan twee krachten genereren. Een pneumatische kracht wordt gebruikt om de massa van de stretcher, de patiënt en eventueel aanwezige medische apparaten te compenseren. Een elektromagnetische kracht wordt gebruikt om de patiënt actief te isoleren van de trillingen van de carrosserie van de ambulance. De eisen die aan deze Active Vibration Isolator (AVI) worden gesteld zijn afgeleid met behulp van een kwart ambulance model.

Door gebruik te maken van een overbrenging tussen de ambulance carrosserie, de AVI en de stretcher, worden de ontwerp eisen van het AVI systeem voordeliger. Een overbrenging van 1:3 geeft de beste afweging tussen de vereiste elektromagnetische kracht, het mechanisch vermogen, de slag en de afmetingen van het complete ontwerp.

Aan de hand van deze ontwerpeisen is de Active Vibration Isolator ontworpen. De belangrijkste eigenschappen van het ontwerp zijn een lage stijfheid van het pneumatische systeem, een lage wrijvingskracht van de interne geleiding, laag energieverbruik wanneer het systeem in rust is en lage kosten. Het ontwerp heeft mogelijk lage slijtage omdat het bestaat uit weinig bewegende onderdelen.

Een regelsysteem is ontworpen om de bewegingen van de AVI aan te sturen. Deze regelaar bestaat uit een skyhook term om de trillingen te reduceren en een term om de AVI en stretcher terug in neutrale positie te brengen en zo de benodigde energie en slag van het systeem te verkleinen. Deze twee acties zijn tegengesteld aan elkaar en daarom moet er een afweging gemaakt worden tussen de benodigde slag en hoe goed de trillingen worden geïsoleerd. Voor dit regelsysteem moet de positie (slag) van de AVI en de absolute snelheid van de stretcher met de patiënt
worden gemeten.

Om experimenteel de prestaties van de AVI te toetsen, is een testoppstelling ontworpen. Met deze opstelling is het mogelijk om de statische en dynamische eigenschappen van de AVI te bepalen. Het is ook mogelijk om verstoringen te introduceren in het systeem en de trilling-isolerende prestaties van het systeem te meten. Er is getracht de dynamica van de testoppstelling met de dynamica van het ambulance model overeen te laten komen.

Het ontwerp en de aansturing van de AVI is gebaseerd op een kwart ambulance model dat enkel de verticale trillingen behandelt. Om inzicht te krijgen in de prestaties van het systeem in de echte wereld wordt een ambulance model met 6 vrijheidsgraden gemaakt. De prestaties van de ambulance met het actieve veer systeem worden vergeleken met de prestaties van een normale ambulance, waar de stretcher vast is bevestigd aan de carrosserie, en met een systeem waar de stretcher passief geveerd wordt.

Simulaties laten zien dat het actieve systeem in staat is om de trillingen van de stretcher en patiënt substantieel (factor 30 gemiddeld) te reduceren voor zowel discrete gebeurtenissen zoals drempels en voor willekeurige weg onregelmatigheden. Het is geschat dat een ambulance met het actieve veer systeem de tijd om het ziekenhuis te bereiken vanaf de plaats van het ongeluk met 60 - 75 procent kan verminderen in vergelijking met een conventionele ambulance, waarbij het comfort van de patiënt gewaarborgd blijft. Het systeem met passive veering geeft ook redelijke trilling isolatie (factor 5 gemiddeld) vergeleken met een normale ambulance.
## Contents

1 Introduction
   1.1 General introduction ................................................. 1
   1.2 Problem statement .................................................. 1
   1.3 Outline report ...................................................... 2

2 Patient vibrations in ambulances ................................. 3
   2.1 Background ......................................................... 3
   2.2 Body vibrations .................................................... 7
   2.3 Road roughness ..................................................... 11

3 Required actuator performance .................................... 15
   3.1 Ambulance model ................................................... 15
   3.2 Reduction ........................................................... 19
   3.3 Actuator requirements .............................................. 27

4 Actuator design ......................................................... 29
   4.1 Concept ............................................................. 29
   4.2 Morphological analysis ............................................. 29
   4.3 Alternatives ......................................................... 35
   4.4 Final design ........................................................ 39

5 Controller design ....................................................... 49
   5.1 Control algorithms .................................................. 49
   5.2 Controller comparison .............................................. 55
   5.3 Controller performance ............................................. 58
   5.4 Controller choice ................................................... 62

6 Test setup ........................................................................ 65
   6.1 Measurements ........................................................ 65
   6.2 Test setup design ...................................................... 71
   6.3 Dynamics of the test setup ......................................... 74

7 Full ambulance model simulation ................................... 79
   7.1 Full ambulance model ............................................... 79
   7.2 Simulations ............................................................ 81

8 Conclusions and recommendations ................................... 87
   8.1 Conclusions ........................................................... 87
   8.2 Recommendations .................................................... 88
<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bibliography</td>
<td>91</td>
</tr>
<tr>
<td>A Quarter ambulance model</td>
<td>97</td>
</tr>
<tr>
<td>A.1 Air spring analysis</td>
<td>97</td>
</tr>
<tr>
<td>A.2 Equations of motion</td>
<td>102</td>
</tr>
<tr>
<td>A.3 Simulation results</td>
<td>104</td>
</tr>
<tr>
<td>A.4 Lagrange equations of motion</td>
<td>106</td>
</tr>
<tr>
<td>B Linear actuator specs</td>
<td>111</td>
</tr>
<tr>
<td>C Parts list</td>
<td>113</td>
</tr>
<tr>
<td>D Test setup calculations</td>
<td>115</td>
</tr>
<tr>
<td>D.1 Fatigue of the eccenter axis</td>
<td>115</td>
</tr>
<tr>
<td>D.2 Forces on the AVI guiding</td>
<td>116</td>
</tr>
<tr>
<td>D.3 AVI guiding eigenfrequency</td>
<td>118</td>
</tr>
<tr>
<td>D.4 Equations of motion models I &amp; II</td>
<td>119</td>
</tr>
<tr>
<td>E Full model evaluation</td>
<td>123</td>
</tr>
<tr>
<td>E.1 Quarter car vs full vehicle model</td>
<td>123</td>
</tr>
<tr>
<td>E.2 Acceleration weighting</td>
<td>125</td>
</tr>
</tbody>
</table>
Chapter 1

Introduction

1.1 General introduction

The word ambulance is derived from the Latin word ambulant, which means walking. Horse-drawn military vehicles, which were equipped to address wounds and accompanied soldiers into the field during the Napoleonic Wars, are considered as the predecessors of modern ambulances. Some were used as field hospitals, and others transported wounded soldiers to hospitals [1].

Transport in these basic vehicles over the poor roads of the time must have been very uncomfortable. While modern ambulances generally travel over much better surfaces, the road-induced vibration experienced by patients during the journey to the hospital can still be unpleasant and/or harmful. The vibration of patients in ambulances has been the subject of a number of papers over the past 30 years, and the research effort in this area emphasizes both that the problem is a very real one and that the quality of the ambulance suspension is an important influence on patient comfort.

The quality of roads has improved over the years. This means the road irregularities have decreased and thus the comfort of transport has increased. However, besides improvement of the road quality, explicit deterioration of the roads is deliberately introduced in the form of speed bumps to control traffic velocities. The number of speed bumps is still increasing because it is an effective traffic calming measure. These speed bumps introduce high peak vibrations to the patient in an ambulance.

1.2 Problem statement

In an ambulance the reduction of vibrations of the stretcher table with the patient is important. Ambulance body vibrations are of less importance. To significantly reduce the patient vibrations an active suspension system can be used. One option is to build a fully active suspension system between all four (or only the rear) wheels and the body of the ambulance. Another option is to place this active suspension or active vibration isolation system between the ambulance body and the stretcher table. This gives the opportunity to provide vibration isolation for the patients at
a potentially lower cost, with a lower energy consumption, and with better performance since the design is freed from vehicle handling constraints.

To create a fully active vibration isolator, an air spring will be used in parallel with an electromagnetic actuator. The air spring can be used to support the mass, while the electromagnetic actuator is used to exert forces to the system. The advantage of this approach is that the system has theoretically zero energy consumption when at rest. The downside of this approach is that the force of the electromagnetic actuator will be counteracted by the change in the force of the air spring when the system is not in its equilibrium position.

The goal of this report is to reduce the vibrations that a (traumatized) person is exposed to, by designing an active vibration isolation system that is capable of reducing the vibrations of the stretcher table and patient in an ambulance to an acceptable level, to design a control strategy that can be practically implemented in an ambulance and the design of a test setup which can be used to test the performance of the active vibration isolator.

These goals are summarized in the problem statement as:

"Design and test an active vibration isolator and control strategy that reduce patient vibrations in an ambulance to an acceptable level."

1.3 Outline report

This report is organized as follows: In chapter 2, a study is made on the vibrations of patients in ambulances. Some background information is treated, vibration limits of human beings and traumatized persons in particular are discussed and road irregularities are analyzed.

In chapter 3 the required performance of the actuator is investigated. A dynamic model of an ambulance is used for various simulations. Different performance criteria are discussed and the actuator requirements resulting from these simulations are discussed.

In chapter 4 the design of the active vibration isolator is treated. Different concepts are studied and the distinguishing features of the design are discussed. In chapter 5 two control strategies are worked out and compared and the active vibration isolator performance is analyzed and compared to conventional and passive suspended ambulance systems.

To evaluate the performance in a laboratory environment a test rig is designed. In chapter 6 the requirements and the design of the test setup are discussed and a comparison is made between the ambulance dynamics and the dynamics of the test setup. In chapter 7 a simulation is made of a full 6-dof ambulance model with the active vibration isolator in place to see how it performs, since it is designed to reduce only vibrations in the vertical direction. The performance is compared to a normal ambulance system and a system with passive stretcher suspension. In chapter 8, the conclusions and recommendations of this research are discussed.
Chapter 2

Patient vibrations in ambulances

2.1 Background

Vehicle dynamics

Vehicle dynamics have been studied by numerous people over the past 50 years. Starting with simple two dimensional models and suspension analysis [2, 3] it has developed into a new field of research. Most papers deal with suspension performance for different types of road irregularities. Simple quarter car vehicle models are extended to full body models, giving information on body roll, pitch, heave and other information in six degrees of freedom. More recent papers show interest in the dynamics and control of (semi-)active suspension systems. Comparison between passive and active suspension systems are made and performance of different controllers is compared [4, 5, 6, 7, 8].

Not only the dynamics of passenger cars are studied, studies of vehicle dynamics include trucks, busses, motor cycles, trains, cranes, earth-moving equipment and also ambulance models. Studies on ambulance dynamics and vibrations are much less common than some of the other subjects mentioned above. However there are studies investigating the dynamics of ambulances and the effects on transported traumatized patients [9, 10, 11].

Ambulance stretcher suspension systems

Many ambulances are based on the chassis of a van. Chassis of such vehicles can give a poor ride compared to that of a typical car [11]. According to [12] amongst others, the dominant floor vibrations in ambulances are generally in the vertical sense and peak around 1.5 - 2 Hz. At higher frequencies, the acceleration levels reduce until resonances are met beyond 10 Hz.

Solutions to reducing the ambulance floor vibrations are found by specifically designing ambulance vehicles instead of basing them on commercial vehicles and adapt the suspension. Using air springs instead of classic coil springs, leaf springs
or other specifically designed suspension systems can reduce the vibrations of the ambulance body.

As an alternative to focusing on reducing ambulance floor vibration, an additional suspension can be provided for the stretcher only. This approach offers no improvement in the ride for the ambulance attendants, but does give the opportunity to provide good isolation for the patient, potentially at a lower cost than if the ambulance suspension was improved. In theory, better isolation is possible by using a stretcher suspension as the designer is freed from the constraints imposed by vehicle handling considerations.

Several designs for stretcher suspensions have appeared in the literature. One paper [14] shows a torsion bar stretcher suspension design, which includes a pendulum and lever system to pivot the suspension about its longitudinal axis when cornering. No mention was made of the ability of the suspension to adapt to patient mass. Reportedly, 10% of those using the suspension in a trial experienced motion sickness. This was thought to be partly a result of the disconcerting visible movement of the ambulance walls relative to the suspended patient.

In [15] a four degree-of-freedom system developed at the University of Delft is described. This passive suspension system is depicted in figures 2.1 and 2.2. Suspension was provided by two independent mechanical systems which used a spring acting through a lever and movable pivot. By adjusting the pivot position with an electric motor, a load-independent ride height was achieved. Roll and transverse suspension were combined in a compound pendulum system which aligned itself with the resultant acceleration associated with vehicle cornering or road camber. Another paper [10] reports on the performance of the so-called Dutch "floating stretcher". This stretcher offered a reduction of 42% in vertical r.m.s. acceleration, and best isolation was realized in the 3-10 Hz range.

In [16] a system is described that uses gas springs and provides isolation in pitch and bounce. The basic principle of operation is similar in some respects to that of the 'floating stretcher', in that variable lever arms and mechanical coil springs are used, but the design is simpler and more compact [17, 18]. Road and shaker tests, which are described by [12], indicate that good levels of isolation are obtained. This suspension is sold commercially in the United Kingdom and used by a number of ambulance services.
Figure 2.2: Delft stretcher in an ambulance [10].

Stretcher suspensions are also used in France and Germany, and are described by [17, 18]. The French system uses a hydro-pneumatic approach developed by BINZ and has a natural frequency in the range 0.5-0.7 Hz. The German system of Miessen appears to be similar to that of the 'floating stretcher' system. A natural frequency of 1 Hz is claimed for a wide range of patient masses.

Active suspension systems

Traditionally automotive suspension designs have been a compromise between the three conflicting criteria of road holding (dynamic tyre load), required working space (suspension travel) and passenger comfort (vertical accelerations) [19]. The suspension system must support the vehicle, provide directional control during handling manoeuvres and provide effective isolation of passengers/payload from road disturbances [20]. Good ride comfort requires a soft suspension, whereas insensitivity to applied loads and good handling requires a stiff suspension.

Due to these conflicting demands, suspension design has had to be something of a compromise, largely determined by the type of use for which the vehicle is designed. A passive suspension system has the ability to store energy via a spring and to dissipate it via a damper. Its parameters are generally fixed, being chosen to achieve a certain level of compromise between road holding, required working space and comfort. Active suspensions are considered to be a way of increasing the freedom one has to specify independently the characteristics of required working space, handling and ride quality.

There are semi-active and active suspensions. A semi-active suspension has the ability to change characteristics of the passive suspension system such as the damping constant of the suspension. This usually requires a small amount of energy. Such a semi-active system is able to operate in two quadrants the force-
An active suspension system has the ability to apply forces using an active element and thus store, dissipate and introduce energy to a system. An active suspension system operates in all four quadrants of the force-velocity diagram (see figure 2.3). Active suspension systems may require a significant amount of energy. (Semi-)active systems can vary its parameters depending upon operating conditions and can have knowledge other than the strut deflection the passive system is limited to.

![Figure 2.3: Force-velocity diagram. Semi active systems are limited to operation in quadrant I and III. Active systems can operate in all four quadrants.](image)

Active suspension systems can be further distinguished between high bandwidth and low bandwidth systems. In low bandwidth systems, in general the actuator will be placed in series with a spring and/or a damper. A low bandwidth system aims to control the suspension over the lower frequency range. At higher frequencies the actuator effectively locks-up and vibrations are isolated passively. These systems can achieve a significant reduction in body roll and pitch during manoeuvres such as cornering and braking, with lower energy consumption and cost than a high bandwidth active system.

A high bandwidth active suspension system is generally considered as an actuator connected between the sprung and unsprung masses of the vehicle. A high bandwidth active system aims to control the suspension over the full bandwidth of the system, thus giving maximum control over the required working space, handling and ride quality. In particular this means that it aims to improve the suspension response around the resonant frequencies of the system. A high bandwidth active system may consume a significant amount of power and will require actuators with a relatively large bandwidth which can be very costly.

Active suspension systems have been successfully implemented on Formula One racing cars by Lotus. An example of a low bandwidth active suspension system is Mercedes’ Active Body Control which is commercially successful. This system is able to control body motions up to 5 Hz using a hydraulic actuator in series with a
High bandwidth active suspension systems are not commercially available yet, mostly due to the high cost of such systems. Bose has developed and demonstrated a high bandwidth active suspension system, which is reported to have a peak power consumption of 50 kW [21]. Initial intentions to take the system in production have been canceled due to the high costs of the system.

2.2 Body vibrations

Human body vibrations

The human body is sensitive to vibrations. This has been the subject of many studies, going back to a study in 1918 considering hand-arm vibrations due to the operation of power tools [22]. Since World War II interest has been shown in whole-body vibrations in passenger cars, trucks, aircraft, ships and earth-moving equipment and numerous studies have been made.

These studies have been reviewed by [23], which studies the mechanical modeling and resonances of the human body, the methods of measuring and testing human performance to vibrations and the physiological and mechanical effects of these vibrations.

Most research has dealt with standing or seated subjects, and has lead to vibration limitations on the human body in the frequency range of 1 - 50 Hz. This is documented in ISO 2631 [24]. This norm sets out the limits for the acceleration as a function of exposure time and vibration frequency. These limits are divided in comfort-ratings, fatigue decreased proficiency and exposure limits. Vibration of the human body is most uncomfortable when the frequencies of the vibrations lie in the resonance frequency range of the human body. Most resonant frequencies of the human body are known, and a multibody model exists which incorporates these resonant frequencies. This model is shown in figure 2.4 and the resonant frequencies are listed in table 2.1. The resonant frequency of the chest and belly cavity (3 - 6 Hz) is very important, as is the resonance frequency of the internal organs (4 - 8 Hz).

Based on the dynamic model and on experimental data, a frequency dependent weighting function is defined. This weighting is different for different axis of motion. The main weighting is defined as follows:

- $W_v$ is the weighting in the vertical direction. This mean head-to-toe for standing and sitting subjects, and front-to-back for supine subjects.
- $W_d$ is the weighting for the directions in the horizontal plane. This means front-to-back and sideways for standing and sitting subjects and head-to-toe and sideways for supine subjects.
- $W_f$ is the frequency weighting concerned with motion sickness, which typically occurs between 0.1 and 0.3 Hz.

The frequency weighting curves are shown in figure 2.5.
Table 2.1: Body resonance frequencies for a standing or sitting person [24].

<table>
<thead>
<tr>
<th>Body part</th>
<th>Resonance frequency [Hz]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Head (axial mode)</td>
<td>± 25</td>
</tr>
<tr>
<td>Eyeball, intraocular structures</td>
<td>30-80</td>
</tr>
<tr>
<td>Shoulder girdle</td>
<td>4-5</td>
</tr>
<tr>
<td>Chest wall</td>
<td>± 60</td>
</tr>
<tr>
<td>Chest &amp; belly cavity</td>
<td>3-6</td>
</tr>
<tr>
<td>Abdominal mass</td>
<td>4-8</td>
</tr>
<tr>
<td>Spinal column (axial mode)</td>
<td>10-12</td>
</tr>
<tr>
<td>Lower arm</td>
<td>16-30</td>
</tr>
<tr>
<td>Hand grip</td>
<td>50-200</td>
</tr>
<tr>
<td>Legs (knees flexed)</td>
<td>± 2</td>
</tr>
<tr>
<td>Legs (stretched)</td>
<td>± 20</td>
</tr>
</tbody>
</table>

This approach is widely accepted, although there are differences between the reported frequency effects of vertical vibration [25]. The exact response of an individual to a particular vibration will depend on other factors in addition to the level of acceleration, its frequency, and the duration of exposure to it such as the stature of a subject [26].

The sensitivity of supine subjects to vibrations has received much less attention than seated or standing subjects. In [27] and [28], a study is made to the response of supine subjects to horizontal and vertical vibrations. It is reported that peak sensitivity to vertical vibrations occurs around 6 Hz.

In [9] vibration limits are presented which define the sensitivity of healthy stretcher-borne subjects to vibrations. According to [12], these limits show that the peak sensitivity to front-to-back (vertical) vibration is largest in the range 4 to 6 Hz. It is also
Figure 2.5: Frequency weighting of different vibration axis according to ISO 2631 [24].

noted that supine subjects are much more sensitive to vertical vibrations compared to standing or sitting subjects. Figure 2.6 (which is reproduced from [12]), exemplifies this and shows that, in the range 4-8 Hz, the level of front-to-back vibration that is tolerated for 2.5 hours by a seated subject is tolerated for only five minutes by a supine subject.

Figure 2.6: Comparison exposure limits for standing and supine persons [12].

The ISO 2631 not only specifies exposure limits, but also comfort ratings for whole-body human vibrations. These ratings, listed in table 2.2, are indications of the perceived comfort. However, for each situation these values can differ, depending
on the expectations of the subject (e.g. in a building or on the bus), the tasks at hand (e.g. writing, reading, drinking or sleeping) and the major axis of vibration. However, the ratings give a good indication of the perceived comfort by a subject exposed to these whole-body vibrations.

<table>
<thead>
<tr>
<th>Acceleration [m/s²]</th>
<th>Scale of discomfort (suggested by ISO 2631)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Less than 0.315</td>
<td>Not uncomfortable</td>
</tr>
<tr>
<td>0.315-0.63</td>
<td>A little uncomfortable</td>
</tr>
<tr>
<td>0.5-1</td>
<td>Fairly uncomfortable</td>
</tr>
<tr>
<td>0.8-1.6</td>
<td>Uncomfortable</td>
</tr>
<tr>
<td>1.25-2.5</td>
<td>Very uncomfortable</td>
</tr>
<tr>
<td>Larger than 2</td>
<td>Extremely uncomfortable</td>
</tr>
</tbody>
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Table 2.2: Comfort ratings.

**Effect of ambulance vibrations on patients**

Ambulance floor vibration has been shown to be detrimental to the condition of injured or seriously ill patients [10]. This is likely to be due in part to:

- The relatively high levels of ambulance floor vibration at frequencies where patient sensitivity is high.
- The reduced tolerance of humans to vibration when in the supine position.
- The reduced tolerance of ill or injured persons to vibrations.

Several reports have been published by various authors dealing with patient deterioration due to ambulance floor vibrations. These reports are mostly subjective, based on the observations of medical personnel, however they provide a good indication of the effect of (ambulance) vibrations on supine patients. Some reports are described below.

In [29] an account is given of a road accident victim dying after being transferred between hospitals. The poor quality of the ambulance ride is regarded as an important secondary factor in his death.

In [11] several cases are described where patient deterioration is associated with ambulance travel. Another report [30] deals with the movement related cardiovascular disturbances (i.e. rise and fall of the patients blood pressure), where it is noted that in 6% of the 430 monitored ambulance journeys the blood pressure was significantly disturbed by the ambulance movement.

In a more recent report [31], the low tolerance to vibrations of patients suffering severe pain is also noted. For example, cancer patients with secondary deposits on the spine need repeated radiotherapy treatments. For these patients, the vibrations associated with the ambulance ride to and from the hospital is often distressing, resulting in some patients refusing to attend treatment.

Another more recent study [32] illustrates that the survival rate of traumatized patients (with the same trauma severeness level) transported by ambulance lies lower.
than those who were transported by helicopter, which is confirmed by other studies [33, 34].

Although some of these reports are somewhat dated, the sensitivity of patients to ambulance floor vibrations is clearly illustrated. Modern ambulance suspension systems are more comfortable, and road conditions have improved. However, there are also much more traffic calming measures (e.g. speed bumps, chicanes, bus lanes etc) for which modern ambulances drastically have to reduce their velocity to prevent shocks and peak vibrations on the ambulance patient. In [35] it is said that on average one speed bump causes a 10 seconds delay. From another study [36] follows that 66% of the ambulance personnel would deviate routes to avoid speed bumps, causing on average 2.5 to 5 minutes delay (depending on the type of trauma) in the transport from trauma scene to the hospital.

Considering the reports stated above, the aim of the Active Vibration Isolator (AVI) design will comprise two main interests:

1. To be able to eliminate the need for velocity reduction when crossing a speed bump, and thus gaining 10 seconds per speed bump and eliminating the need for alternative routes.

2. To evoke extra ride comfort to patients undergoing transport, especially those who have to travel between hospitals, which can be a stressing and time consuming enterprise.

2.3 Road roughness

Introduction

The vertical axle loads of a vehicle are composed of a static and a dynamic component. The static component follows from the distribution of the vehicle weight over the vehicle axis. The dynamic component is induced by the road unevenness, that subjects the vehicle to vertical oscillations and depends on the road profile, the vehicle characteristics and the vehicle velocity.

The road unevenness is defined as the deviation of a traveled surface from a true planar surface with characteristic dimensions that affect ride quality, vehicle dynamics, dynamic pavement loads and pavement drainage [37]. Road unevenness can be either random or discrete. Discrete road unevenness is only present at limited distance along the road, while the random road unevenness corresponds to the overall unevenness along the road. For a two-dimensional vehicle model, only the longitudinal unevenness profile \(z_r(x)\) that represents the deviation at a point \(x\) along the road is considered.

Discrete as well as random unevenness can be described by a deterministic function \(z_r(x)\). Following the ISO 8608 [38], random road unevenness can also be described in a statistical way by a power spectral density function. This approach is not suited for discrete or periodic road unevenness [39].
A forward Fourier transformation of the coordinate \( x \) along the road to the wavenumber \( k_x \) reveals the wavenumber content \( \tilde{z}_r(k_x) \) of the unevenness profile \( z_r(x) \), where \( x \) is the coordinate in the direction of travel:

\[
\tilde{z}_r(k_x) = \int_{-\infty}^{\infty} z_r(x)e^{ik_x x} dx.
\]  

(2.1)

Table 2.3 shows a classification of road roughness according to the wavelength \( \lambda_x = 2\pi/k_x \) of the road unevenness \[40\]. The range of the road unevenness which is important for vehicle dynamics is characterized by wavelengths \( \lambda_x \) between 0.5 and 50 m or wavenumbers \( k_x \) between 0.12 and 12 rad/m. The largest wavelengths are only relevant at high vehicle velocities.

<table>
<thead>
<tr>
<th>Class</th>
<th>Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>Microtexture</td>
<td>( \lambda_x &lt; 5 \cdot 10^{-4} ) m</td>
</tr>
<tr>
<td>Macrotecture</td>
<td>( 5 \cdot 10^{-4} &lt; \lambda_x &lt; 5 \cdot 10^{-2} ) m</td>
</tr>
<tr>
<td>Megatecture</td>
<td>( 5 \cdot 10^{-2} &lt; \lambda_x &lt; 0.5 ) m</td>
</tr>
<tr>
<td>Unevenness</td>
<td>( 0.5 &lt; \lambda_x &lt; 50 ) m</td>
</tr>
</tbody>
</table>

Table 2.3: Classification of road roughness.

**Random road unevenness**

The random road surface roughness is a typical example of a random process. In a classical paper \[39\], the road surface unevenness is described as a two-dimensional Gaussian random process with a single power spectral density as a function of the wavenumber \( k_x \). In the case where the road shows discrete or periodic irregularities such as joints or potholes, a description by a power spectral density function (PSD) is not suitable. The approach of the paper is proposed as a basis for the ISO 8608 standard. The classification is based on a comparison of measured, single-sided PSD and eight categories labeled from A to H, that are defined by a range of artificial PSD of the following form:

\[
S_r(k_x) = S_r(k_0) \left( \frac{k_x}{k_{x0}} \right)^{-w},
\]  

(2.2)

where \( w \) equals 2 and the value of \( S_r(k_0) \) depends on the unevenness class, defined in table 2.4. The wavenumber content decreases with an increasing wavenumber or a decreasing wavelength. It is indicated in ISO 8608 that the profiles in table 2.4 can be used as the input for theoretical parametric studies.

The characterization of the road surface roughness by a PSD can be used in two ways. It can be used to calculate the PSD and statistical properties in a direct way. It can also be used to generate a set of artificial unevenness profiles. The latter can be done by applying the inverse fast Fourier transformation (FFT) on \( S_r(k_x) \) \[41\] to give:

\[
z_r(x) = \sum_{k=1}^{N} \left( \frac{4S_r(k_0)}{NLc} \left( \frac{2\pi k}{NLc k_{x0}} \right)^{-2} \right)^{1/2} \cos \left( \frac{2\pi k}{NLc} x + \phi_k \right),
\]  

(2.3)

12
Roughness

\[ S_r(k_0) \cdot 10^{-6} \text{ m}^4 \]

<table>
<thead>
<tr>
<th>Class</th>
<th>Minimum</th>
<th>Mean</th>
<th>Maximum</th>
</tr>
</thead>
<tbody>
<tr>
<td>A (very good)</td>
<td>0</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>B (good)</td>
<td>2</td>
<td>4</td>
<td>8</td>
</tr>
<tr>
<td>C (average)</td>
<td>8</td>
<td>16</td>
<td>32</td>
</tr>
<tr>
<td>D (poor)</td>
<td>32</td>
<td>64</td>
<td>128</td>
</tr>
<tr>
<td>E (very poor)</td>
<td>128</td>
<td>256</td>
<td>512</td>
</tr>
<tr>
<td>F</td>
<td>512</td>
<td>1024</td>
<td>2048</td>
</tr>
<tr>
<td>G</td>
<td>2048</td>
<td>4096</td>
<td>8192</td>
</tr>
<tr>
<td>H</td>
<td>8192</td>
<td>16384</td>
<td>32768</td>
</tr>
</tbody>
</table>

\((k_{x0} = 1 \text{ rad/m})\)

Table 2.4: ISO 8608 road roughness classification.

where \(L_c\) is the distance interval between successive ordinates of the surface profile, \(N\) is the number of data points, \(\varphi_k\) is a set of independent random phase angles uniformly distributed between 0 and \(2\pi\) and \(x\) is the coordinate along the direction of travel.

Using (2.3) different road profiles can be generated. An example of a road profile is shown in figure 2.7.

![Sample of a generated road profile (class A).](image)

Figure 2.7: Sample of a generated road profile (class A).

Discrete road unevenness

Traffic plateaus, poorly filled trenches and joints in a road surface are examples of discrete road unevenness profiles that generate vibrations. Profiles for traffic bumps are described by [42, 43], which is a Dutch guideline created by CROW for traffic bump design. A traffic bump is sine-shaped and can be described by:

\[
    z_r(x) = \frac{H}{2} \left[ 1 - \cos \left( \frac{2\pi x}{L} \right) \right], \quad 0 < x < L. \tag{2.4}
\]

Where \(H\) is the bump height in meters and \(L\) is the bump length in meters. Traffic bumps can be designed for different velocities and heights. In table 2.5(a) these variables are listed for different velocities.

Another guideline for traffic bumps comes from the Institute of Transportation Engineers (ITE) [44]. This guideline defines a Watts (TRRL) profile and a Seminole
As mentioned in section 2.2, traffic bumps are the cause of much delay when an ambulance travels from the site of an accident to the hospital. The average traffic bump causes vertical accelerations in the range of 5.6 m/s² (with a standard deviation of 1.1 m/s²) [45]. Compared to the comfort ratings in table 2.2, this is a very high value, in the ‘extremely uncomfortable’ category. Especially for traumatized patients it will be very important to reduce the vertical accelerations induced by traffic bumps.
Chapter 3

Required actuator performance

3.1 Ambulance model

Vehicle model

The most general and useful automotive suspension system design information can be derived from a single wheel station, quarter car representation [46].

The quarter car model contains no representation of the geometric effects of having four wheels (wheelbase filtering and its lateral equivalent) and offers no possibility of studying longitudinal interconnection, the use of front suspension state information to improve the performance at the rear, or the resonant excitation of the body roll motion.

For the ambulance case the primary interest lies in the vertical accelerations and suspension forces. Therefore, the quarter car model does appear to contain the most basic features of the problem. It is the simplest model which has these features and possesses particular advantages over more complex models in terms of:

- Being described by few design parameters,
- having few performance parameters,
- having only a single input, the road unevenness disturbance acting on one wheel, leading to ease of computation of performance and ease of application of optimal control theory to derive control laws and
- ease of mapping and understanding of the relationships between design and performance.

When full advantage of the simplicity of the quarter car model has been taken and studying it further offers no benefits, more elaborate models will become cost beneficial and of course are essential at any stage to assess features which are implicitly omitted from simpler approaches.
To create a quarter ambulance model, the quarter car vehicle model is extended with an extra degree of freedom. This extra mass represents the stretcher table, the stretcher, the patient and possibly some medical equipment attached to the stretcher. In normal ambulances this extra mass is rigidly attached to the ambulance floor. In this case, the only isolation to ambulance body vibrations comes from the stretcher mattress on which the patient lies. To reduce the vibrations of the patient, an AVI will be placed between the car body mass and the stretcher mass.

![Figure 3.1: Quarter car ambulance model.](image)

As mentioned in section 1.2, the actuator will consist of a low-stiffness air spring and an electromagnetic actuator. To get an indication of the requirements of the actuator, it will be modeled as a low stiffness spring and a small damper, representing the air spring, in parallel with an ideal force generator, which represents the electromagnetic actuator. The properties of the air spring are described in the next section. The ambulance model is shown in figure 3.1.

**Air spring design**

The concept of the AVI design is to create an air spring with a very low stiffness. The air spring stiffness can be calculated by:

$$k_a = \frac{\gamma p A^2}{V},$$

(3.1)
where $\gamma$ is the specific heat ratio of air, $A$ is the effective air spring area, $p$ is the pressure difference between the internal pressure of the air spring and the ambient pressure and $V$ is the volume of the air spring. The analysis of the air spring is described in appendix A.1.

Keeping the force of the air spring constant requires $pA$ to be constant. Without changing the medium, the air spring stiffness can be changed by adjusting $A/V$. Thus decreasing the air spring effective area or increasing the air spring volume. Decreasing the effective air spring area $A$ is bounded by the maximum internal pressure $p$ of the air spring since the force $pA$ must be constant. The only option that remains when the maximum internal pressure is reached, is to increase the volume $V$ of the air spring. This is done by means of a buffer volume. Adding a buffer volume introduces damping to the system due to the airflow from the buffer volume to the air spring volume. The derivation of the air spring stiffness and the damping of the system is described in appendix A.1.

**Tyres**

Pneumatic tyres contact the ground over a finite length and envelope short wavelength irregularities. For a typical car tyre contact length of 0.25 m, enveloping will provide a significant filtering of the road profile input for wavelengths shorter than 0.75 m [47]. For vehicle velocities in excess of 30 m/s, the input frequencies corresponding to these wavelengths are larger than 40 Hz ($f = v/\lambda$), which is beyond the main focus of the research. For input frequencies larger than 80 Hz, tyre carcass resonances can occur [48], and dramatic increases in tyre model complexity to properly represent real behavior are then necessary. In the velocity and frequency ranges of interest, therefore, it will be considered adequate to think of the tyre making contact with the ground at a single point.

**Comparison of stretcher vibrations**

The parameters of the ambulance model are listed in table 3.1. These values are scaled from other quarter car model parameters to match the size of a typical ambulance. The air spring stiffness and damping coefficient are calculated by using a spring volume of 2.6 liter and a buffer volume of 15 liter, that are connected by a capillary tube with a diameter of 7.5 mm and a length of 500 mm.

<table>
<thead>
<tr>
<th>Description</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wheel mass</td>
<td>$m_w$</td>
<td>67 kg</td>
</tr>
<tr>
<td>Tyre springrate</td>
<td>$k_w$</td>
<td>400 kN/m</td>
</tr>
<tr>
<td>Body mass</td>
<td>$m_b$</td>
<td>900 kg</td>
</tr>
<tr>
<td>Suspension springrate</td>
<td>$k_b$</td>
<td>80 kN/m</td>
</tr>
<tr>
<td>Suspension dampingcoefficient</td>
<td>$c_b$</td>
<td>6000 Ns/m</td>
</tr>
<tr>
<td>Stretcher mass</td>
<td>$m_s$</td>
<td>100 kg</td>
</tr>
<tr>
<td>Air spring springrate</td>
<td>$k_a$</td>
<td>737 N/m</td>
</tr>
<tr>
<td>Air spring dampingcoefficient</td>
<td>$c_a$</td>
<td>8.9 Ns/m</td>
</tr>
</tbody>
</table>

Table 3.1: Quarter car ambulance model.
The equations of motion for this system are derived in Appendix A.2. Three different ambulance models are compared:

- An ambulance model with no suspension between the ambulance body and the stretcher table. It is assumed these are rigidly attached to one another.

- An ambulance model with a passive suspension system between the ambulance body and the stretcher table. This passive suspension consists of the air spring with an additional damper and is tuned such that it has a dimensionless damping coefficient $\xi$ of 0.35.

- An ambulance model with the air spring and an electromagnetic actuator for active vibration isolation.

For the latter case, a simple PID controller is implemented. The properties of the controller are discussed in chapter 5. This controller generates a force between the body mass and the stretcher mass, as shown in figure 3.1. It is assumed that it is possible to measure the ambulance stretcher table height $z_s$, which is used as a feedback signal to the controller.

**Performance criteria**

For the three cases different road unevenness scenario’s are simulated and the performance is compared. To compare the different systems the following performance criteria are evaluated:

- Patient discomfort (accelerations of the stretcher mass $\ddot{z}_s$).

- Required working space (difference between $z_s$ and $z_b$).

- Actuator force levels $F_{LM}$.

- Mechanical power consumption ($P_{mech} = F_{LM}v$).

In all suspension systems, both passive and active, a trade off must be made between the required working space and the amount of vibrations reduction it is capable of. To find the potential performance of the AVI’s vibrations reduction and from that the requirements of the AVI, no limits are imposed on the required working space in this comparison. The PID controller is tuned in such a way that for all the different road unevenness scenario’s the acceleration of the ambulance stretcher mass will be below 0.315 m/s² (comfort criteria), thus providing very good vibration reduction.

Using the simple quarter car model, the performance parameters can all be calculated. Simulations are made with this model with the various road irregularities discussed in section 2.3 as input. The results of these simulations are listed in Appendix A.3. In table 3.2 the results of a simulation over a rough road (class D) and a 100 mm Watts profile are listed.

As can be seen, the accelerations of the stretcher when it is rigidly attached to the ambulance body are very high. Introduction of a passive spring and damper system reduce these vibrations by a large part. However the remaining vibrations are
Table 3.2: Results of simulations.

<table>
<thead>
<tr>
<th></th>
<th>Rigid</th>
<th>Passive</th>
<th>Active</th>
</tr>
</thead>
<tbody>
<tr>
<td>Watts profile</td>
<td>100 mm, 25 km/u (peak values)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$z_s$ m/s$^2$</td>
<td>6.1</td>
<td>2.2</td>
<td>0.31</td>
</tr>
<tr>
<td>stroke m</td>
<td>-</td>
<td>0.13</td>
<td>0.13</td>
</tr>
<tr>
<td>$F_{LM}$ N</td>
<td>-</td>
<td>-</td>
<td>90</td>
</tr>
<tr>
<td>$P_{mech}$ W</td>
<td>-</td>
<td>-</td>
<td>59</td>
</tr>
<tr>
<td>Road class D</td>
<td>60 km/u (RMS values)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$z_s$ m/s$^2$</td>
<td>1.3</td>
<td>0.22</td>
<td>0.03</td>
</tr>
<tr>
<td>stroke m</td>
<td>-</td>
<td>0.02</td>
<td>0.05</td>
</tr>
<tr>
<td>$F_{LM}$ N</td>
<td>-</td>
<td>-</td>
<td>35</td>
</tr>
<tr>
<td>$P_{mech}$ W</td>
<td>-</td>
<td>-</td>
<td>3.0</td>
</tr>
</tbody>
</table>

still quite large with peak accelerations reaching 2.2 m/s$^2$, which are still very uncomfortable. Using an active force element to reduce the vibrations, the resulting stretcher accelerations can be reduced to a level where the vibrations are no longer perceived as uncomfortable by a subject.

Two observations can be made from the simulation data:

- Because no limits are imposed on the working space of the active and passive suspension system, the required working space of the actuator lies between -130 and +130 mm. This is quite much considering the air spring and the electromagnetic actuator will have to make a stroke of 260 mm.

- The required electromagnetic force of the active system is largely dependent on the stroke of the actuator and the stiffness of the air spring. For example, in the Watts profile simulation the required force $F_{LM}=90$ N. The change in the net air spring force due to the displacement of the air spring equals $k \cdot \text{stroke} = 737 \cdot 0.13 = 96$ N. Thus a large part of the electromagnetic motor force is to compensate the change of the net air spring force.

A smaller required working space would be beneficial to both the required AVI stroke (and thus the overall AVI dimensions) and the required electromagnetic motor force. However, this will be at the cost of the AVI’s vibration isolation capacities. It is possible to get around this trade off by introducing a reduction to the ambulance system. This is discussed in the next section.

### 3.2 Reduction

By introducing a reduction $r$ to the system between the stretcher mass and the AVI, the required static force becomes $r$-times larger. This is no problem, because an air spring can be selected which is able to deliver this static force. The advantage is that the required stroke is reduced by approximately a factor $r$.

As mentioned in the previous section, the force of the electromagnetic motor is for a large part compensating the change of the net air spring force, $\delta F = k \cdot \text{stroke}$.

So a reduction of the required stroke will result in a smaller force. However, to
keep the comparison between the different reductions $r$ fair, it is assumed that the air spring and the buffer volume have fixed dimensions. Thus, when the static force on the air spring is increased, the internal pressure of the air spring must be increased and by that the stiffness of the air spring is increased as well (see appendix A.1). If the air spring stiffness is $k_0$ and the required stroke $stroke_0$ when the reduction $r=1$, then for any reduction $r$:

$$
\delta F = k \cdot stroke = (k_0 \cdot r) \cdot \left( \frac{stroke_0}{r} \right) = k_0 \cdot stroke_0,
$$

and thus the change of the net air spring force is equal for any chosen reduction $r$, and the minimum required electromagnetic motor force is bounded by this force.

The dynamic model of the ambulance with reduction is depicted in figure 3.2. This system is modeled using the SimMechanics toolbox in Matlab and the equations of motion are derived using the Lagrange equations. This derivation is given in the appendix A.4. The goal is to find an optimum value for the reduction $r$ and the length of the arm of the lever $l$, that gives the best trade off between the different performance criteria listed in section 3.1. With the use of a reduction, there is a larger scope for the trade off between the required working space and the amount of vibration reduction.

It should be noted that by introducing a reduction in the system, one important property of the vibration isolator is lost. Namely the ability to be able to filter out high frequent vibrations. With a reduction mechanism present, high frequent vibrations will be transmitted through the reduction mechanism. For the ambulance stretcher isolation this imposes no problem because a typical road profile does not contain much energy in the high frequent range.

**Lagrange equations of motion**

The ambulance model with the reduction is depicted in figure 3.2. The equations of motion of this system are derived in appendix A.4. The equations of motion of this system are:

$$
m_w \ddot{z}_w + k_w (z_w - z_r) - k_b (z_b - z_w) - c_b (\dot{z}_b - \dot{z}_w) = F_1,
$$

$$
(m_b + m_s + m_{red}) \ddot{z}_b - lr (m_s + \frac{1}{2} m_{red}) \ddot{\theta} + k_b (z_b - z_w) + c_b (\dot{z}_b - \dot{z}_w) - F_{LM} = F_2,
$$

$$
l^2 \ddot{\theta} + lr (m_s + \frac{1}{2} m_{red}) \ddot{z}_b + k_a l^2 \ddot{\theta} + c_a l^2 \ddot{\theta} - F_{LM} l = T_3.
$$

The equations of motion are rewritten in state space form and compared to the SimMechanics model. The state space model is derived in appendix A.4. There is a difference between the outputs of these two models. This difference increases when $\theta$ becomes larger and the linearization techniques used to derive the state space model ($\cos(\theta) = 1$ and $\sin(\theta) = \theta$) no longer apply.
To compare the SimMechanics and the state space model some simulations are made. While both systems are at rest and thus in steady state, a 1 Hz sine shaped signal \( z_r(t) = A \sin(2\pi t) \) is introduced as a disturbance to the model, with different amplitudes. For small amplitudes of \( z_r(t) \) the difference between the two models is very small. For larger amplitudes of \( z_r(t) \) a significant difference occurs between the output variable \( \theta \) of both models. The difference between the two models for output variables \( z_w \) and \( z_b \) is small for all amplitudes.

This is shown in figure 3.3, where the difference between the output variables of both models is plotted as a percentage of the maximum values of these signals versus the amplitude \( A \) of the disturbance signal \( z_r(t) \). From these simulation results it can be said that for small disturbances the representation of the ambulance model as a state space system is accurate.
Figure 3.3: Error between the SimMechanics and state space model for different amplitudes of the disturbance signal $z_r$. 
Optimum reduction

A simple PID controller is designed that uses the absolute position of the stretcher as a feedback signal. The PID controller mainly uses the proportional and derivative gains to suppress the vibrations and a small integral gain to return to the equilibrium position of the absolute height. More details about this controller are discussed in chapter 5. It should be noted that the practical implementation of this controller is difficult since the absolute height of the stretcher cannot be directly measured inside the vehicle.

Two scenario’s are simulated. One where the ambulance drives over a 100 mm Watts profile at 25 km/u and one where the ambulance drives over a rough road (road class D) at 60 km/u. For the first scenario the peak values of the relevant signals are considered, where for the latter case the rms values are considered. Several simulations are made, where the lever length \( l \) is varied between 0.2 and 1.0 m and the reduction \( r \) is varied between 0.5 and 10. From the simulations results, the best values for \( l \) and \( r \) are determined.

As mentioned in section 3.1 the performance criteria are: the remaining stretcher accelerations \( \ddot{z}_s \), the electromagnetic motor force \( F_{LM} \), the mechanical power consumption of the system \( P_{mech} \) and the required working space. These will be addressed in the following sections.

Stretcher acceleration

The controller gain is scaled such that the remaining acceleration of the stretcher mass \( \ddot{z}_s \) is more or less constant and within the limit of the comfort criterium \((\ddot{z}_s < 0.315 \text{ m/s}^2)\) whenever possible. The stretcher accelerations are shown in figure 3.4 for the bump profile and the rough road profile. Note that for \( l = 1 \text{ m} \) and \( r \geq 8 \) the stretcher accelerations could not be suppressed below 0.315 m/s\(^2\).

![Figure 3.4: Stretcher mass accelerations \( \ddot{z}_s \). Left: Watts profile (peak values), Right: Road profile D (rms values).](image-url)
Electromagnetic force

The required maximum actuator force can now be compared for different values of \( l \) and \( r \), this is shown in the figure 3.5 below.

![Diagram showing electromagnetic force comparison](image)

Figure 3.5: Required electromagnetic force \( F_{LM} \). Left: Watts profile (peak values), Right: Road profile D (rms values).

For static forces, the air spring force is proportional with the reduction \( r \). As can be seen in figure 3.5, the electromagnetic motor force is not proportional with the reduction \( r \). This is caused by several relations between the geometry of the ambulance model and the required force \( F_{LM} \).

First of all, as mentioned in the beginning of this section, the part of the electromagnetic force that compensates the change in net air spring force is constant, independent of the reduction \( r \) and lever length \( l \) (the lever length does actually affect the static load, but its contribution to the total load is small).

Secondly, the mass of the lever is dependent on both \( l \) and \( r \), as the mass \( m_{\text{red}} \) is defined by \( m_{\text{red}} = 10 \cdot l \cdot r \) in kg. This means that the mass of the lever is not constant, and thus influences the required electromagnetic motor force (and for a small part the air spring force).

When dynamics are considered, the mass of the stretcher \( m_s \) and the mass of the lever \( m_{\text{red}} \) are not proportional to the reduction \( r \), but the equivalent masses can be calculated by:

\[
m_{s,eq} = m_s \cdot r^2 \quad \text{and} \quad m_{\text{red,eq}} = m_{\text{red}} \cdot \left(\frac{1}{2}r\right)^2.
\]

Thus the part of the electromagnetic motor force that compensates the movement of the masses is proportional to \( r^2 \).

Because the equivalent mass is proportional to \( r^2 \) and the air spring stiffness is
proportional to $r$, the eigenfrequency of the air spring changes with the square root of $1/r$:

$$\omega_n = \frac{1}{2\pi} \sqrt{\frac{k_0 \cdot r}{(m_s + \frac{1}{r} m_{red}) \cdot r^2}} = \frac{1}{2\pi} \sqrt{\frac{k_0}{(m_s + \frac{1}{r} m_{red}) \cdot r^2}}. \quad (3.7)$$

It is difficult to determine how the different contributions mentioned above relate to the total electromagnetic motor force $F_{LM}$. The static terms (e.g., mass and stiffness variations) can easily be expressed in terms of $r$ and $l$. The contribution of the dynamic forces are not so transparent and cannot be expressed in terms of $r$ and $l$, since they are dependent on the actions of the (scaled) PID controller, and the accelerations and velocities of the different masses in the ambulance model.

**Power requirement**

Not only the maximum force is important, but also the maximum power consumption of the actuator. The mechanical power of the electromagnetic motor can be expressed as the product of force and velocity:

$$P_{mech} = F \cdot v = F_{LM} (\dot{z_a} - \dot{z_b}). \quad (3.8)$$

Normally, the mechanical power is constant when a reduction is used, since the required force $F$ becomes $r$ times larger and the velocity becomes $r$ times smaller. However, the required force $F$ in this case is the electromagnetic motor force $F_{LM}$ which is not proportional to the reduction $r$ as mentioned in the section above. The velocity of the actuator is proportional to $1/r$. This results in the power requirements shown in figure 3.6 for different values of $r$ and $l$. As $r \to 0$, $P_{mech} \to \infty$ because the actuator velocity goes to infinity.

![Figure 3.6: Mechanical power $P_{mech}$. Left: Watts profile (peak values), Right: Road profile D (rms values).](image)

To create the required force the electromagnetic actuator requires electric power.
The total power consumption of the electric motor can be calculated by adding the mechanical and electric power that are required:

\[ P = P_{\text{mech}} + P_{\text{el}} = Fv + I^2R = F_{LM}((\dot{z}_a - \dot{z}_b) + \left(\frac{F_{LM}}{K}\right)^2R. \quad (3.9) \]

Where \( I \) is the electromagnetic motor current in A, \( R \) is the electromagnetic motor resistance in \( \Omega \) and \( K \) is the electromagnetic motor force constant in N/A.

As can be seen, the total power is dependent on \( F_{LM}^2 \). The force constant \( K \) and resistance \( R \) are unique for every electromagnetic actuator and in general become larger for electromagnetic motors that produce larger forces. Therefore any conclusions about the total power consumption can only be made when these properties are known for a given actuator. What can be concluded is that the total power still depends on the squared value of the electromagnetic motor force and minimizing this force thus has a large effect on the required electric power.

The energy used by the system can be calculated by integrating the power over time. It is assumed that the electromagnetic motor cannot regenerate energy, and thus only the positive values of the power signal are used to calculate the energy use.

\[ E = \int_0^T P(t) \, dt, \quad P(t) > 0. \quad (3.10) \]

The time span over which the electromagnetic force is applied does not change with different values of \( r \) and \( l \), thus the energy consumption is proportional to the power requirement. Therefore the energy consumption of the system will not be considered, because the power requirement will suffice to give insight on the energy consumption as well.

**Working space**

Another important factor for the design of the vibration isolator is the required working space since it influences the overall dimensions of the design. For the different values of \( r \) and \( l \) the required actuator stroke is plotted in figure 3.7. As can be seen in the figures, the required stroke is proportional to \( 1/r \) and (almost) independent of the lever length \( l \). When \( r \) goes to zero, the required stroke goes to infinity.

**Selection**

Considering the performance criteria the following conditions are favorable:

- **Low force** \( F_{LM} \). A lower force generally requires a smaller and cheaper actuator. Furthermore a low force implies a low electric power requirement.
- **Low mechanical power** \( P_{\text{mech}} \). Lowest mechanical power means the system operates at its most efficient point.
- **Small stroke**. A small stroke means that the overall dimensions of the actuator can be kept small.
Small value of $l \cdot r$. A small total length of the reduction mechanism results in small dimensions of the complete system.

Since these conditions do not all point in the same direction to choose optimum values of $r$ and $l$, a trade off must be made. With a small reduction $r$, the required force $F_{LM}$ is small, but the required mechanical power $P_{mech}$ becomes very large, as well as the required stroke and thus a small reduction is not very favorable. A large reduction results in a high force $F_{LM}$, and thus a very large (and expensive) electromagnetic motor.

The lever length $l$ does show consistency, as in general a smaller lever length results in a lower force and power, and keeps the dimensions small without increasing the required stroke or sacrificing performance. It should be noted that a very small lever length can result in impractical situations, since it must be possible to physically mount the actuator in the reduction mechanism.

For the design of the ambulance, the following reduction $r$ and lever length $l$ are chosen because these give a good trade off between the criteria mentioned above:

$$r = 3, \quad l = 0.2 \text{ m}.$$
most actuator requirements.

<table>
<thead>
<tr>
<th>Requirement</th>
<th>Old (no reduction)</th>
<th>New (with reduction)</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Static force</td>
<td>1000</td>
<td>3000</td>
<td>N</td>
</tr>
<tr>
<td>Dynamic RMS force</td>
<td>35</td>
<td>40</td>
<td>N</td>
</tr>
<tr>
<td>Dynamic peak force</td>
<td>95</td>
<td>110</td>
<td>N</td>
</tr>
<tr>
<td>Working space</td>
<td>300</td>
<td>100</td>
<td>mm</td>
</tr>
<tr>
<td>Velocity</td>
<td>1.1</td>
<td>0.4</td>
<td>m/s</td>
</tr>
<tr>
<td>Acceleration</td>
<td>14</td>
<td>5</td>
<td>m/s²</td>
</tr>
<tr>
<td>Bandwidth</td>
<td>&gt;20</td>
<td>&gt;20</td>
<td>Hz</td>
</tr>
</tbody>
</table>

Table 3.3: Actuator requirements, with and without reduction.

These requirements follow from simulations with the linearized ideal model, e.g. no bandwidth limit or friction force is taken into account. The maximum values for the actuator velocity and acceleration are determined for the simulated scenario’s. These results are shown in appendix A.3 in figure A.7.

Power spectrum analysis of the simulation data shows that accelerations of the ambulance body are the most significant in the range from 1 - 100 Hz, but up until 10 Hz already 75 percent of the energy is delivered, and 90 percent at 20 Hz. This can be seen in figure 3.8, where the power spectrum of the ambulance body acceleration is plotted for a simulated ride over a class D road profile with 60 km/u. From the frequency data it can be concluded that the actuator will have to do most work in the range from 1 - 40 Hz. Higher actuator bandwidth is preferable, but will not influence the performance significantly. As mentioned in section 2.2, the human body is most sensitive to vibrations in the range between 4 - 8 Hz. The actuator bandwidth should at least be higher than this frequency range.

![Power Spectrum of Ambulance Body Acceleration](image1.png)
![Power Spectrum of Percentage of Maximum Body Acceleration](image2.png)

(a) PSD of ambulance body accelerations $\ddot{z}_b$. (b) Power spectrum of percentage of maximum body acceleration $\ddot{z}_s$.

Figure 3.8: Required actuator bandwidth.
Chapter 4

Actuator design

4.1 Concept

The concept of the AVI is that it is capable to support a large mass, that is to be isolated by using an active force element. This force element should be an electromagnetic motor, because this type of actuator has a very important property; it has zero stiffness. Furthermore, an electromagnetic motor is capable of smooth operation, silent motion, has a lack of internal moving parts and a lack of backlash.

By creating an air spring with a low stiffness (by using a buffer volume as discussed in section 3.1), the force of the electromagnetic motor can be kept small. Using input and output valves for the air inside the air spring, the air spring itself can be used for low-frequent actuation, e.g. when weight changes of the load occur.

The combination of these two elements will result in an active vibration isolator, which can support a large mass. It can actively reduce vibrations with a relative low dynamic force of an electromechanical motor. Potentially the system will have a low energy use, theoretically zero when in equilibrium.

4.2 Morphological analysis

Requirements

In chapter 3, the requirements of the actuator are analyzed. These requirements are listed in table 3.3. These requirements follow from different functions that must be integrated in the actuator design. The different functions of the actuator are:

- Static force generation
- Dynamic force generation
- Reciprocating motion/guiding

The requirements of the actuator can be subdivided into these three functions. This division is listed in table 4.1. For each of these functions possible solutions that meet the requirements are selected. These different solutions are listed in a morphological overview in table 4.2.
Semi-static force

The semi-static force must compensate the payload on the AVI. A pneumatic spring is chosen to generate this force. The use of a pneumatic system has several advantages over a hydraulic system or a traditional spring:

- No problems with leakage.
- Low stiffness (with the use of a buffer volume).
- Low weight.
- No fluid return tube needed.

<table>
<thead>
<tr>
<th>Function</th>
<th>Requirement</th>
</tr>
</thead>
<tbody>
<tr>
<td>Semi-static force generation</td>
<td>$F_{\text{static}}$ 3000 N</td>
</tr>
<tr>
<td></td>
<td>$k$ 750 N/m</td>
</tr>
<tr>
<td></td>
<td>$c$ 10 Ns/m</td>
</tr>
<tr>
<td>Dynamic force generation</td>
<td>stroke 100 mm</td>
</tr>
<tr>
<td></td>
<td>$F_{\text{peak}}$ 110 N</td>
</tr>
<tr>
<td></td>
<td>$F_{\text{rms}}$ 40 N</td>
</tr>
<tr>
<td></td>
<td>$v_{\text{el max}}$ 0.4 m/s</td>
</tr>
<tr>
<td></td>
<td>$a_{\text{cc max}}$ 5 m/s²</td>
</tr>
<tr>
<td>Guiding system</td>
<td>stroke 100 mm</td>
</tr>
<tr>
<td></td>
<td>$v_{\text{el max}}$ 0.4 m/s</td>
</tr>
<tr>
<td></td>
<td>$a_{\text{cc max}}$ 5 m/s²</td>
</tr>
<tr>
<td></td>
<td>$F_{\text{fric}}$ &lt; 5 N</td>
</tr>
</tbody>
</table>

Table 4.1: Actuator requirements by function.

Dynamic force

There are many ways to create a dynamic, reciprocating motion and/or force. This can be done by using a hydraulic system, a ball screw and motor, a voice coil actuator, a linear motor or a pneumatic actuator. The choice for an electromagnetic force module (e.g. voice coil or linear motor) is made, because it has several advantages over the other actuating principles:

- High bandwidth.
- Long stroke.
- Frictionless operation.
- Zero stiffness.

Especially the zero stiffness in combination with the frictionless operation and the very high bandwidth offer the ability to create a highly efficient system with low power dissipation.

Guiding system

For the guiding system there are no initial preferences for a certain system, all possibilities will be considered.
<table>
<thead>
<tr>
<th>Function</th>
<th>Method 1</th>
<th>Method 2</th>
<th>Method 3</th>
<th>Method 4</th>
<th>Method 5</th>
<th>Method 6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pneumatic force</td>
<td>Rolling lobe</td>
<td>Single convoluted</td>
<td>Double convoluted</td>
<td>Reversible sleeve</td>
<td>Steel bellow</td>
<td>roc/piston</td>
</tr>
<tr>
<td>Electromagnetic force</td>
<td>Voice coil (moving magnet)</td>
<td>Voice coil (moving coil)</td>
<td>Linear shaft motor</td>
<td>Linear motor (ironless core)</td>
<td>Linear motor (iron core)</td>
<td></td>
</tr>
<tr>
<td>Linear guiding system</td>
<td>Flexures</td>
<td>Ball bearing</td>
<td>Air bearing</td>
<td>Wheel guiding</td>
<td>Magnetic bearing</td>
<td>Polymer bearing</td>
</tr>
</tbody>
</table>

Table 4.2: Morphological overview.
Pneumatic force

For the generation of the pneumatic force, several alternatives are considered. Four types of air bellows are considered; the rolling lobe air spring, the single and double convoluted air springs and the reversible sleeve type. Furthermore a steel bellow and a custom designed rod and piston are considered.

The linearity of the spring stiffness is very important. Besides the compression of the air inside the air spring the force can vary by a change of the effective area of the air spring. This occurs with the reversible sleeve air spring and the single and double convoluted air springs. When the height of the air spring changes, the geometry and thus the effective area of the spring change as well, causing changes up to a factor two of the air spring force under constant pressure. The rolling lobe air spring, the steel bellow and the custom rod and piston do not suffer from this phenomena, since their geometry is constant with their height. Because of this property the reversible sleeve air spring and the single and double convoluted air springs are discarded.

Other selection criteria besides the ones listed in table 4.1 and the air spring linearity are the dimensions, weight and price. The remaining three alternatives are listed in table 4.3.

<table>
<thead>
<tr>
<th>Method</th>
<th>Rolling lobe</th>
<th>Steel bellow</th>
<th>Rod/piston</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model</td>
<td>Alumalift 10014 VR</td>
<td>Convat 316L DN50</td>
<td>custom design</td>
</tr>
<tr>
<td>Stroke (mm)</td>
<td>150</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>$D_i$ (mm)</td>
<td>82.5</td>
<td>52</td>
<td>80</td>
</tr>
<tr>
<td>$D_o$ (mm)</td>
<td>101</td>
<td>95</td>
<td>90</td>
</tr>
<tr>
<td>$h_{nom}$ (mm)</td>
<td>250</td>
<td>108</td>
<td>150</td>
</tr>
<tr>
<td>$h_{max}$ (mm)</td>
<td>325</td>
<td>124.5</td>
<td>200</td>
</tr>
<tr>
<td>Linearity</td>
<td>good</td>
<td>good</td>
<td>good</td>
</tr>
<tr>
<td>$p$ @ 3kN (bar)</td>
<td>4.4</td>
<td>4.3</td>
<td>4.0</td>
</tr>
<tr>
<td>$F$ @ 1 bar (N)</td>
<td>670</td>
<td>460</td>
<td>500</td>
</tr>
<tr>
<td>$F$ @ 7 bar (N)</td>
<td>6200</td>
<td>3220</td>
<td>3500</td>
</tr>
<tr>
<td>$k$ (kN/m)</td>
<td>20</td>
<td>75</td>
<td>10</td>
</tr>
<tr>
<td>Price (€)</td>
<td>60</td>
<td>1000</td>
<td>100 (est.)</td>
</tr>
</tbody>
</table>

Table 4.3: Static force alternatives.

The stiffness of the rubber bellow and the rod/piston can be reduced by means of a buffer volume, as discussed in section 3.1. The stiffness of the steel bellow is determined not by the air volume but by the material properties of the bellow and can therefore not be reduced. Therefore this option is rejected.
From the properties listed in table 4.3 it is concluded that a custom built cylinder is the best option. However, a rod and piston will require dynamic seals and a separate guiding to prevent the air from escape and guide the reciprocating movement of the rod in the piston. This introduces friction to the system. The rubber bellow does not suffer from this problem and is therefore also a good option to use as a pneumatic spring.

**Electromagnetic force**

The ironless core linear motor and the moving coil voice coil actuator are both very fast systems, but the force density is in general low because there is no use of a solid core. A high force density is preferable since it will enable a small and compact design. Thus, of the original five electromagnetic force actuators shown in table 4.2, the ironless core linear motor and the moving coil voice coil actuator are rejected.

The remaining three (moving magnet voice coil, iron core linear motor and linear shaft motor) are compared. Other selection criteria besides the ones listed in table 4.1 are the motor constant, time constants, thermal resistance, dimensions, mass, price and whether or not they have a built in position sensor. These specifications are listed in table 4.4.

<table>
<thead>
<tr>
<th>Method</th>
<th>Voice coil</th>
<th>Linear motor</th>
<th>Linear motor</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>moving magnet</td>
<td>iron core</td>
<td>shaft</td>
</tr>
<tr>
<td>Model</td>
<td>H2W Tech</td>
<td>Tecnotion</td>
<td>Copley</td>
</tr>
<tr>
<td></td>
<td>NCM30-25-090</td>
<td>TM 3 S</td>
<td>STB2504</td>
</tr>
</tbody>
</table>

| $F_{\text{peak}}$ [N] | 120 | 120 | 312 |
| $F_{\text{rms}}$ [N]  | 40.1 | 30 | 50 |
| Stroke [mm]            | 76.2 (100)* | ∞ | ∞ |
| $v_{\text{max}}$ [m/s] | 10 | 10 | 10 |
| $a_{\text{max}}$ [m/s$^2$] | 200 | 100 | 100 |
| $K_T$ [N/$\sqrt{\text{W}}$] | 5.2 | 9.48 | 6.5 |
| $\tau_e$ [ms]          | ? | 6.5 | 0.65 |
| $\tau_m$ [ms]          | ? | 0.19*J | 0.14*J |
| $R_{\text{th}}$ [°C/W] | ? | 1.4 | 1.33 |
| Height [mm]            | 195.6 | 93 | 160 |
| Volume [cm$^3$]        | 619 | 186 | 321 |
| Mass [kg]              | 2.7 | 1.1 | 2.4 |
| Pos. sensor            | no | no | yes |
| Price [€]              | 1200 (3000)* | 608 | 1128 |

*In brackets the specs of the version with 100 mm stroke.

Table 4.4: Electromagnetic actuators.
The iron core linear motor has the best specifications. The linear shaft motor comes out second. The voice coil actuator is rejected because it has a limited stroke. It is possible to build a custom voice coil actuator with a 100 mm stroke, but that is very expensive. Furthermore the weight and dimensions of the voice coil actuator are relatively high.

The advantage of the linear shaft motor is that it has a built in position sensor, which can be used for feedback. The iron core linear motor can optionally also be delivered with a position sensor. With this option, the difference in price and dimensions between the two actuators is reduced, and the linear shaft motor is interesting since it has a cylindrical symmetric design.

Guiding system

Out of the six original options for the guiding system, three are worked out into more detail, namely the wheel guiding, the linear ball bearing and the polymer bearing. The air bearing, the (electro-)magnetic bearing and the flexures are discarded for being either too heavy, expensive, large, complex or a combination of these. The properties of the remaining three guiding systems are listed in table 4.5. For the guiding system, mass, volume and price are considered, as well as operating temperature range, need for lubrication and the maximum axial load.

<table>
<thead>
<tr>
<th>Method</th>
<th>Wheel guiding</th>
<th>Polymer bearing</th>
<th>Ball bearing</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model</td>
<td>TPA RV18-3 + MR18</td>
<td>Trelleborg Zurcon</td>
<td>SKF</td>
</tr>
<tr>
<td>stroke [mm]</td>
<td>∞</td>
<td>∞</td>
<td>∞</td>
</tr>
<tr>
<td>$v_{max}$ [m/s]</td>
<td>4</td>
<td>2</td>
<td>5</td>
</tr>
<tr>
<td>$F_f$ [N]</td>
<td>low</td>
<td>medium</td>
<td>14</td>
</tr>
<tr>
<td>Volume [cm$^3$]</td>
<td>25</td>
<td>3</td>
<td>24</td>
</tr>
<tr>
<td>Mass [gr]</td>
<td>medium</td>
<td>low</td>
<td>110</td>
</tr>
<tr>
<td>$T$ [$^\circ$C]</td>
<td>-30 - 80</td>
<td>-60 - 80</td>
<td>-20 - 80</td>
</tr>
<tr>
<td>Surf. rough. [$\mu$m]</td>
<td>100</td>
<td>1-4</td>
<td>20</td>
</tr>
<tr>
<td>Axial load [N]</td>
<td>1200</td>
<td>4400</td>
<td>2500</td>
</tr>
<tr>
<td>lubrication</td>
<td>no</td>
<td>no</td>
<td>yes</td>
</tr>
<tr>
<td>Price [€]</td>
<td>?</td>
<td>?</td>
<td>?</td>
</tr>
</tbody>
</table>

Table 4.5: Guiding system.

The polymer bearing is very attractive because of the small dimensions, mass and low cost. However, it does cause friction to the system. The ball bearing is quite large and heavy and because the balls within the bearing slip against each other some friction is introduced. The wheel guiding has very little friction when loaded.
axially, however it is quite large and when the wheel guiding is asymmetrically loaded, friction will occur. As an alternative to the wheel guiding, track rollers can be used as a guiding. These do not require an extra frame but can use existing surfaces of the construction as contact area. This provides more freedom in the design while still providing the advantage of a wheel guided system.

4.3 Alternatives

In the latter section for each function one option came out best. Combining the three best options however, might not lead to the best design, since some options are easier to integrate together than others. With the preceding in mind, three designs are made with different compositions, listed in table 4.6. These configurations are also shown in the morphological overview in table 4.2.

<table>
<thead>
<tr>
<th>Alternative</th>
<th>Pneumatic force</th>
<th>Electromagnetic force</th>
<th>Guiding</th>
</tr>
</thead>
<tbody>
<tr>
<td>alternative 1</td>
<td>rod and piston</td>
<td>iron core linear motor</td>
<td>wheel guiding</td>
</tr>
<tr>
<td>alternative 2</td>
<td>rod and piston</td>
<td>linear shaft motor</td>
<td>polymer bearing</td>
</tr>
<tr>
<td>alternative 3</td>
<td>air bellow</td>
<td>linear shaft motor</td>
<td>wheel guiding</td>
</tr>
</tbody>
</table>

Table 4.6: Alternative designs.

**Alternative 1**

This concept has all three options that came out best in the previous section. The smallest linear motor is combined with the wheel guiding system. These are built into a custom designed rod and piston. This concept design is shown in figure 4.1. As can be seen, some problems arise with this design. The force of the linear motor is not applied to the centerline of the actuator. This will introduce a momentum on the wheel guiding and will result in a high friction force.

Furthermore, the linear motor and wheel guiding are compact, but have a square form. Putting these into a cylinder and piston will require a large diameter piston which diminishes the advantage of the compact linear motor. The piston and cylinder need dynamic sealing and these seals only work properly on a cylindrical surface. The seals will also introduce more friction to the system. This alternative, although it has the best options available for each function, does not seem to be the best solution to the design problem. It is still compact with its low height, but this is because there is no built in position sensor present.
Figure 4.1: Design 1, height 280 mm, diameter 115 mm.
Alternative 2

The second design considered consists of a linear shaft motor, built into a custom rod and piston and guided with polymer bearings. The advantage of the linear shaft motor is that it is axis-symmetric and the force comes from the centerline of the actuator. This way, there will be no internal momentum.

![Diagram of Alternative 2 design](image)

Figure 4.2: Design 2, height 370 mm, diameter 90 mm.

The linear shaft motor is built into the custom cylinder. To seal the rod and piston, dynamic seals are needed. It also needs two polymer bearings to avoid contact of the rod and piston and guide the shaft through the linear motor forcer. The complete design is shown in figure 4.2.

The downside of this design is the friction force of the dynamic seals and the polymer bearings. This friction will introduce hysteresis to the system. The durability of the seals and bearings is dependent on the friction force. The order of magnitude of this friction force is difficult to determine and therefore it is hard to determine what the lifespan and the dynamic behavior of this design will be. The friction will also cause power dissipation of the electromechanical actuator.

Alternative 3

The third design uses the same linear shaft motor, but now uses guiding wheels as a bearing mechanism and a rubber bellow instead of the custom rod and pis-
The advantage of the rubber bellow is that there are no dynamic seals needed. This way, the only friction present, is the rolling stiffness of the air bellow, which is zero in theory.

The wheel guiding will be in direct contact with the shaft of the linear shaft motor. This way, no extra rail or guiding is needed, thus saving costs, weight and space. The guiding wheels will have bearings to reduce the friction. With this design the friction forces are very low and thus low wear and energy use is achieved. This design is shown in figure 4.3.

Figure 4.3: Design 3, height 375 mm, diameter 100 mm.

**Alternatives evaluation**

In the previous section, the best method for each function is identified. The combination of these three in alternative 1 however, does not give the best integral design. Because the linear motor generates an asymmetric force, and thus introduces an internal momentum, the friction force of the guiding wheels increases. The design is compact, however, this is mostly due to the lack of a position sensor integrated in the linear motor. Adding a position sensor diminishes this advantage.

The second alternative does not have the problem of the asymmetric force, the linear shaft motor generates an axisymmetric force. The polymer bearings enable a design with a small diameter. However, these bearings, together with the dynamic
seals will introduce friction to the system. Wear of the bearings and seals can also prove to be a problem. Both the friction force and the wear of these polymer bearings and dynamic seals are difficult to determine and can vary with different (side-)loads on the actuator and temperature changes. The friction will introduce hysteresis to the system, which will result in dissipation of the energy created by the linear shaft motor.

The third alternative has no problems concerning polymer bearings, dynamic seals and wear. The wheels provide guiding with little friction and will have little wear. The rubber bellow has a very low stiffness (theoretically zero) and thus a low overall friction force will bring little hysteresis to the system. This system has the least moving parts and only the guiding wheels and the shaft of the linear motor make moving contact.

This last design is chosen for the final design. This design will be treated in more detail in the next section.

4.4 Final design

In this section the concept of the third alternative is worked out into more detail. Different parts of the design are highlighted and calculations are made. Illustrations provide some insight in the working principles of the design.

Wheel guiding

As mentioned in section 4.2, wheels can provide a good guiding with very low friction if they are properly loaded. By using three wheels at an angle of 120°, the shaft can be guided. This setup is sketched in figure 4.4. The guiding system will be installed on two sides of the linear shaft motor. This way the movements in x, y, θ and φ direction are fixed. The reciprocating motion in the z-direction is free and the rotation round the z-axis ψ is semi free, only fixed by the sideways friction force of the wheels.

Two of the wheels will be fixed, and the third wheel is pressed against the shaft with a spring. This pre-tension assures that all the wheel are continuously in contact.
with the shaft and thus backlash will be avoided. The wheel guiding is only there to guide the shaft, so that it will not touch the wiring inside linear shaft motor. The guiding is not designed to take care of external forces that act on the actuator. The actuator will have to be installed in such a way that the external forces in the x and y direction are not introduced to the actuator. This can be done by mounting two Y-bearing plummer blocks on the top and bottom of the AVI.

The wheels will be pre-loaded by the spring with a force of 40 N. The spring is brought to tension by a set screw. The contact pressure between the wheel and the shaft is a Hertzian contact stress. When two cylinder shaped surfaces with radii \( r \) and \( R \), perpendicular to each other make contact, there exists an elliptic Hertzian contact stress. The contact radii \( a \) and \( b \) of this contact surface can calculated by:

\[
a = 2^{-1/3} \omega^{11/24} \left( \frac{3Fr}{Er} \right)^{1/3}, \\
b = 2^{-1/3} \omega^{-4/21} \left( \frac{3Fr}{Er} \right)^{1/3}.
\]

Where \( F \) is the contact force, \( \omega = R/r \) and

\[
Er = \left[ \frac{1}{2} \left( \frac{1 - \nu_1^2}{E_1} + \frac{1 - \nu_2^2}{E_2} \right) \right]^{-1}.
\]

Where \( E_i \) is the modulus of elasticity and \( \nu_i \) is the Poisson ratio of the respective material. With these equations the contact stress can be calculated:

\[
\sigma_{max} = 1.5 \frac{F}{\pi ab},
\]

and the suppression is:

\[
\delta = \frac{a^2}{2R} + \frac{b^2}{2r}.
\]

The shaft of the linear motor is made of steel. The wheels are made of POM (Polyoxymethylene). POM has a lower modulus of elasticity (\( E=2.8 \) GPa) compared to steel (\( E=210 \) GPa) and therefore the contact stress will be lower when POM wheels are used instead of steel wheels. When the contact stress exceeds the \( \sigma_{0.2} \) limit, the material will deform plastically. Since the shaft of the linear motor contains magnets and should not be deformed, it must be assured that if plastic deformation occurs, it will be the wheels that deform. By choosing POM as the wheel material, not only will the contact stress be reduced, but when plastic deformation occurs, the wheels will deform. This deformation will then create a new, bigger contact area and the contact stress will be reduced to a level where no plastic deformation will occur, without affecting the performance of the guiding system or damaging the linear motor shaft.

From the calculations above and with the dimensions of the final design \( r=9.5 \) mm and \( R=12.5 \) mm, the initial maximum contact stress will be \( 83 \) N/mm\(^2\), which is smaller than the maximum allowable stress for steel (\( 400 \) N/mm\(^2\)) but larger than that of POM (\( 65 \) N/mm\(^2\)). This is no problem, because it means the wheels will undergo a small plastic deformation. The final design of the wheel guiding system.
The guiding wheel that is tensioned by the spring is fixed to an axis (see figure 4.5(a)). This axis rotates round a pin. The axis is positioned in a groove. This groove is dimensioned such that the axis will hit the walls of the groove when the tension force of the spring is exceeded. This way, the shaft of the linear motor cannot hit the internals of the linear motor forcer. This provides the linear shaft motor with protection against excessive external side forces.

**Air spring**

The linear shaft motor will be located inside the air spring. The air spring is too short to accommodate the whole shaft. Therefore the bottom and top of the air spring need to be extended. The top of the air spring will be replaced by a new, higher one. The foot of the spring will be hollowed out and an extra body will be attached to it. This will extend the total length of the air spring. Thereby there is enough space for the linear shaft motor and the wheel guiding system to be incorporated inside the air spring. This redesign is shown in figure 4.6.

**Linear shaft motor**

Traditionally, linear electric motors have been designed by ‘opening out flat’ their rotary counterparts. It is the same as the imaginary equivalent of cutting through a conventional motor rotary armature and rotary stator, and laying it out flat. Although this does provide a solution, a number of inherent disadvantages arise.

The linear shaft motor consists of a moving coil assembly (forcer), which encircles a round magnetic shaft. The magnets in the shaft assembly are not exposed. With this design, the forcer can make full use of the magnetic flux. Because of this, the air gap is not critical and there is typically 0.5 - 1.75 mm nominal annular clearance between the forcer and the shaft. The difference between a normal iron core linear motor and a linear shaft motor is illustrated in figure 4.7.
In addition to the obvious geometric differences between the cylindrical and flat configurations, the other difference is that the magnetic circuits of most motors, including linear motors, are made of magnetic iron while linear shaft motors are not. Because of this, the linear shaft motor has no core containing iron, thus there is no magnetic attraction between the forcer and the shaft. There is no absorption force between the shaft and the forcer (coil), thereby eliminating cogging. The coils of the linear shaft motor themselves form the core thus providing the stiffness expected in an iron core linear motor.

Because the shaft style linear motors make full use of the magnetic flux, they are highly efficient. Linear shaft motors do not require any type of cooling for normal operation, thereby eliminating the need to incorporate any type of cooling (heatsink, liquid or forced air-cooling) into the design.

Other advantages of a linear shaft motor and electromagnetic actuators in general are the lacking of any backlash, smoothness of operation (no velocity fluctuation), silent motion, lack of internal moving parts and low inertia.

The linear shaft motor is equipped with an analog Hall encoder. This encoder is used for position feedback for the controller of the linear motor. A Hall encoder is a sensor that uses the magnetic field of the shaft for feedback. When a current-carrying conductor is placed into a magnetic field, a voltage will be generated perpendicular to both the current and the field. This principle is known as the
Hall effect.

Figure 4.8(a) illustrates the basic principle of the Hall effect. It shows a thin sheet of semiconducting material (Hall element) through which a current is passed. The output connections are perpendicular to the direction of current. When no magnetic field is present, current distribution is uniform and no potential difference is seen across the output.

When a perpendicular magnetic field is present, as shown in figure 4.8(b), a Lorentz force is exerted on the current. This force disturbs the current distribution, resulting in a potential difference (voltage) across the output. This voltage is the Hall voltage ($V_H$).

The Hall voltage is proportional to the vector cross product of the current $I$ and the magnetic field $B$ [49]:

$$V_H \propto IB.$$ \hspace{1cm} (4.6)

It is on the order of $7 \mu V/\text{Vs/gauss}$ in silicon and thus requires amplification for practical applications. This amplification is provided by the controller of the linear shaft motor. The built-in analog Hall sensor of the linear shaft motor has a repeatability of $12 \mu m$ with an absolute accuracy of $350 \mu m$.

The linear shaft motor fits inside the air spring. The protective cap that is posi-
tioned over the Hall sensors is removed, to reduce the required installation diameter. The motor frame is used as part of the load bearing construction. The two frames of the wheel guiding system are attached to it and the lower section is connected to the air spring with two screws, unifying the air spring, wheel guiding and the linear shaft motor to one rigid body. A spec sheet of the linear shaft motor is listed in appendix B.

**Wire plug**

The linear motor is placed within the air spring. The power cable and the sensor must be led out of the pressurized spring to the power supply and controller. The wires will be led through a plug to the outside. This way it is still possible to remove the wiring without damaging the air spring structure.

The wire will be glued to the plug with synthetic resin, thereby making sure it is air tight. The plug is inserted in the bottom of the air spring. With an o-ring the gap between the plug and the air spring structure is closed. The plug does not have to be fixed to the air spring structure, since the pressure inside the spring will keep the plug in its place. This is sketched in figure 4.9(a) and in figure 4.9(b) a photo of the plugs is shown.

![Wire plug sketch](image1.png)

(a) Sketch ($p_1 > p_0$).

![Wire plug photo](image2.png)

(b) Photo.

Figure 4.9: Wire plug.

**End stops**

When the actuator is subject to extreme loads, beyond the design scope, it needs protection to avoid damage. This is done by introducing two end stops to limit the stroke. These can transduce the force when the stroke hits its upper or lower limit. These end stops are built in the actuator by putting two thick o-rings on the two ends of the shaft of the linear motor. When the actuator reaches an end stop, the o-ring will provide progressive stiffness to reduce the impact force and velocity. The end stop is shown in figure 4.10.
Figure 4.10: Photo of the o-ring end stops.

Overview

In figure 4.11 and overview of all the parts of the design are shown. In table 4.7 the names of all the important parts are listed. In appendix C a complete list of all components is included. In figures 4.12(a), (b) and (c) a section view of the design is shown in its minimum, middle and maximum position respectively.

In figure 4.13 the AVI is shown fixed in a lathe, filled with pressurized air at 1.1 bar. Here, the AVI generates a static force of approximately 1000 N. The total weight of the AVI is 5.52 kg.

This design is a prototype. This means that the standard parts that are used, most important the air spring and the linear shaft motor, are not perfectly matched to the design. The air spring needs to be extended and the protective cap of the linear shaft motor Hall sensor must be removed. If higher quantities of the AVI must be produced, a customized air spring and linear shaft motor can be advantageous when the complexity and cost of the design are considered.

<table>
<thead>
<tr>
<th>#</th>
<th>Description</th>
<th>#</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Rubber bellow</td>
<td>7</td>
<td>Guiding wheels</td>
</tr>
<tr>
<td>2</td>
<td>Linear motor forcer</td>
<td>8</td>
<td>Axis guiding wheel</td>
</tr>
<tr>
<td>3</td>
<td>Linear motor shaft</td>
<td>9</td>
<td>Spring guiding wheel</td>
</tr>
<tr>
<td>4</td>
<td>Wheel guiding frame</td>
<td>10</td>
<td>O-ring end stops</td>
</tr>
<tr>
<td>5</td>
<td>Top</td>
<td>11</td>
<td>Wire plug</td>
</tr>
<tr>
<td>6</td>
<td>Bottom</td>
<td>12</td>
<td>Clamp</td>
</tr>
</tbody>
</table>

Table 4.7: Description of AVI parts.
Figure 4.11: Photo of all the actuator parts.
Figure 4.12: Section view of AVI at minimum, maximum and mid stroke.

Figure 4.13: Photo of the pressurized AVI fixed in a lathe.
Chapter 5

Controller design

5.1 Control algorithms

There are various ways to control a dynamic system, which are extensively desc-

1. To use the information of the system states for controller feedback to mini-
2. The design of a controller consisting of two parts; a skyhook damper and a
3. The design of a PID controller to control the absolute position of the stretcher

These three control methods each have their own advantages and disadvantages. These are further investigated and explained below. The different control methods are compared to see which one gives the best performance regarding the performance criteria discussed in section 3.1 and which one is most practical to implement in the ambulance system.

State feedback

If all the states of the state space model are known, the acceleration of the stretcher mass \( \ddot{z}_s \) can be calculated. If the linear motor force \( F_{LM} \) is defined as:

\[
F_{LM} = -m_s r \ddot{z}_s, \tag{5.1}
\]

then the resulting force on the ambulance stretcher should equal zero in the ideal case.

The acceleration of the stretcher mass can be expressed as:

\[
\ddot{z}_s = f(l, r, z_w, \dot{z}_w, z_b, \dot{z}_b, \theta, \dot{\theta})
= \frac{rm_{red}\left(k_b(z_b - z_w) + c_b(\dot{z}_b - \dot{z}_w)\right) + l(2m_{red} + 4m_b)(k_a\theta + c_a\dot{\theta})}{r(m_s m_{red} + 4m_b m_s + m_b m_{red})}. \tag{5.2}
\]
This type of feedback requires knowledge of all the system states. By direct measurement this will be hard if not impossible to achieve. An observer can be designed to reconstruct the system states that cannot be measured directly. The stability of state space controllers has to be considered, and robust or adaptive control techniques might prove necessary. The implementation of this kind of model based controller is interesting since it theoretically can provide optimal performance. But it will not be treated further in this report because this type of control can result in very complex controller designs to reach optimal performance and simpler, more straightforward and intuitive solutions are available, which are discussed below.

**Skyhook and level control**

Skyhook control is a common control technique for active suspension systems and has been discussed by many, e.g. see [52, 53]. Supposedly there is a damper connected between the stretcher mass and an inertial reference in the sky (i.e., a ceiling that remains vertically fixed relative to a ground reference). Notice that this is a purely fictional configuration, since for this to actually happen, the damper must be attached to a reference in the sky that remains fixed in the vertical direction, but is able to translate in the horizontal direction. Ignoring this problem at the moment, the focus will be on the performance of this configuration.

The force generated by this skyhook damper is equal to:

$$F_{sky} = -c_{sky} \ddot{z}. \quad (5.3)$$

By tuning the damping ratio $c_{sky}$, the transmissibility of the resonance frequency of the air spring and stretcher mass can be decreased. For sufficiently large skyhook damping ratios (i.e., above $\xi > 0.707$), it is possible to isolate even at the resonance frequency. The advantage is that the skyhook damper does not increase the transmissibility above the resonant frequency, where a normal, passive damper does increase the transmissibility above the resonant frequency when the damping ratio is increased.

One method of generating the skyhook damping force is to replace the skyhook damper with an active force generator, e.g. an electromagnetic motor, positioned between the stretcher and body mass of the ambulance. Now, the force generated by the electromagnetic motor can be described in Laplace form by:

$$F_{LM}(s) = c_{sky} s \dot{z}. \quad (5.4)$$

Not only is damping of the vibrations important, the system should also return to its horizontal position ($\theta = 0$), to make sure the system does not run out of working space and to reduce the energy used by the electromagnetic motor. Therefore an extra term is added to the equation, an integral action for $\theta$. The electromagnetic motor force can now be written as:

$$F_{LM}(s) = c_{sky} s \dot{z} + k_{level} \frac{1}{s} \theta. \quad (5.5)$$

This controller requires two input signals; first, the absolute velocity of the stretcher mass. This velocity can be measured using an acceleration or velocity sensor. Secondly, the angle of the reduction arm, which corresponds to the position of the
Figure 5.1: Bode diagram skyhook controller, solid: leveling term, dashed: skyhook term.

The actuator position can easily be measured using the built in hall sensors of the linear motor.

The controller is tuned such that it gives good vibration isolation and has good leveling capacities. This is achieved when $c_{\text{sky}} = 2000 \, \text{Ns/m}$ (this corresponds to a dimensionless damping constant $\xi$ of 0.71) and $k_{\text{level}} = 2000 \, \text{N/s}$. The bode diagram of this controller is depicted figure 5.1. It can be seen that the leveling term is dominant in the low frequency range up to 0.16 Hz, whereas the skyhook term is dominant in the frequency range from 0.16 Hz onward.

The leveling term and the skyhook term in (5.5) oppose each other; e.g. good leveling of the stretcher will lead to bad vibration isolation and vice versa. Because of this, at the intersection point where the magnitude of the skyhook term is equal to that of the leveling term, these two terms cancel each other and the result is that the controller output is zero. This point lies for the given controller parameters at 0.16 Hz. A problem arises because the resonant frequency of the air spring and stretcher mass system lies at 0.25 Hz. Therefore the resonance that occurs at 0.25 Hz is not isolated properly because the controller action is very low since the two terms almost cancel each other. Two things can be done to solve this problem; decrease the frequency where the actions of the controller parameters coincide or increase the resonant frequency of the system.

Lowering the frequency of the break point by reducing the gain of the leveling term $k_{\text{level}}$ will result in good vibration isolation of the resonance frequency, but will also deteriorate the leveling capabilities of the controller. The classic case of
vibration isolation vs. required working space comes into play. Increasing the skyhook damping term \( c_{sky} \) results in too much damping at the lower frequencies, again resulting in an increase in the required working space.

The approach here is to increase the resonant frequency of the system. This frequency can be calculated by:

\[
\omega_n = \frac{1}{2\pi} \sqrt{\frac{k}{m}}. \tag{5.6}
\]

Thus, by increasing the air spring stiffness, the resonance frequency is increased. The damping term \( c_{sky} \) can be adjusted accordingly to keep the dimensionless damping constant \( \xi \) equal to 0.71. The stiffness of the air spring can easily be increased by making the buffer volume of the air spring smaller. The downside of this approach is that the electromagnetic motor will have to work against this increased air spring stiffness, and thus the increase in required electromagnetic motor force is proportional to the increase in air spring stiffness (see section 3.2).

Tuning of the system leads to almost a doubling the air spring stiffness to 4000 N/m and an increase of the skyhook damping term \( c_{sky} \) to 2700 Ns/m. This corresponds to an eigenfrequency of 0.33 Hz. With these changes, the vibration isolation of the system is good as are its leveling capabilities. The required electromagnetic motor peak force is increased to 140 N. The linear motor selected in chapter 4 is capable of generating these peak forces.

The difference between these two controllers is illustrated in the figures 5.2 and 5.3. In figure 5.2 a step response of the ambulance system is plotted. As can be seen from the data, the system with increased air spring stiffness and damping provides better damping of the eigenfrequency of the air spring and the stretcher mass.

![Figure 5.2: Step response. Thick line: input, solid: response with \( c_{sky}=2700 \) Ns/m and \( k_a=4000 \) N/m, dashed: response with \( c_{sky}=2000 \) Ns/m and \( k_a=2000 \) N/m.](image)

In figure 5.3 the difference between the controller action for both controllers is
shown for the step response simulation. The contribution of the leveling term and the damping term are plotted separately. The resulting controller output \(F_{LM}\) is the difference between these two signals. As can be seen, in the upper part of the graph, the leveling and damping action are almost equal and the resulting output \(F_{LM}\) is small. In the lower graph, the difference between the two controller actions is larger, resulting in a larger controller output force \(F_{LM}\) which provides sufficient vibration isolation and has good leveling capacities.

![Graph showing step response simulation](image)

Figure 5.3: Skyhook controller output. Top: system with \(c_{sky}=2000\) Ns/m and \(k_a=2000\) N/m, bottom: system with \(c_{sky}=2700\) Ns/m and \(k_a=4000\) N/m.

**PID control**

A standard PID controller is designed that uses the absolute position of the stretcher as a feedback signal. The PID controller mainly uses the proportional and derivative gains to suppress the vibrations and a small integral gain to return to the absolute height. This can be written as:

\[
F_{LM}(s) = \left( P_c + \frac{I_c}{s} + D_c s \right) z(s). \tag{5.7}
\]

Where \(P_c\), \(I_c\) and \(D_c\) are the proportional, integral and derivative gains respectively. It should be noted that the practical implementation of this controller is difficult since the absolute height of the stretcher cannot be directly measured inside the vehicle.

The controller is tuned such that it gives good performance. First, damping is provided by setting the derivative term to \(D_c=2000\). Analogue to the skyhook controller, to limit the required working space, integral action is added; \(I_c=20\). Again
the combination of the integral and derivative action result in a ‘notch’ at their intersecting point. This is overcome by adding a proportional term $P_c=200$. The resulting bode diagram of the controller is shown in figure 5.4.

![Bode diagram PID controller.](image)

In contrast to the skyhook controller, this controller has no issue of contradictorily controller signals that cancel each other. The air spring stiffness does not have to be increased thus giving an advantage because the low air spring stiffness will result in a low electromagnetic force.

The disadvantage of this controller is that it uses the absolute position of the stretcher mass as a reference signal. This means that the leveling or integral action of the PID controller only works if the average road levitation is zero. When large and/or low frequent variations in the road levitation occur, the leveling action of the controller is not adequate anymore and the integral action of the controller actually causes an increase in the required working space, since it tries to keep the stretcher mass at an absolute height. This is illustrated further in the next section.

This controller requires only one input signal, the absolute position of the stretcher mass. Measuring the absolute height is difficult, since the stretcher is located inside the moving ambulance body. There are optical systems that are capable of measuring the absolute height [54], but no simple, cheap, compact solutions are readily available. Another option is to design an observer to reconstruct the absolute position of the stretcher mass. Designing a good observer can be difficult. In [55] it is mentioned that 6 measured parameters are required for position control of an active suspension system.
5.2 Controller comparison

Looking at the properties of the two controllers, the main distinction is the difference between the kind of input signals used. The PID controller uses the absolute value of the stretcher mass position, whereas the skyhook controller uses the absolute velocity and the angle of the reduction arm. For the latter case, these signals can be measured with relative ease, while it is very difficult to measure the absolute height of the stretcher mass.

Another advantage for the use of relative signals over absolute signals is the tracking of the road input profile. While the objective is to reduce vibrations and thus filtering out the road irregularities, the ambulance should follow the average road levitation. When the absolute height of the stretcher mass is used as an input, the controller will attempt to keep the height of the stretcher on the virtual ‘zero’ level, irrespective of the nominal or average height of the road profile.

Using relative coordinates the controller will attempt to follow the average road levitation, since the height of the stretcher mass is averaged to the average height of the ambulance body. Because of these properties, the skyhook controller will require less working space, and a lower electromagnetic force. This is illustrated by simulating a ride over a hilly road and a rough road profile (class D).

For the hilly ride simulation, a road class B profile is superimposed on a hill profile with 20 m levitation and a maximum gradient of 15% at a velocity of 30 km/u. The rough road profile D is simulated for a velocity of 25 km/u.

The rms and peak values of the performance criteria for both controllers are listed in tables 5.1 and 5.2 for both of these simulations. The profiles of the road irregularities $z_r$ and stretcher height $z_s$ are plotted in figures 5.5 and 5.6 for the hilly road and the rough road respectively.

As can be seen from the data, for the hilly road the PID controller has very bad performance. Even though it gives good vibration reduction, it required a very large force and stroke due to its bad trajectory following capabilities. Such a design is not feasible in practice.

For the rough road simulation, the performance of the PID controller and skyhook controller lie closer together. The PID controller provides better vibration reduction, and the skyhook controller provides better trajectory following, as can also be seen in figure 5.6. Even though the air spring of the skyhook controlled system has a much higher stiffness, the required RMS force, power and the total energy is lower compared to the PID controller albeit its higher peak values.

Thus, although the PID controller in general gives better vibration isolation, the skyhook controller proves to be a better solution for the practical case. It has slightly worse vibration isolation, but has good leveling capabilities resulting in a small working space without increasing the required force, power and energy use of the system.
Figure 5.5: Trajectory following of controllers. Solid: road profile, dashed: skyhook controller (overlaps with road profile), dash-dotted: PID controller.

<table>
<thead>
<tr>
<th></th>
<th>PID controller</th>
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<th>Skyhook controller</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>rms</td>
<td>peak</td>
<td>rms</td>
</tr>
<tr>
<td>$\ddot{z}$ [m/s²]</td>
<td>0.13</td>
<td>0.27</td>
<td>0.11</td>
</tr>
<tr>
<td>$F$ [N]</td>
<td>5186</td>
<td>7792</td>
<td>88</td>
</tr>
<tr>
<td>$P$ [W]</td>
<td>363</td>
<td>783</td>
<td>0.94</td>
</tr>
<tr>
<td>$E$ [J]</td>
<td>18169</td>
<td></td>
<td>29</td>
</tr>
<tr>
<td>stroke [m]</td>
<td>2.39</td>
<td>3.58</td>
<td>0.03</td>
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</tbody>
</table>

Table 5.1: Comparison of peak and rms values of performance variables for the skyhook and PID controller for a hilly road.

<table>
<thead>
<tr>
<th></th>
<th>PID controller</th>
<th></th>
<th>Skyhook controller</th>
</tr>
</thead>
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<tr>
<td></td>
<td>rms</td>
<td>peak</td>
<td>rms</td>
</tr>
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<td>$\ddot{z}$ [m/s²]</td>
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<td>$P$ [W]</td>
<td>0.84</td>
<td>5.15</td>
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<td>$E$ [J]</td>
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<tr>
<td>stroke [m]</td>
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<td>0.035</td>
<td>0.009</td>
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</table>

Table 5.2: Comparison of peak and rms values of performance variables for the skyhook and PID controller for a rough road (class D).
Figure 5.6: Trajectory following of controllers, road class D. Grey: road profile, dashed: skyhook controller, dash-dotted: PID controller.
5.3 Controller performance

To determine the performance of the controllers, the transfer functions between the input \( z_r \) and the output \( \ddot{z}_s \) have to be determined for the different ambulance systems. This is done for the skyhook and PID controlled ambulance systems, for the ambulance model where the stretcher is rigidly attached to the ambulance body and for the ambulance model with a passive suspension connected between the ambulance body and the stretcher mass. These systems are discussed in section 3.1.

These systems can be represented by the standard plant setup shown in figure 5.7:

![Standard plant setup](image_url)

Figure 5.7: Standard plant setup.

The different signals of the standard plant setup are:

- \( w \): the exogenous variables (disturbances, sensor noise, setpoints).
- \( z \): the regulated variables (ideally, \( z = 0 \)).
- \( v \): the measured variables (\( v \) and \( z \) do not need to be equal).
- \( u \): the manipulated variables.

\( P \) represents the plant and \( K \) the controller. For the ambulance system, the various states can be described by:

\[
\begin{align*}
  w &= [z_r], \\
  u &= [F_{LM}], \\
  z &= [\ddot{z}_s]
\end{align*}
\]

and \( v_{sky} = \begin{bmatrix} \theta \\ \dot{z}_s \end{bmatrix} \) for the skyhook controller and \( v_{pid} = [z_s] \) for the PID controlled system.

The plant \( P \) can be described by:

\[
\begin{bmatrix}
  z \\
  v
\end{bmatrix} = \begin{bmatrix}
P_{11} & P_{12} \\
P_{21} & P_{22}
\end{bmatrix} \begin{bmatrix}
w \\
u
\end{bmatrix}.
\] (5.8)
The transfer function $M$ that describes the closed loop behavior of the plant can be calculated by:

$$M = P_{11} + P_{12}K(I - P_{22}K)^{-1}P_{21}. \quad (5.9)$$

The controller design objective in the standard plant setup is to design $K$ such that the closed-loop transfer function $M$ from $w$ to $z$ is "small". Ideally, $M = 0$ in $z = Mw$. A certain norm $||M||_{norm}$ can be used to quantify "small". This is called norm based control. An attempt was made to rewrite the equations of motion to the standard plant setup using Matlab. However, this resulted in numerical errors (e.g. 348th order system with several infinite terms). Therefore a different approach is used to find the transfer function between the road input $z_r$ and the stretcher accelerations as output.

The equations of motion ((A.35), (A.36) and (A.37)), described in section A.4 are rewritten in Laplace form and rearranged as a combination of transfer functions. The controller output $F_{LM}$ is substituted using (5.5) for the skyhook controller and (5.7) for the PID controller. This gives:

$$z_w = H_{z_w,z_b}(s)z_b + H_{z_w,z_r}(s)z_r, \quad (5.10)$$

$$z_b = H_{z_b,\theta}(s)\theta + H_{z_b,z_w}(s)z_w, \quad (5.11)$$

$$\theta = H_{\theta,z_b}(s)z_b. \quad (5.12)$$

This is visualized in figure 5.8.

![System transfer functions diagram](image)

Figure 5.8: System transfer functions.

With these equations the transfer functions from $z_r$ to $z_b$ and $\theta$ can be calculated:

$$z_b = H_{z_b,z_r}(s)z_r = \left( H_{z_b,z_r}(s)H_{z_w,z_r}(s) + H_{z_b,\theta}(s)H_{\theta,z_w}(s) \right) \frac{1}{1 - H_{z_b,z_w}(s)H_{z_w,z_b}(s)} z_r, \quad (5.13)$$

and:

$$\theta = H_{\theta,z_r}(s)z_r = \left( H_{\theta,z_b}(s)H_{z_b,z_r}(s)H_{z_w,z_r}(s) \right) \frac{1}{1 - H_{\theta,z_w}(s)H_{z_w,z_b}(s)} \left( 1 - \frac{H_{\theta,z_b}(s)H_{z_w,\theta}(s)}{1 - H_{z_b,z_w}(s)H_{z_w,z_b}(s)} \right)^{-1} z_r, \quad (5.14)$$
Now, the transfer function from road disturbance \( z_r \) to the ambulance stretcher acceleration \( \ddot{z}_s \) can be calculated by:

\[
\ddot{z}_s = \ddot{z}_b - lr \ddot{\theta} = s^2 \left( H_{z_b,z_r}(s) - lr H_{\theta,z_r}(s) \right) z_r.
\] (5.15)

The bode diagrams of the transfer functions of the skyhook controlled, the PID controlled, the passive suspended and the rigid system are shown in figure 5.9. In this figure it can be seen that the PID controlled system has the best suppression of the road irregularities. The skyhook controller has bad performance in the region where the magnitude of the damping term and the leveling term intersect (0.16 Hz), but even so it gives good vibration reduction. The system with a passive suspension between the ambulance stretcher and body also gives a significant reduction of the vibrations compared to the rigid body case, but the addition of an active element helps to reduce the vibrations even further. As mentioned in section 3.1, the passive suspension does not provide enough isolation to meet the required performance.

![Bode diagram](image)

Figure 5.9: Bode diagram \( H_{\ddot{z}_s,z_r}(s) \) of different systems. Solid: skyhook controlled, dashed: PID controlled, dash-dotted: passive system, dotted: rigid system.

For insight in the frequencies that are still transmitted to the stretcher mass, the power spectral density is plotted in figure 5.10 for the skyhook controlled system, the PID controlled system, the passive suspension system and the rigid system of the simulation of a ride over a class D road at 25 km/u.

60
It can be seen that the transmitted frequencies of the PID- and skyhook controlled systems are predominant in the range between 0.1 and 2 Hz. As mentioned in section 2.2, the human body is most sensitive to vibrations between 1 and 10 Hz. The active system is able to reduce the vibrations in this region with a factor 30 on average. Again, it can be seen that the PID controlled system gives better performance compared to the skyhook controlled system at the point where the magnitude of the damping and leveling term of the skyhook controller intersect. The passive system gives good isolation in the higher frequency range while still transmitting significant vibrations in the range from 0.1 to 5 Hz. At frequencies above 20 Hz, the passive system provides better isolation than the two active systems. This is because the lever that is used for the reduction in the active systems attenuates high frequency vibrations through its rotation point where it is fixed to the ambulance body mass.

In figure 5.11 the frequency content of the actuator stroke for the skyhook and PID controlled systems and the stroke of the passive suspension system are plotted. As can be seen in this figure the actuator stroke of the PID controlled system is very large for low frequencies. This comes from integral action of the controller, that tries to keep the ambulance stretcher mass at a constant, absolute height.

The skyhook controlled system requires much working space at the point where the magnitude of the leveling and damping action of the controller intersect. At the same time, the stretcher mass accelerations of this system at this frequency

Figure 5.10: Power spectral density of stretcher accelerations $\ddot{z}_s$. 
are peaking. Some attempts were undertaken to find a solution to the diminishing of the controller output at the intersecting frequency of the leveling and damping term; e.g. the introduction of a notch on the leveling action or an inverse notch on the damping term. However, these attempts resulted in unstable systems.

Figure 5.11: Power spectral density of the actuator and passive suspension stroke.

### 5.4 Controller choice

Two control algorithms are considered, the skyhook controller with leveling action and the PID controller. The selection criteria for the controller, mentioned in section 3.1, are patient discomfort, required working space, required electromagnetic motor force and the mechanical power consumption. The practical implementation of the controller is also taken into account.

In the previous sections, various aspects of the two controllers are discussed. The choice is made to use the skyhook controller with the leveling action. This controller has several advantages over the PID controlled system. The skyhook controller requires less working space, a lower electromagnetic motor force and has the lowest power consumption. The controller requires two feedback signals, the absolute velocity of the stretcher mass and the position of the AVI. These signals are relatively easy to measure. The negative aspect of this controller is the trade off that must be made between the amount of skyhook damping and leveling action.
This results in a frequency band where the reduction of stretcher mass vibrations is less effective.

The PID controller provides better vibration isolation, mainly because it does not have the problem of two opposing controller actions. This controller requires the absolute position of the stretcher mass as a feedback signal. This signal is difficult to measure, but it can be done with advanced optical systems or by means of an observer to reconstruct the signal. However, even when the signal is available, this controller requires a large working space since it uses the absolute instead of the relative height of the stretcher mass as a feedback signal. Because it requires a large working space, it also requires a larger electromagnetic force and more mechanical power. Because of these disadvantages and the advantages of the skyhook controller with leveling action, the latter controller is chosen at the expense of the PID controller.
Chapter 6

Test setup

To identify the static and dynamic properties of the AVI and verify its performance, experiments must be performed. The experimental results give insight in the AVI properties and its behavior and the results can be used for evaluation of the design and the control algorithm. First the required measurements are discussed, which will result in the identification of the relevant system parameters of the AVI. Next, the design of the test setup that is able to perform the necessary tests is described. Finally the dynamics of the test setup are analyzed and compared to the ambulance system dynamics. The goal is to represent the ambulance dynamics as well as possible. However, some concessions will be made, one being that no reduction between the AVI and the load will be present in the test setup design. The other main difference is that the rotation point of the lever is fixed instead of moving together with the input disturbances. This is done to reduce the cost and dimensions of the setup. The requirements for the test setup are:

- Identify the system parameters of the AVI under load.
- Identify the system dynamics of the AVI under load.
- Capable to simulate the ambulance dynamics as well as possible.
- Test the control algorithm.
- Test the performance of the AVI.

6.1 Measurements

System identification and experiments

To get insight in the properties and behavior of the AVI, experiments have to be made. Parameters of interest are:

- Air spring stiffness.
- Stiffness of the end stops.
- Static friction force.
• Dynamics of the linear shaft motor; e.g. transfer function between the current and force.

• Dynamics of the pneumatic system; e.g. transfer function between control valves and force.

• Dynamics of the AVI; e.g. transfer function between the disturbances and the accelerations of the load.

This will be explained below. Estimates of these parameters will be made and the experiments to obtain these parameters are described.

**Air spring stiffness**

The stiffness of the air spring is one of the most important properties of the AVI. To verify if the calculations that were made are correct, this parameter has to be measured. Using the equations from appendix A.1, the air spring stiffness can be calculated by:

\[ k = \frac{\gamma p_0 A_c^2}{V_0}. \]  

(6.1)

The internal pressure equals \( p_0 = m \cdot g/A_c \). With a load of 100 kg, the air spring stiffness is estimated to be 700 N/m.

**End stop stiffness**

The o-rings situated at the ends of the linear motor shaft are there to absorb the energy in case the AVI is overloaded. The stiffness of the o-rings is important because it can be used in the modeling of the AVI and because the total energy that can be absorbed with the o-rings can be calculated from this data, namely by:

\[ U_k = \frac{1}{2} k x^2. \]  

(6.2)

The stiffness of the o-rings at the end stops can only be roughly estimated without doing advanced FEM analysis. The force to compress an o-ring with 8 mm diameter by 20 % is 213 N. This equals to a stiffness of \( k = 213/(8 \cdot 0.20) = 133 \) N/mm. Because of the round shape of the o-ring, the stiffness will increase progressively with further compression and is thus nonlinear. It is estimated that the average stiffness of the o-ring lies round 175 N/mm or \( 175 \times 10^3 \) N/m.

**Static friction force**

One of the goals of the AVI design was to introduce as little friction as possible. However, some friction will always be present and introduce hysteresis to the system. This friction will mainly come from the rolling resistance of the guiding wheels. The bearings in the guiding wheels have a starting torque of 0.254 Nmm. Using six wheels with a radius of 9.5 mm, the total friction force by the guiding wheels equals:

\[ F_{f, \text{wheels}} = n \frac{M}{r} = 6 \cdot \frac{0.264}{9.5} = 0.16 \text{ N}. \]

The rolling resistance of the rubber bellow is in theory zero, but in practice it is not. The energy required to bend the rubber is approximately equal to the energy
released by the bent part of the bellow that is straightened. Therefore, little energy needs to be introduced to the system and thus a small force is needed to change the height of the air spring.

The magnets in the shaft of the linear shaft motor introduce a cogging force to the system. This force must be measured, and varies between -2.8 and +2.8 N over a length of 104 mm, which is twice the length of the magnets inside the shaft. The cogging introduces hysteresis to the system.

The three parameters discussed above can be identified with one experiment. A force-displacement characteristic must be obtained. This can be done by loading the air spring with a nominal load. When the system is in rest, it is moved from its equilibrium to its upper and lower height. Meanwhile, the force and the displacement are measured. With the estimated values mentioned above, the force-displacement diagram shown in figure 6.1 should be obtained.

![Graphs showing force-displacement characteristics.](image)

(a) Air spring and o-ring stiffness.  
(b) Air spring stiffness, friction and cogging force.

Figure 6.1: Estimate of the force-displacement characteristic.

**Dynamics of the linear shaft motor**

The dynamics of the linear shaft motor are an important property of the system. The bandwidth of the linear shaft motor will determine the bandwidth of the AVI. The transfer function of the input current to the output force is of interest. The linear shaft motor by itself with no load applied has a bandwidth of approximately 2000 Hz. How this bandwidth is affected when the linear shaft motor is built into the AVI and it is loaded needs to be investigated.

**Dynamics of the pneumatic system**

With two pneumatic valves, one for the input and one for the output of air, the pressure inside the air spring can be controlled. The dynamics of this pneumatic system are of interest. The bandwidth that can be achieved by pneumatic control is approximately 1 Hz [56]. This means that when the pneumatics of the AVI are controlled, the linear shaft motor can be relieved, thus minimizing its energy consumption. The eigenfrequency of the air spring and the Helmholtz frequency must
be measured as well. To obtain this information the transfer function between the pressure inside the air spring and the exerted force must be measured.

**Dynamics of the AVI**

Most important are the dynamics of the complete AVI system. The transfer function between the disturbances acting on the base of the AVI and the resulting accelerations of the supported mass must be measured. This will show what the performance of the system is and up to which frequency vibration isolation can be achieved.

To verify that the AVI design meets its design criteria, it must also be subjected to typical ambulance body vibrations.

The different transfer functions discussed above can be obtained by measuring the relevant input and output signals of the transfer functions of interest. Using and measuring bandwidth limited white noise as a random input to the system and measuring the output parameter of interest, the transfer function can be obtained using FRF techniques.

To simulate typical ambulance body vibrations, two experiments can be done. Simulation of the ambulance crossing a speed bump and the simulation of a ride over a rough road. When the ambulance drives over a speed bump, the ambulance body will start to vibrate in its (damped) eigenfrequency. This frequency is the main component of the vibrations that the AVI has to absorb. The eigenfrequency of the ambulance body can be calculated by:

\[ f_e = \frac{1}{2\pi} \sqrt{\frac{k}{m_b}}, \]  

(6.3)

and equals 1.5 Hz. The amplitude of this vibration should be 40 mm, since that is the stroke the AVI needs to make when the ambulance would drive over a 100 mm Watts profile.

The other experiment is to simulate the ride over a rough road. Road profiles have a slope of -20 dB/decade, which means that the intensity of the vibration is reduced when the frequency increases. The vibrations of the ambulance body show another resonance besides the body resonance. That is the resonance of the wheel vibrations that act on the AVI through the ambulance body. This resonance has a frequency of 12.3 Hz. At this frequency, the amplitude of the vibrations for a very rough road are approximately 20 mm.

**Measurement system**

**Sensor selection**

Sensors need to be selected to measure the signals discussed in the previous section. These sensors and their properties are listed in table 6.1. The linear shaft motor is capable of generating a force with a resolution of 0.14 N. The position of the AVI is indirectly measured, with a reduction of 1:100 (see section 6.2). The resolution and accuracy of the LVDT sensor must therefore be multiplied by a factor 100 to find the accuracy and resolution of the position signal of the load on the AVI.
Table 6.1: Test setup sensor information.

<table>
<thead>
<tr>
<th>Type</th>
<th>Force</th>
<th>Acceleration</th>
<th>Position</th>
<th>Position</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sensor</td>
<td>HBM-S9</td>
<td>Si-Flex S1500 capacitive sensor</td>
<td>Copley Controls Hall sensor</td>
<td>Solarton DP1 LVDT sensor</td>
</tr>
<tr>
<td>I/O</td>
<td>18-bit ADC</td>
<td>18-bit ADC</td>
<td>12-bit Quad</td>
<td>14-bit</td>
</tr>
<tr>
<td>Range</td>
<td>0 - 2 kN</td>
<td>-34 - 34 m/s²</td>
<td>0 - 51 mm</td>
<td>0 - 1 mm</td>
</tr>
<tr>
<td>Resolution</td>
<td>0.008 N</td>
<td>0.26 mm/s²</td>
<td>12 µm</td>
<td>60 nm</td>
</tr>
<tr>
<td>Accuracy</td>
<td>1 N</td>
<td>0.02 mm/s²</td>
<td>35 µm</td>
<td>0.1 µm</td>
</tr>
</tbody>
</table>

With the theoretical accuracy of all the sensors known, an estimate can be made of the accuracy of the measurements.

Under the assumption that the air spring stiffness equals 700 N/m, the error in the air spring stiffness measurement can be calculated. When the AVI makes a stroke of 50 mm, the maximum and minimum measured values, taking the accuracy of the force transducer and the LVDT’s into account are:

\[
\frac{\delta F_{\text{low}}}{\delta x_{\text{high}}} < k_{a,nom} < \frac{\delta F_{\text{high}}}{\delta x_{\text{low}}}
\]

\[
\frac{35 - 1}{(50 + 0.01) \cdot 10^{-3}} < 700 < \frac{35 + 1}{(50 - 0.01) \cdot 10^{-3}}
\]

\[
680 < 700 < 720 \text{ N/m.}
\]

The stiffness of the air spring can thus be measured with an accuracy of ± 20 N/m or 3 %. Analogously the stiffness of the o-rings can be measured. Assuming a stiffness of \(175 \cdot 10^{-3} \text{ N/m}\), and a stroke of 8 mm, the stiffness can be determined with an accuracy of ± 350 N/m or 0.4 %.

The friction force of the guiding wheels is lower than the accuracy of the force transducer. With a separate force transducer which has an accuracy of 0.1 N, the friction force of the wheels is measured to lie between 0.0 and 0.2 N. This is not the only friction present. The magnets in the shaft of the linear motor introduce a cogging force. This force is much larger than the friction force of the guiding wheels, and is measured to vary between -2.8 and +2.8 N. The controller of the linear shaft motor does not eliminate this cogging force by its internal controller. The cogging force can be measured and then be used as a feedforward signal to increase the performance of the system.

With a resolution of 0.26 mm/s², the accelerometer has a dynamic resolution of 100 dB. This is sufficient for the dynamic experiments, where the transfer functions discussed in the previous section are measured.

**Electronics**

To connect the sensors, the linear shaft motor and the pneumatic valves, a PC with a data acquisition card is used. The configuration of the system is shown in figure...
6.2. With the hardware it is possible to do real-time control using the Matlab and Simulink environment.

The static experiments can be conducted using the position signal from the LVDT’s and the force signal from the force transducer. The dynamic experiments can be conducted using the LVDT’s or the accelerometer as a feedback signal and the current for the linear shaft motor as an input signal. The Pneumatic valves can be used as an input for low frequent pneumatic control.

The next step will be to design dedicated electronics that have sensor and actuator electronics and the control algorithm integrated in one package. This custom board can be used drive the AVI in the ambulance. This version only needs the electronics to process the accelerometer signal, the linear shaft motor signal and the signals for the pneumatic valves. The force transducer and the LVDT’s are not needed for normal operation of the AVI.

Figure 6.2: Diagram of I/O connections of the test setup.
6.2 Test setup design

A setup is designed that will make it possible to do all the experiments described in section 6.1. This setup is shown in figure 6.3. The test setup will not have a reduction between the load and the AVI. This is to reduce the dimensions and the cost of the test setup. The other simplification is the rotation point that is fixed to the world, instead of moving together with the input disturbance position. Because of this simplification, only the bottom of the AVI has to be excited instead of the complete setup. The AVI will be loaded with a 100 kg mass. This mass consists of 15 steel strips, thus making it possible to vary the load on the AVI. These strips are fixed using bolts and nuts.

The mass is guided by a lever which rotates round a fixed point. The AVI is positioned below the center of gravity of the load. Since the AVI is constructed to accept only high loads in the vertical direction, two bearing blocks are fitted on the top and bottom to prevent any side forces or torques acting on the system. Only the friction force of the bearing blocks introduces forces perpendicular to the reciprocating motion direction, but these are well below the 40 N that internal guiding system of the AVI is capable to withstand.

The bottom of the AVI is also connected with a lever to the frame. Using these two levers, the load and the AVI itself are able to make vertical movements with little rotation round the horizontal axis. This way, a very simple yet efficient guiding is created that has little friction. The movement of the mass and the AVI will not be strictly vertical, but for small displacements the rotation and side motion can be
neglected. This is illustrated in figure 6.4, where the dimensions of the test setup are schematically shown in three configurations.

On the left the AVI is retracted to its minimum length and the eccenter is in its lowest position. In the middle drawing the setup is shown in its nominal position, e.g. the AVI at mid-stroke and the lower lever horizontally. In the right drawing the AVI is depicted in its extended position with the eccenter at its highest point. Note that the drawings are not to scale. With the chosen dimensions the maximum rotation of the AVI with respect to the vertical axis is 1.5° and there is enough space for the 100 kg mass on top of the lever.

The rotation of the top lever, and thus the displacement of the mass, is measured using two LVDT's. Since the LVDT's have a small stoke of 1 mm they are positioned close to the rotation axis. The two LVDT's are each positioned on one side of the center of the rotation axis. This way, the rotation of the lever can be measured differentially and the distance from the axis center to the point where the LVDT measures the position is not critical. The distance between the two LVDT's is critical for the measurement. Since these are positioned close to each other and in one body, it is not difficult to position the LVDT's accurately. This is depicted in figure 6.5.

The accelerations of the mass will be measured with the accelerometer. This sensor will be located on the top of the AVI, as can be seen in figure 6.6. The sensor will measure the vertical accelerations of the top of the AVI and thus also the mass.

To introduce a disturbance to the test setup, an eccenter is used. This eccenter is driven by an electric motor coupled to a reduction. The torque of the motor is transferred to the input shaft of the AVI, where it is acting as a disturbance displacement. This will introduce a sign-changing torque to the axis. The shaft is
also subject a bending force. Therefore it is important to determine the minimal diameter of the axis. These calculations are made in appendix D.1. The equivalent bending moment is 80 Nm. This requires a minimal axis diameter of 25 mm of normal construction steel to prevent fatigue. In the test setup a shaft with 30 mm diameter is used for extra safety margin. This part of the test setup is shown in figure 6.7.

As mentioned in section 6.1, the eccentric excites the system with a 1.5 Hz and with a 12.3 Hz signal, to simulate a bump and a rough road. The movement of the AVI is not strictly vertical, but also small horizontal movements are present. These horizontal movements introduce forces on the internal guiding system of the AVI due to the inertia of the masses. The forces that are introduced to the AVI due to these horizontal movements are calculated in appendix D.2.

For the given experiments, the maximum force on the guiding equals 14.6 N. The AVI guiding can withstand forces up to 40 N. The lowest eigenfrequency of the guiding equals 244 Hz (see appendix D.3). This ensures that no internal resonances of the AVI occur when the given experiments are conducted because this
frequency is higher than the excitation frequency of 12.3 Hz.

To determine the force-displacement characteristic, the forces generated by the AVI need to be measured. To do this, the eccenter is replaced by a force transducer, as depicted in figure 6.8. This way the force and displacement of the AVI can be measured.

A thick stud bolt is used to protect the AVI is some error or failure occurs. With nuts the movement of the top lever can be limited to the desired range. The static force that the stud bolt can carry equals:

$$F_b = \frac{\pi^2 EI}{l_b^2}.$$  \hfill (6.4)

Where $E$ is the modulus of elasticity of the material, $I$ is the second moment of area of the stud bolt and $l_b$ is the equivalent beam length. For the stud bolt, the maximum buckling force equals 14.5 kN.

### 6.3 Dynamics of the test setup

The test setup that is designed to verify the actuator performance must represent the ambulance system as close as possible. As mentioned before, to reduce costs and dimensions of the test setup some simplifications are made. To determine how this influences the dynamic behavior of the test setup, a comparison between two models is made.

In figure 6.9 the dynamic model of the ambulance (model I) is shown, without the wheel and road interaction. This model can be used when the input $u$ represents the body movement $z_b$ instead of the road levitation $z_r$. The test setup can be modeled according to figure 6.10 (model II). The main difference is that the point of rotation of the lever is fixed to the world, whereas in the ambulance its movement is equal to the movement of the ambulance body.
The equations of motion for both models are derived in appendix D.4. The resulting equations of motion are described below:

Model I

\[
m_s l^2 r^2 \dddot{\theta} - m_s l r \dddot{u} + k_a l^2 \theta + c_a l^2 \dot{\theta} - F_{LM} l = T. \tag{6.5}
\]

Model II

The equations of motion of model II can be written as:

\[
m_s l^2 r^2 \dddot{\theta} + k_a (l^2 \theta + lu) + c_a (l^2 \dot{\theta} + l \dot{u}) - F_{LM} l = T. \tag{6.6}
\]
Model comparison

Using the Laplace transform of the equations of motion for both models, the transfer functions of the input $u$ to the stretcher acceleration $\ddot{z}_s$ can be calculated (see appendix D.4).

The reduction $r$ for the test setup is chosen to be one, and the arm length $l$ equal to 0.34 m. This is done from a practical point of view, to keep the dimensions and cost of the test setup low.

The open loop transfer functions of the two models described above (with $r = 1$ and $l=0.34$ m) and the uncontrolled model of the ambulance are shown in figure 6.11. As can be seen, due to the different air spring stiffness and reduction $r$ of the different models, the eigenfrequency of the air spring and stretcher mass system has different values, varying from 0.33 - 0.42 Hz.

Note that in the low frequent region all models behave the same, and that the behavior of models I and II only differs for frequencies of approximately 2 Hz and higher. Below this frequency both models I and II behave identically.

Analogous to section 5.3, the closed loop transfer functions of the two models of the test setup, model I and II, are determined using the skyhook controller with the leveling action to regulate the electromagnetic force $F_{LM}$.

In figure 6.12 the transfer functions of these systems are plotted when the skyhook controller is applied. This is done for model I and II and for the ambulance model. Looking at these controlled systems and at the uncontrolled systems, some remarks can be made:

- The dynamics of all models are the same in the low frequent region for the uncontrolled models.
The resonances of the body mass and the wheel mass of the ambulance model are, logically, not present for both models I and II.

Model I and II have the same eigenfrequency and start to differentiate at frequencies of 2 Hz and higher.

The skyhook controller with the leveling term causes a difference in the low frequent region between model II and model I.

The transfer function of the ambulance model has a negative slope at high frequencies, while model I and II have positive slopes.

The test setup is designed to simulate the behavior of the ambulance dynamics. As can be seen in figure 5.10, for the rigid ambulance system, the transmission of road irregularities to the ambulance body is highest between 0.3 and 20 Hz. Lower and higher frequencies are mostly filtered out by the dynamics of the tyre stiffness and wheel mass and the body suspension and body mass.

Due to the fact that for the ambulance system, the ambulance body will mostly transmit vibrations in this frequency region, it would be favorable if the test setup would have a low vibration transmission outside this region. From that point of view, the test setup, model II, shows better behavior than model I in the higher frequency region because it better coincides with the ambulance model transfer function. The two dynamic tests discussed in section 6.1 will excite the system at 1.5 Hz and at 12.3 Hz. At these frequencies the dynamics of model II, the test setup model, are better matched to the ambulance system than model I.

Below approximately 0.2 Hz, the dynamics of the controlled model II are different. This is caused by the leveling term of the skyhook controller in combination with the fixed point of rotation of the lever. In the ambulance model and model

---

**Figure 6.11:** Bode diagram of open loop transfer functions. Solid: $H_{\ddot{z}, z}$ ambulance system, dashed: $H_{\ddot{z}, u}$ model I, dotted: $H_{\ddot{z}, u}$ model II.
I, the two terms that define the skyhook controller force, the skyhook damping term and the leveling term, oppose each other. However, in model II, the leveling term amplifies the damping term, since both these term aim to minimize the lever movement. This explains why the skyhook controlled model II has better isolation in the low frequent region, where the leveling term is dominant over the damping term. Since no experiments are performed in this frequency range, this is considered to be no issue.

From this analysis it can be said that with the proposed test setup it is possible to determine the potential performance of the actuator, although the dynamics of the system are not identical to that of the ambulance model. The air spring has a different eigenfrequency, and the low (<0.3 Hz) and high (>20 Hz) frequent behavior is different. However, to identify the AVI’s potential performance this rudimental test setup will give adequate results.
Chapter 7

Full ambulance model simulation

7.1 Full ambulance model

The design and analysis of the AVI are based on a quarter car model. As mentioned in section 3.1, there are good arguments to choose this approach. The AVI is designed with one degree of freedom in mind, the movements along the vertical axis. As mentioned in section 2.2, the dominant vibrations in a car or ambulance are in the vertical direction. It is interesting to verify the performance of the AVI, which is designed with one degree of freedom in mind, in a 3D world, with six degrees of freedom.

For this purpose, a full vehicle model of the ambulance is created using the SimMechanics toolbox in Matlab. The dimensions of the ambulance model are depicted in figure 7.1. The parameters of the model are listed in table 7.1. The parameters and dimensions are derived from a VW T5 Tyche ambulance [57].

![Figure 7.1: 3D Model of the ambulance. Dimensions in m.](image)
The parameters of the ambulance suspension and the AVI properties are identical to those used for the quarter car ambulance model. A comparison between the full ambulance model and the quarter ambulance model is made in appendix E.1. The quarter ambulance model shows good resemblance with the full ambulance model.

<table>
<thead>
<tr>
<th>Description</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wheel mass</td>
<td>( m_w )</td>
<td>67 kg</td>
</tr>
<tr>
<td>Tyre springrate</td>
<td>( k_w )</td>
<td>400 kN/m</td>
</tr>
<tr>
<td>Body mass</td>
<td>( m_b )</td>
<td>3600 kg</td>
</tr>
<tr>
<td>Suspension springrate</td>
<td>( k_b )</td>
<td>80 kN/m</td>
</tr>
<tr>
<td>Suspension damping coefficient</td>
<td>( c_b )</td>
<td>6000 Ns/m</td>
</tr>
<tr>
<td>Stretcher mass</td>
<td>( m_s )</td>
<td>100 kg</td>
</tr>
<tr>
<td>Air spring springrate</td>
<td>( k_a )</td>
<td>4000 N/m</td>
</tr>
<tr>
<td>Air spring damping coefficient</td>
<td>( c_a )</td>
<td>10 Ns/m</td>
</tr>
<tr>
<td>Reduction</td>
<td>( r )</td>
<td>3 -</td>
</tr>
<tr>
<td>Lever length</td>
<td>( l \cdot r )</td>
<td>0.6 m</td>
</tr>
<tr>
<td>Lever mass</td>
<td>( m_{red} )</td>
<td>6 kg</td>
</tr>
</tbody>
</table>

Table 7.1: 3D ambulance model parameters.

The reduction is modeled with a lever system. The stretcher mass is guided by a parallelogram that only allows motion of the stretcher in the direction perpendicular to the base of the ambulance body. Translations in the other two directions and the rotations along any of the axis are not possible. It is assumed that joints of the parallelogram are frictionless. This system is depicted in figure 7.2. The tyres of the ambulance are 215/65R16 tyres, which are modeled with the MF-Tyre & MF-Swift 6.1 software, using estimated values.

![Figure 7.2: 3D Model of the stretcher guiding.](image)

The performance of the ambulance with the active suspension is compared to a model of a conventional ambulance and an ambulance with a passive suspension located between the body and stretcher mass. In the conventional model, the stretcher is rigidly attached to the ambulance body. For the passive suspended
model, the air spring is positioned under the center of gravity of the stretcher mass. An extra damper is added to the air spring, tuned such that $\xi = 0.35$ to provide adequate damping of the vibrations. All other parameters of the three ambulance models are identical.

The accelerations of the stretcher mass are compared, and the comfort rating is calculated using the acceleration weighting functions described in ISO 2631 [24]. Acceleration weighting is applied for the vertical vibrations as well as for the vibrations that cause motion sickness. These weighting functions can be found in appendix E.2. The comfort rating depending on the comfort index is shown in table 7.2.

<table>
<thead>
<tr>
<th>Acceleration [m/s²]</th>
<th>Scale of discomfort (suggested by ISO 2631)</th>
</tr>
</thead>
<tbody>
<tr>
<td>&lt; 0.315</td>
<td>Not uncomfortable</td>
</tr>
<tr>
<td>0.315-0.63</td>
<td>A little uncomfortable</td>
</tr>
<tr>
<td>0.5-1</td>
<td>Fairly uncomfortable</td>
</tr>
<tr>
<td>0.8-1.6</td>
<td>Uncomfortable</td>
</tr>
<tr>
<td>1.25-2.5</td>
<td>Very uncomfortable</td>
</tr>
<tr>
<td>&gt; 2</td>
<td>Extremely uncomfortable</td>
</tr>
</tbody>
</table>

Table 7.2: Comfort ratings.

### 7.2 Simulations

With these models, two simulations are made, a ride with a discrete event, e.g., a speed bump and a ride over a road with random irregularities, e.g., a rough road. To control the active suspension of the ambulance model, the skyhook controller discussed in chapter 5 is used.

For each event, the minimal or maximum velocity that can be reached to traverse the bump or road while the stretcher vertical accelerations remain in the ‘not uncomfortable’ category.

#### Speed bump simulation

For this simulation a speed bump with a Watts-profile shape is used. The height of the bump is 100 mm. The design velocity for this speed bump is 25 km/u. The speed bump profile is shown in figure 7.3.

In table 7.3 the results are listed for the simulation where the ambulance drives over the speed bump at its design velocity of 25 km/u. As can be seen, the stretcher accelerations for the rigid case are high. The comfort index is 1.10 m/s² for the rigid case, which can be identified as ‘uncomfortable’ using the criteria of ISO 2631. For the rigid case, a rating of ‘not uncomfortable’ for the vertical stretcher accelerations is achieved when the velocity of the ambulance is reduced to 10 km/u.

The passive system reduces the vibrations by approximately a factor 2.5 compared
to the rigid case. The comfort index falls just in the 'little uncomfortable' category, but vertical stretcher accelerations peak at 3.2 m/s². The active system is able to reduce the stretcher vertical vibrations by approximately a factor 25 compared to the rigid case, with peak accelerations being not larger than 0.3 m/s².

### System Comparison

<table>
<thead>
<tr>
<th>System</th>
<th>Rigid</th>
<th>Passive</th>
<th>Active</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\ddot{z}_s$ (rms)</td>
<td>1.45</td>
<td>0.66</td>
<td>0.07</td>
</tr>
<tr>
<td>Comfort index</td>
<td>1.10</td>
<td>0.34</td>
<td>0.03</td>
</tr>
<tr>
<td>Motion sickness index</td>
<td>0.03</td>
<td>0.05</td>
<td>0.02</td>
</tr>
<tr>
<td>$\ddot{z}_s$ (peak)</td>
<td>7.6</td>
<td>3.2</td>
<td>0.3</td>
</tr>
<tr>
<td>Stroke (peak)</td>
<td>-</td>
<td>80</td>
<td>27</td>
</tr>
<tr>
<td>$F_{LM}$ (peak)</td>
<td>-</td>
<td>-</td>
<td>79</td>
</tr>
</tbody>
</table>

Table 7.3: Simulation results for a ride over a speed bump, 25 km/u.

The ambulance with the active suspension system is able to cross the speed bump with 50 km/u, while the comfort index still remains in the 'not uncomfortable' category and the peak acceleration is 0.5 m/s². Velocities higher than twice the design velocity of the speed bump are not investigated because it is assumed that such a high velocity is not safe in an environment where 25 km/u speed bumps are present.

**Road simulation**

Besides discrete road irregularities, the behavior of the stretcher mass when the ambulance is subject to random road irregularities is also of interest. For these simulations, two roads are considered, which are depicted in figure 7.4. These road profiles come from an actual measured road profile which is scaled to increase or decrease the roughness. Power spectral analysis shows that the rough profile falls in the roughness class of D (poor) and E (very poor) and the smoother profile varies between the roughness categories A (very good), B (good) and C (average). This can be seen in figure 7.5.

The rough road simulations are made with a velocity of 40 km/u, which is considered an average velocity when traveling over a poor to very poor road. The results of these simulations are shown in table 7.4. As can be seen, the rigid ambulance comfort index falls in the ‘uncomfortable’ to ‘very uncomfortable’ category. The
passive and active system both fall in the 'not uncomfortable' category. The passive system has a factor 8 higher comfort index and high peak accelerations compared to the active system.

When the rigid ambulance model is simulated with a velocity of 10 km/u, the comfort index almost equals 0.315 m/s² and is thus 'not uncomfortable'. The passive system is not able to transverse the road at velocities higher than 40 km/u without exceeding the comfort criterium. The ambulance model with the AVI system is capable to undertake this road with a velocity of 80 km/u. Again, higher velocities are not investigated because it is assumed that such high velocities are not safe.

The smooth road, that varies between the 'average' and 'very good' classification along its frequency range, is considered as an average road, which is normally traveled with a velocity of 80 km/u. With this velocity the active and passive ambulance systems again show good vibration isolation whereas the comfort index of the rigid ambulance model is high, in the 'uncomfortable' category. To make the rigid am-
Table 7.4: Simulation results for a ride over a rough road, 40 km/u.

<table>
<thead>
<tr>
<th>System</th>
<th>Rigid</th>
<th>Passive</th>
<th>Active</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\ddot{z}_s$ (rms)</td>
<td>1.73</td>
<td>0.40</td>
<td>0.12</td>
</tr>
<tr>
<td>comfort index</td>
<td>1.68</td>
<td>0.25</td>
<td>0.03</td>
</tr>
<tr>
<td>motion sickness index</td>
<td>0.07</td>
<td>0.08</td>
<td>0.09</td>
</tr>
<tr>
<td>$\ddot{z}_s$ (peak)</td>
<td>11.3</td>
<td>2.4</td>
<td>0.5</td>
</tr>
<tr>
<td>stroke (peak)</td>
<td>-</td>
<td>72</td>
<td>87</td>
</tr>
<tr>
<td>$F_{LM}$ (peak)</td>
<td>-</td>
<td>-</td>
<td>205</td>
</tr>
</tbody>
</table>

The different simulations show similar results. With the rigid ambulance model it is not possible to address the bumps or road irregularities at their nominal velocity if it is desired to provide high comfort to the patient in the ambulance. If the vibrations of the patient have to be below the ‘not uncomfortable’ criterium that is discussed in ISO 2631, the ambulance must reduce its velocity. The required velocity reduction is quite large, ranging from 15 to 60 km/u or 60 to 75 % of the nominal velocity. This means that the ambulance has to travel at significantly lower velocities and thus it takes more time to reach the hospital from a trauma scene.

The ambulance model with the active suspension system is able to bring the comfort index well below the ‘not uncomfortable’ criterium when the road irregularities are overcome. With the active suspension system, the ambulance model is able to stay below this criterium even when the velocity is increased. This means that the ambulance is able to travel faster to the hospital than the rigid or passive ambulance systems.

Observations

The different simulations show similar results. With the rigid ambulance model it is not possible to address the bumps or road irregularities at their nominal velocity if it is desired to provide high comfort to the patient in the ambulance. If the vibrations of the patient have to be below the ‘not uncomfortable’ criterium that is discussed in ISO 2631, the ambulance must reduce its velocity. The required velocity reduction is quite large, ranging from 15 to 60 km/u or 60 to 75 % of the nominal velocity. This means that the ambulance has to travel at significantly lower velocities and thus it takes more time to reach the hospital from a trauma scene.

The ambulance system with the passive suspension provides on average a factor 3 to 4 better comfort rating compared to the rigid model. The remaining vibrations for most scenario’s fall just in the ‘not uncomfortable’ category when the roads or bumps are crossed at their nominal velocity. The peak vibrations of the passive system are still very high.

The ambulance model with the active suspension system is able to bring the comfort index well below the ‘not uncomfortable’ criterium when the road irregularities are overcome. With the active suspension system, the ambulance model is able to stay below this criterium even when the velocity is increased. This means that the ambulance is able to travel faster to the hospital than the rigid or passive ambulance systems.
If an accident has occurred 5 km from a hospital and it is assumed that an ambulance with the active suspension system is able to travel from the trauma scene to the hospital with an average velocity of 50 km/u. Then the ambulance will take 6 min to reach the hospital. If an ambulance without the (active-) suspension system is used, it is assumed to travel 60% slower on average, at 20 km/u. So this ambulance would take 15 minutes to reach the hospital. This means that by using the active stretcher suspension system the time to reach the hospital is significantly reduced.

Some remarks must be made. First of all, the modeling of the active suspension system is rather basic. Sensor dynamics, actuator bandwidth limitation and friction in the guiding system are not accounted for. These will aggravate the performance of the active ambulance stretcher suspension system. The passive suspension system is also modeled with an idealized, frictionless guiding system. The simulation results mentioned above should therefore be regarded as indicating the potential performance of the systems.

Secondly, the reader should be aware that at present day, ambulances travel faster from the trauma scene to the hospital than the velocities mentioned above where the ambulance system with no suspension is able to reach a ‘not uncomfortable’ comfort index. In practice, the ambulance personnel must make a compromise between the comfort of the patient and the necessity to quickly reach the hospital.

Finally, the comfort index is a guideline and the perceived comfort of a passenger is not only dependent on the vertical accelerations but is also dependent on the expectations of the subject and the physique of the subject (e.g., mass, body dimensions). Furthermore, it can readily be assumed that a wounded, traumatized person is more sensitive to vibrations than a healthy subject. However, no data is available nor is any research done that gives insight in the correlation between the severity of the trauma and the sensitivity to vibrations.
Chapter 8

Conclusions and recommendations

As mentioned in the introduction of this work, the goal of this research is to:

"Design and test an active vibration isolator and control strategy that reduce patient vibrations in an ambulance to an acceptable level."

The results of this research and the conclusions from these results are discussed below. After the conclusions, the recommendations for further work are treated.

8.1 Conclusions

The level of vibrations that a patient is subjected to in an ambulance is often too high. Measures have to be taken by the ambulance personnel to reduce these vibrations. This is done by either reducing the velocity of the ambulance or by deviating from the quickest or shortest route to avoid speed bumps and other obstacles that cause high vibration peaks. These vibrations are most severe in the vertical direction. The human body is most sensitive to vibrations in the range from 0.1 to 80 Hz, and it is within this range that the dominant vertical vibrations of an ambulance stretcher occur.

It is possible to design an Active Vibration Isolator (AVI) that is able to reduce the stretcher vertical vibrations by approximately a factor 30. The concept of the AVI design is to use a pneumatic force to support the mass and an electromagnetic force as an active element. This results in a simple design, with low energy consumption and possibly low wear. Using a reduction of 1:3 between the stretcher mass and the AVI results in less required working space, whilst maintaining good performance of the AVI.

The performance of the AVI can be tested with a test setup that is designed. This test setup uses two levers, one at the top to guide the mass loaded onto the AVI and one at the bottom to guide the disturbances introduced to the AVI. The AVI is positioned between these two levers using two plummer blocks. Disturbances are
introduced to the bottom of the AVI using an electric motor with an eccentric. This test setup differs from the ambulance system because no reduction is used (e.g. the center of gravity of the load is positioned directly above the AVI) in the test setup and the disturbances are introduced to the AVI in a different manner (the rotation point of the top lever does not move along with the disturbances introduced at the bottom). These differences between the test setup and the real ambulance model introduce no significant changes when the performance of the AVI is verified.

A control algorithm that uses a skyhook damping term and a leveling term is well suited for the control of the AVI. This algorithm gives a good trade off between the required working space and the vibration isolation capabilities of the system. This controller requires input signals that can easily be measured inside the ambulance. The downside of this controller is that it has low performance in the frequency region where the magnitude of the skyhook damping term and the leveling term intersect.

In a full, 6-dof ambulance model the AVI and the control algorithm provide good isolation of the vertical stretcher mass vibrations. The vertical stretcher vibrations of this system are a factor 30 lower compared to a normal ambulance. This means the ambulance does not have to slow down or take alternative routes to reach the hospital, thereby reducing the time to reach the hospital by 60 to 70%.

An ambulance equipped with a passive suspension system between the ambulance body and the stretcher mass provides reasonable vibration isolation, reducing the vertical stretcher mass vibrations by a factor 5 on average.

8.2 Recommendations

This report deals with the question of how the vibrations that patients are subjected to in ambulances can be reduced. An Active Vibration Isolator (AVI) is designed and a control strategy is formulated. Simulations are made and a test rig is designed to verify the performance of the AVI by experiment. In the future, several actions can and must be taken to improve the AVI performance and to take the next steps that lead to the development of a prototype.

To improve the performance of the AVI, other control techniques can be implemented. The main goal should be to reduce the impact of the trade off between the required working space and the level of vibration reduction. The skyhook controller presented in this report provides a good balance between the required working space and the isolation of vibrations. However, the controller performance can be improved in the frequency region where the leveling and damping term of the skyhook controller cancel each other out. This can be done by working with non-linear control techniques, e.g. make the gain of the leveling action dependent on the position of the AVI or by adding (inverse-)notch filters. Important is to provide good vibration reduction at the eigenfrequency of the air spring and strong leveling when the AVI reaches the extremes of its stroke length while maintaining system stability.

In this report only the design of the AVI itself is treated. A construction must
be designed around the AVI which can be built into an ambulance. Special attention must be paid to create a guiding system with a low friction force to take full advantage of the AVI performance. This can achieved by using a parallelogram for instance. Another goal can be to integrate this construction below and within the bounds of the ambulance stretcher table. This prevents anything from protruding that can harm the patient or ambulance personnel or obstruct the ambulance attendee from taking proper care of the patient. A prototype can be built to do road tests to measure real world performance of the AVI system.

In this report only vertical vibrations are considered. Vibrations and rotations in other directions could be reckoned with as well. High disturbance forces occur when an ambulance is braking, accelerating or cornering. These are in general motions with a lower frequency. A study can be made to investigate the possibilities how two or three AVI’s or one AVI assisted by other actuators, springs and/or dampers can be used to control multiple degrees of freedom and thus reducing the vibrations that patients are subjected to even further, controlling vertical accelerations, roll and pitch.

Finally, when the AVI prototype will be produced in larger quantities (100+), the development of a custom air spring and a customized electromagnetic motor will help to attain an even simpler design. This can help to reduce the costs because less custom parts will be needed and to simplify the assembly of the Active Vibration Isolator.
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Appendix A

Quarter ambulance model

A.1 Air spring analysis

From [58] the stiffness $k$ of a pneumatic cylinder can be derived. The thermal time constant is expected to be in the orders of minutes, so all movements can be seen as dynamic in a thermo-dynamical sense. In that case the process is seen as adiabatic. The pressure in the cylinder $p_c$ and the volume $V_c$ of the cylinder for a given displacement $\delta x$ are related to the static pressure $p_0$ and the initial volume $V_0$ by:

$$p_c V_c^\gamma = p_0 V_0^\gamma,$$

(A.1)

where

$$V_c = V_0 + A_c \delta x.$$  

(A.2)

With $A_c$ the effective cylinder piston area and $\gamma$ the specific heat ratio of air. The stiffness of the spring is given by the rate of change of force with respect to spring displacement:

$$k = -\frac{d}{d(\delta x)}(p_c A_c).$$

(A.3)

Using (A.1) and (A.2), this can be rewritten as:

$$k = \frac{\gamma p_0 A_c^2}{V_0} \left( \frac{1}{1 + \frac{A_c \delta x}{V_0}} \right)^{\gamma+1}.$$ 

(A.4)

If the small non-linear term (in the brackets) is ignored, this can be rewritten as:

$$k = \frac{\gamma p_0 A_c^2}{V_0}.$$ 

(A.5)

For best vibration isolation it is essential that the air spring will have a low stiffness. From (A.5) it can be seen that to lower the air spring stiffness, either the internal pressure or the effective area have to be reduced or the volume of the air spring has to be increased. Since the air spring force is defined by:

$$F = (p_c - p_a) A_c,$$

(A.6)
where $p_a$ is the ambient pressure. Keeping the force of the air spring constant requires $p_c A_c$ to be constant. Without changing the medium, the air spring stiffness can be changed by adjusting $A_c / V_0$. Thus decreasing the air spring effective area or increasing the air spring volume. Decreasing the effective air spring area $A_c$ is bounded by the maximum internal pressure $p_c$ of the air spring since the force $p_c A_c$ must be constant. The only option that remains when the maximum internal pressure is reached, is to increase the volume $V_0$ of the air spring. To increase the volume, a buffer tank can be connected to the air spring by means of a capillary tube. This is shown in figure A.1.

![Figure A.1: Pneumatic cylinder with buffer volume.](image)

The extra buffer volume will lower the air spring stiffness, but will also introduce damping to the system because the air has to flow from the air spring volume to the buffer volume.

### Capillary damping

A capillary restriction can be formed in a number of ways. Typically, parallel plates \[58\], tubes \[59\], or sections of foam \[60\] are used. When the flow of air through a capillary is laminar, the equations modeling it can be linearized and analysis is simple.

For a restriction formed by a tube of diameter $d_c$ and length $L_c$, the capillary resistance $C_R$ is given by:

$$C_R = \frac{\pi d_c^4}{128 \mu L_c}.$$ \hspace{1cm} (A.7)

This is valid when $L_c / d_c > 10$ and $Re < 2300$. 

98
A capillary-damped isolator can be modeled by an equivalent system which consists of two linear springs and a viscous damper, shown in figure A.2 [62]. The spring stiffnesses are given by:

\[ k_1 = \frac{\gamma p_0 A_c^2}{V_c 0}, \tag{A.8} \]

and

\[ k_2 = \frac{\gamma p_0 A_t^2}{V_t}, \tag{A.9} \]

and the damping coefficient is given by:

\[ c_c = \frac{A_c^2}{C_R}. \tag{A.10} \]

When the capillary resistance is low, the isolator behaves as a low frequency system which is virtually undamped. Conversely, if the capillary resistance is high, the surge tank has virtually no effect; the isolator stiffness is high, and there is very little damping. This means that at low excitation frequencies, the effective system volume is that of the tank and cylinder, and the isolator stiffness \( k \) is low:

\[ k(\omega \to 0) = \frac{\gamma p_0 A_c^2}{V_c 0 + V_t}. \]

At higher frequencies, there is minimal flow between the cylinder and tank and the tank is effectively blocked off from the cylinder. As a result, the air spring behaves as a stiffer system with little damping:

\[ k(\omega \to \infty) = \frac{\gamma p_0 A_t^2}{V_c 0}. \]

In [58] it is shown that this undamped behavior at high frequencies is advantageous and gives rise to an attenuation rate of 40 dB/decade, whereas the attenuation rate for a conventional isolation system (linear spring and viscous damper in parallel) is 20 dB/decade.

The air in the capillary between the spring and the buffer starts to resonate at a
given frequency, the so-called Helmholtz frequency. This frequency can be calculated by:

$$f_{Hs} = \frac{v_c}{2\pi} \sqrt{\frac{A_c}{V_c l_c}}, \quad (A.11)$$

where $v_c = \sqrt{\frac{\gamma p_0}{\rho}}$ is the velocity of sound inside the capillary.

For the initial ambulance model the air spring stiffness is calculated using (A.5) and the damping coefficient is calculated by using (A.10).

With a cylinder volume of 2.6 liter, a buffer volume of 15 liter, a capillary tube with a diameter of 7.5 mm and a length of 500 mm, the air spring stiffness and damping can be calculated. These properties are listed in table A.1 below.

<table>
<thead>
<tr>
<th>Equation</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k_1$</td>
<td>4970</td>
<td>N/m</td>
</tr>
<tr>
<td>$k_2$</td>
<td>865</td>
<td>N/m</td>
</tr>
<tr>
<td>$k$</td>
<td>737</td>
<td>N/m</td>
</tr>
<tr>
<td>$c_c$</td>
<td>8.9</td>
<td>Ns/m</td>
</tr>
<tr>
<td>$M$</td>
<td>100</td>
<td>kg</td>
</tr>
</tbody>
</table>

Table A.1: Air spring parameters.

To simplify the modeling of the air spring it is modeled as a single spring with a damper in parallel. The stiffness of this single spring is the air spring stiffness $k$. The damping coefficient is the capillary damping term $c_c$. Because the air spring has little damping, these two models are almost identical. In figure A.3 the bode diagram is shown for the capillary damping model and the simplified air spring model. As can be seen, the capillary damping model has better damping from frequencies of 50 Hz and higher. In the frequency region of 0.1 to 40 Hz, which is of interest for this research, the two models are identical.
Figure A.3: Bode diagram of the two air spring models. Solid: Simplified model, dashed: capillary model.
A.2 Equations of motion

The equations of motion of the quarter car ambulance model (depicted below, identical to figure 3.1) can be described by:

\[ m_w \ddot{z}_w + k_w(z_w - z_r) - k_b(z_b - z_w) - c_b(\dot{z}_b - \dot{z}_w) = F_1. \]  
(A.12)

\[ m_b \ddot{z}_b + k_b(z_b - z_w) + c_b(\dot{z}_b - \dot{z}_w) - k_a(z_a - z_b) - F_{LM} = F_2. \]  
(A.13)

\[ m_s \ddot{z}_s + k_a(z_a - z_b) + c_a(\dot{z}_a - \dot{z}_b) - F_{LM} = F_3. \]  
(A.14)

These equations of motion can be rewritten in matrix form, giving:

\[ \dot{\psi} = A\psi + Bu, \]  
(A.15)

\[ y = C\dot{\psi} + Du. \]  
(A.16)

Where:

\[ \psi = \begin{bmatrix} z_w & z_b & z_s \\ \dot{z}_w & \dot{z}_b & \dot{z}_s \end{bmatrix}^T, \]

\[ A = \begin{bmatrix} O & I \\ -M_s^{-1}K_s & -M_s^{-1}C_s \end{bmatrix}, \]

\[ B = \begin{bmatrix} O \\ -M_s^{-1}F \end{bmatrix}; \]

\[ M_s = \begin{bmatrix} m_w & 0 & 0 \\ 0 & m_b & 0 \\ 0 & 0 & m_s \end{bmatrix}, \]

\[ K_s = \begin{bmatrix} k_w + k_b & -k_b & 0 \\ -k_b & k_b + k_a & -k_a \\ 0 & -k_a & k_a \end{bmatrix}, \]

\[ C_s = \begin{bmatrix} c_b & -c_b & 0 \\ -c_b & c_b + c_a & -c_a \\ 0 & -c_a & c_a \end{bmatrix}, \]

\[ F = \begin{bmatrix} k_w & 0 \\ 0 & -1 \end{bmatrix}, \]

\[ u = \begin{bmatrix} z_r \\ F_{LM} \end{bmatrix}, \]

and \( C \) is a 6x6 identity matrix and \( D = 0 \).

This model is used to run simulations of the different bump and road profiles. The results of these simulations are listed in the figures in the next section (section A.3).
Figure A.4: Quarter car ambulance model.
### A.3 Simulation results

<table>
<thead>
<tr>
<th>#</th>
<th>Profile</th>
<th>Velocity (km/u)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Watts, 100 mm</td>
<td>25</td>
</tr>
<tr>
<td>2</td>
<td>Watts, 75 mm</td>
<td>30</td>
</tr>
<tr>
<td>3</td>
<td>Seminole, 100 mm</td>
<td>40</td>
</tr>
<tr>
<td>4</td>
<td>Seminole, 75 mm</td>
<td>45</td>
</tr>
<tr>
<td>5</td>
<td>Sine-shaped, 80 mm</td>
<td>20</td>
</tr>
<tr>
<td>6</td>
<td>Sine-shaped, 120 mm</td>
<td>20</td>
</tr>
<tr>
<td>7</td>
<td>Sine-shaped, 80 mm</td>
<td>30</td>
</tr>
<tr>
<td>8</td>
<td>Sine-shaped, 120 mm</td>
<td>30</td>
</tr>
<tr>
<td>9</td>
<td>Sine-shaped, 80 mm</td>
<td>50</td>
</tr>
<tr>
<td>10</td>
<td>Sine-shaped, 120 mm</td>
<td>50</td>
</tr>
<tr>
<td>11</td>
<td>Sine-shaped, 80 mm</td>
<td>60</td>
</tr>
<tr>
<td>12</td>
<td>Sine-shaped, 120 mm</td>
<td>60</td>
</tr>
</tbody>
</table>

| A  | Road class A                    | 120             |
| B  | Road class B                    | 100             |
| C  | Road class C                    | 80              |
| D  | Road class D                    | 60              |
| E  | Road class E                    | 40              |
| F  | Road class F                    | 20              |
| G  | Road class G                    | 10              |
| H  | Road class H                    | 5               |

Table A.2: Simulated scenario’s.

![Graphs of performance criteria](image)

Figure A.5: Peak values of the performance criteria for the bump profile simulations listed in table A.2.
Figure A.6: RMS values of the performance criteria for the road profile simulations listed in table A.2.

Figure A.7: Peak values of the required actuator acceleration and velocity for the different road and bump profile simulations listed in table A.2.
A.4 Lagrange equations of motion

The Lagrange equations of motion of a dynamic system can be described by:

\[
\frac{d}{dt} \left( \frac{\partial T}{\partial \dot{q}_i} \right) - \frac{\partial T}{\partial q_i} + \frac{\partial U}{\partial q_i} + \frac{\partial D}{\partial \dot{q}_i} = F_i, \quad (A.17)
\]

where:

- \( T \) is the kinetic energy,
- \( U \) is the potential energy,
- \( D \) is the dissipative energy and
- \( F \) are the external forces or torques acting on the system.

The dynamics of the system shown in figure A.8 can be described by the following generalized coordinates:

\[
q = \begin{bmatrix} z_w \\ z_b \\ \theta \end{bmatrix}, \quad (A.18)
\]

With these coordinates the original coordinates of the system can be rewritten:

\[
\begin{align*}
z_s &= z_b - lr \theta, \\
z_{red} &= z_b - \frac{lr}{2} \theta, \\
z_a &= z_b - \theta.
\end{align*} \quad (A.19)
\]

Using these coordinates the kinetic energy of the system \( T \) can be described by:

\[
T = \sum_{i=1}^{N_m} \frac{1}{2} m_i z_i^2,
\]

\[
= \frac{1}{2} m_w z_w^2 + \frac{1}{2} m_b z_b^2 + \frac{1}{2} m_s \left( \dot{z}_b - lr \dot{\theta} \right)^2 + \frac{1}{2} m_{red} \left( \dot{z}_b - lr \frac{1}{2} \dot{\theta} \right)^2,
\]

\[
= \frac{1}{2} m_w z_w^2 + \frac{1}{2} \left( m_b + m_s + m_{red} \right) z_b^2 + \frac{1}{2} l^2 r^2 \left( m_s + \frac{1}{4} m_{red} \right) \dot{\theta}^2 - lr \left( m_s + \frac{1}{4} m_{red} \right) \dot{z}_b \dot{\theta}. \quad (A.20)
\]

From the kinetic energy \( T \) the first two terms of the Lagrange equations of motion can be derived:

\[
\frac{d}{dt} \left( \frac{\partial T}{\partial \dot{z}_w} \right) = m_w \ddot{z}_w, \quad (A.21)
\]

\[
\frac{d}{dt} \left( \frac{\partial T}{\partial \dot{z}_b} \right) = \left( m_b + m_s + m_{red} \right) \ddot{z}_b - lr (m_s + \frac{1}{2} m_{red}) \ddot{\theta}, \quad (A.22)
\]

\[
\frac{d}{dt} \left( \frac{\partial T}{\partial \dot{\theta}} \right) = l^2 r^2 (m_s + \frac{1}{4} m_{red}) \ddot{\theta} - lr (m_s + \frac{1}{2} m_{red}) \ddot{z}_b. \quad (A.23)
\]

And:
\[ \frac{\partial T}{\partial z_w} = 0, \]  
(A.24)

\[ \frac{\partial T}{\partial z_b} = 0, \]  
(A.25)

\[ \frac{\partial T}{\partial \theta} = 0. \]  
(A.26)

Figure A.8: Ambulance model.
The potential energy can be described by:

\[ U = \sum_{i=1}^{N_p} \frac{1}{2} k_i z_i^2 + \sum_{i=1}^{N_m} m_i g z, \]

\[ = \frac{1}{2} k_w (z_w - z_r)^2 + \frac{1}{2} k_b (z_b - z_w)^2 + \frac{1}{2} k_a (z_b - l \theta - \dot{z}_b)^2, \]

\[ = \frac{1}{2} k_w (z_w^2 + z_r^2 - 2 z_r z_w) + \frac{1}{2} k_b (z_b^2 + z_w^2 - 2 z_w z_b) + \frac{1}{2} k_a l^2 \dot{\theta}^2. \]  

(A.27)

It is assumed that the system is in steady state, and thus the gravitational potential energy is not taken into account.

The third term of the Lagrange equations of motion can be described by:

\[ \frac{\partial U}{\partial z_w} = k_w (z_w - z_r) - k_b (z_b - z_w), \]  

(A.28)

\[ \frac{\partial U}{\partial z_b} = k_b (z_b - z_w), \]  

(A.29)

\[ \frac{\partial U}{\partial \theta} = k_a l^2 \dot{\theta}. \]  

(A.30)

The dissipative energy can be described by:

\[ D = \sum_{i=1}^{N_c} \frac{1}{2} c_i \dot{z}_i^2, \]

\[ = \frac{1}{2} c_b (\dot{z}_b - \dot{z}_w)^2 + \frac{1}{2} c_a (\dot{z}_b - l \dot{\theta} - \dot{z}_b)^2, \]

\[ = \frac{1}{2} c_b (\dot{z}_b^2 + \dot{z}_w^2 - 2 \dot{z}_w \dot{z}_b) + \frac{1}{2} c_a (l^2 \dot{\theta}^2). \]  

(A.31)

And the last term of the Lagrange equations of motion can be described by:

\[ \frac{\partial D}{\partial \dot{z}_w} = -c_b (\dot{z}_b - \dot{z}_w), \]  

(A.32)

\[ \frac{\partial D}{\partial \dot{z}_b} = c_b (\dot{z}_b - \dot{z}_w), \]  

(A.33)

\[ \frac{\partial D}{\partial \dot{\theta}} = c_a l^2 \dot{\theta}. \]  

(A.34)

With these formula’s the equations of motion of the ambulance system can be described by:

\[ m_w \ddot{z}_w + k_w (z_w - z_r) - k_b (z_b - z_w) - c_b (\dot{z}_b - \dot{z}_w) = F_1. \]  

(A.35)

\[ (m_b + m_s + m_{red}) \ddot{z}_b - lr (m_s + \frac{1}{4} m_{red}) \ddot{\theta} + k_b (z_b - z_w) + c_b (\dot{z}_b - \dot{z}_w) - F_{LM} = F_2. \]  

(A.36)

\[ l^2 \ddot{z} (m_s + \frac{1}{4} m_{red}) \ddot{\theta} - lr (m_s + \frac{1}{4} m_{red}) \ddot{z}_b + k_a l^2 \dot{\theta} + c_a l^2 \dot{\theta} - F_{LM} l = T_3. \]  

(A.37)
These equations of motion can be rewritten in matrix form, giving:

\[
\dot{\psi} = A\psi + Bu, \quad y = C\psi + Du. \tag{A.38}
\]

Where:

\[
\psi = \begin{bmatrix} q \\ \dot{q} \end{bmatrix}, \quad A = \begin{bmatrix} O & I \\ -M_s^{-1}K_s & -M_s^{-1}K_s \end{bmatrix}, \quad B = \begin{bmatrix} O \\ F \end{bmatrix},
\]

\[
M_s = \begin{bmatrix} m_w & 0 & 0 \\ 0 & m_b + m_s + m_{\text{red}} & -lr(m_s + \frac{1}{2}m_{\text{red}}) \\ 0 & -lr(m_s + \frac{1}{2}m_{\text{red}}) & l^2r^2(m_s + \frac{1}{2}m_{\text{red}}) \end{bmatrix},
\]

\[
K_s = \begin{bmatrix} k_w + k_b & -k_b & 0 \\ -k_b & k_b & 0 \\ 0 & 0 & k_a l^2 \end{bmatrix},
\]

\[
C_s = \begin{bmatrix} c_b & -c_b & 0 \\ -c_b & c_b & 0 \\ 0 & 0 & c_a l^2 \end{bmatrix}, \quad F = \begin{bmatrix} k_w/m_w & 0 \\ 0 & -1 \\ 0 & -l \end{bmatrix}, \quad u = \begin{bmatrix} u \\ F_{LM} \end{bmatrix},
\]

and \( C \) is a 6x6 identity matrix and \( D = 0 \).
Appendix B

Linear actuator specs
## ELECTRICAL SPECIFICATIONS

<table>
<thead>
<tr>
<th>FORCER TYPE</th>
<th>2504</th>
<th>2506</th>
<th>2508</th>
<th>2510</th>
<th>units</th>
</tr>
</thead>
<tbody>
<tr>
<td>S (N)</td>
<td>312</td>
<td>468</td>
<td>524</td>
<td>780</td>
<td>N</td>
</tr>
<tr>
<td>P (N)</td>
<td>166</td>
<td>234</td>
<td>312</td>
<td>390</td>
<td>N</td>
</tr>
<tr>
<td>Peak force @ 25°C ambient for 1 sec</td>
<td>312</td>
<td>468</td>
<td>524</td>
<td>780</td>
<td>N</td>
</tr>
<tr>
<td>Peak current @ 25°C ambient for 1 sec</td>
<td>20</td>
<td>20</td>
<td>20</td>
<td>20</td>
<td>Apk</td>
</tr>
</tbody>
</table>

**With 25 x 25 x 2.5cm heatsink plate**
- Continuous stall force @ 25°C ambient | 51.2 | 69.5 | 96.4 | 102.4 | N |
- Continuous stall current @ 25°C ambient | 2.31 | 4.62 | 2.10 | 4.20 | 1.96 | 3.92 | 1.66 | 3.72 | Ams |
- Without heatsink plate
  - Continuous stall force @ 25°C ambient | 42.5 | 59.5 | 75.1 | 90.0 | N |
  - Continuous stall current @ 25°C ambient | 1.92 | 3.84 | 1.80 | 3.60 | 1.70 | 3.40 | 1.63 | 3.26 | Ams |

**Force constant (sine commutation)**
- 22.1 | 11.0 | 33.1 | 16.5 | 44.1 | 22.0 | 55.2 | 27.6 | N/Arms |
- 15.6 | 7.8  | 23.4 | 11.7 | 31.2 | 15.6 | 39.0 | 19.5 | N/Apk |

**Back EMF constant (phase to phase)**
- 18.0 | 9.0  | 27.0 | 13.5 | 36.0 | 18.0 | 45.0 | 22.5 | Vp/pl/m/s |

**Fundamental forcer constant**
- 6.47 | 7.92 | 9.13 | 10.24 | N/V |

**Eddy current loss**
- 9.51 | 12.65 | 15.58 | 18.61 | N/J/s |

**Resistance @ 25°C (phase to phase)**
- 6.02 | 1.50 | 9.02 | 2.25 | 12.03 | 3.01 | 15.04 | 3.76 | Ohm |

**Inductance @ 1kHz (phase to phase)**
- 3.90 | 0.97 | 5.95 | 1.46 | 7.90 | 1.95 | 9.75 | 2.44 | mH |

**Electrical time constant**
- 0.65 | 0.65 | 0.65 | 0.65 | ms |

**Maximum working voltage**
- 380 | 380 | 380 | 380 | V dc |

**Pole pitch (one electrical cycle)**
- 51.2 | 51.2 | 51.2 | 51.2 | mm |

**Peak acceleration (1)**
- 223 | 111 | 223 | 111 | 235 | 117 | 256 | 128 | m/s² |

**Maximum speed (1)**
- 8.7 | 7.3 | 6.5 | 7.2 | 5.4 | 7.6 | 4.6 | 7.0 | m/s |

**Notes:**
1. S-series forcer phases, P-parallel forcer phases
2. Reduce continuous stall force to 69% at 40°C ambient
3. Based on a moving forcer and no payload
4. Based on a moving forcer with triangular move over maximum stroke and no payload

## THERMAL SPECIFICATIONS

<table>
<thead>
<tr>
<th>FORCER TYPE</th>
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<th>2506</th>
<th>2508</th>
<th>2510</th>
<th>units</th>
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<tbody>
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<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>°C</td>
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<tr>
<td>Thermal resistance Rth (Phase to ground)</td>
<td>0.41</td>
<td>0.27</td>
<td>0.20</td>
<td>0.16</td>
<td>°C/Watt</td>
</tr>
</tbody>
</table>

**With 25 x 25 x 2.5cm heatsink plate**
- Power dissipation @ 25°C ambient | 62.3 | 77.0 | 89.2 | 100.2 | Watt |
- Thermal resistance Rth (Phase to ground) | 0.79 | 0.69 | 0.64 | 0.59 | °C/Watt |

**Without heatsink plate**
- Power dissipation @ 25°C ambient | 43.1 | 56.4 | 67.5 | 77.3 | Watt |
- Thermal resistance Rth (Phase to ground) | 1.33 | 1.06 | 0.91 | 0.91 | °C/Watt |

**Thermal time constant**
- 1186 | 1276 | 1377 | 1486 | s |

## MECHANICAL SPECIFICATIONS

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<th>2506</th>
<th>2508</th>
<th>2510</th>
<th>units</th>
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<tbody>
<tr>
<td>Maximum stroke</td>
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<td>1129</td>
<td>1078</td>
<td>1027</td>
<td>mm</td>
</tr>
<tr>
<td>Force mass (including bearings)</td>
<td>1.40</td>
<td>2.10</td>
<td>2.65</td>
<td>3.05</td>
<td>kg</td>
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<tr>
<td>Force mass (excluding bearings)</td>
<td>1.15</td>
<td>1.60</td>
<td>2.15</td>
<td>2.55</td>
<td>kg</td>
</tr>
<tr>
<td>Thrust rod mass/length</td>
<td>3.5</td>
<td>3.5</td>
<td>3.5</td>
<td>3.5</td>
<td>kg/m</td>
</tr>
</tbody>
</table>

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DS010891  www.copleycontrols.com  Page 2
### Appendix C

### Parts list

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<thead>
<tr>
<th>Name</th>
<th>Description</th>
<th>Qty</th>
<th>Retailer</th>
<th>Price (€)</th>
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<tbody>
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<td>Pneumatic vent (SMC)</td>
<td>2</td>
<td>Almotion</td>
<td>4,40</td>
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<tr>
<td>10014VR</td>
<td>Rolling lobe air spring</td>
<td>1</td>
<td>Alumalift</td>
<td>50,00</td>
</tr>
<tr>
<td>STB2504</td>
<td>Linear shaft motor and controller</td>
<td>1</td>
<td>Copley</td>
<td>1684,00</td>
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<tr>
<td>628-6-2Z</td>
<td>Bearing</td>
<td>6</td>
<td>Eriks</td>
<td>94,86</td>
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<tr>
<td>11024241</td>
<td>NBR 70 O-ring 20x8</td>
<td>2</td>
<td>Eriks</td>
<td>1,81</td>
</tr>
<tr>
<td>10028068</td>
<td>NBR 70 O-ring 6,5x1</td>
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<td>Eriks</td>
<td>0,64</td>
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<td>0,84</td>
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<td>NBR 70 O-ring 9x1</td>
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<td>10027591</td>
<td>NBR 70 O-ring 9x2</td>
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<td>Eriks</td>
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<td>25-1-2 90-100</td>
<td>Power Clamp RVS</td>
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<td>Eriks</td>
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<td>Fabory</td>
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<td>Coil spring</td>
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<td>Tevema</td>
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<td></td>
<td></td>
<td><strong>2410,91</strong></td>
</tr>
</tbody>
</table>

Table C.1: Actuator parts list.
Appendix D

Test setup calculations

D.1 Fatigue of the eccentric axis

The force and torque acting on the axis of the eccentric drive are large. Therefore it is necessary to calculate the minimum axis diameter, to prevent fatigue in the material of the axis.

In figure D.1 the forces and torque acting on the axis are depicted, along with the force- moment- and torque diagrams. The force \( F \) is the force introduced by the AVI to the axis. The torque \( T \) is the torque introduced to the shaft by the electric motor. The forces \( F_{r1} \) and \( F_{r2} \) are the reaction forces in the two bearing blocks.

![Figure D.1: Loads on the eccentric axis.](image-url)
The shaft is subject to bending forces and torque. The torque changes sign in each revolution. The bending force does not change sign. These two loads can be combined to a comparable bending moment $M_c$:

$$M_c = \sqrt{M^2 + 0.75 \left( \frac{\sigma_b}{\phi \cdot \tau_t} \right)^2}, \quad (D.1)$$

where:
- $M$ is the maximum bending moment of the shaft,
- $T$ is the torsion moment of the drive shaft,
- $\sigma_b/(\phi \cdot \tau_t)$ is a factor which depends on the type of the load, and equals 1.5 for this specific case [63].

With the comparable bending moment, the minimum shaft diameter can be calculated by:

$$d \approx 3.4 \sqrt{\frac{M_c}{\sigma_{b,D}}}, \quad (D.2)$$

where:
- $\sigma_{b,D}$ is the maximum allowable stress for the specific load case for normal construction steel and equals $200 \cdot 10^6$ N/m$^2$.

From these formulas it follows that the equivalent bending moment $M_c$ is 79.4 Nm and the minimum shaft diameter $d$ is 25 mm. In test setup, a shaft diameter of 30 mm is used, which is able to withstand a bending moment of 137 Nm, for extra margin on the safety.

### D.2 Forces on the AVI guiding

The AVI can be schematically depicted according to figure D.2 to analyze the forces acting on the guiding system. The two pre-stressed guiding wheels are represented by the two springs $k_1$ and $k_2$. The linear motor frame, the upper and lower guiding frame, the bottom of the AVI and the bearing block are represented as a rigid construction with mass $M=3.17$ kg. The linear motor shaft and the top of the AVI are represented as a rigid construction with mass $m=1.83$ kg. It is assumed that the AVI rotates round point $A$ and the accelerations introduced by the eccenter are introduced to the AVI in point $B$.

As mentioned in section 6.1, the eccentric will excite the system with a 1.5 Hz sine signal with an amplitude of 40 mm and a 12.3 Hz signal with a 20 mm amplitude. With an amplitude of 40 mm, the displacement in horizontal direction of the AVI lies between 0 and 4.6 mm. With a 20 mm amplitude the maximum displacement is 0.6 mm. The maximum acceleration of the horizontal vibrations in point $B$ can be calculated by:

$$x_B = A \sin(2\pi ft), \quad (D.3)$$
$$\dot{x}_B = 2\pi fA \sin(2\pi ft), \quad (D.4)$$
$$\ddot{x}_B = 4\pi^2 f^2 A \sin(2\pi ft). \quad (D.5)$$

Where $A$ is the amplitude of the vibration, $f$ is the frequency and $t$ is the time. The maximum absolute value of the acceleration occurs when $\sin(2\pi ft) = 1$. Then the
maximum horizontal acceleration equals $\ddot{x}_B = 4\pi^2 f^2 A = 0.4 \text{ m/s}^2$ for the 1.5 Hz experiment and analogously 3.5 m/s$^2$ for the 12.3 Hz experiment.

The forces on the springs of the guiding system can now be calculated. Using the dimensions shown in figure D.2, the equivalent force at the location of the center of gravity $M$ equals:

$$F_M = M\ddot{x}_M = M \frac{400}{500} \ddot{x}_B = 8.86 \text{ N}. \quad (D.6)$$

The force on spring 1 and 2 can now be calculated by balancing this force with the spring forces. Using figure D.3, the force $F_1$ and $F_2$ can be calculated:

$$F_1 = F_M \frac{188+87}{188} = 13.0 \text{ N}, \quad (D.7)$$
$$F_2 = F_M \frac{87}{188} = 4.1 \text{ N}. \quad (D.8)$$

The force of the mass $m$ on the guiding system can be calculated in the same way. The force $F_m$ in point $m$ equals:

$$F_m = m\ddot{x}_m = m \frac{200}{500} \ddot{x}_B = 2.55 \text{ N}. \quad (D.9)$$

The forces on the guiding system can now be represented by:

$$F_1 = F_m \frac{70}{113} = 1.76 \text{ N}, \quad (D.10)$$
$$F_2 = F_m \frac{113}{188} = 1.53 \text{ N}. \quad (D.11)$$
These forces are lower than the maximum allowable force of 40 N that the springs of the guiding wheel exert on the masses. The maximum possible acceleration in point \( B \) that can be taken by the AVI guiding before the force on the guiding wheels exceeds 40 N can be calculated by:

\[
\ddot{x}_{B,\text{max}} = \frac{F_{\text{max}}}{\left(\frac{400}{500} \frac{188+87}{188} + \frac{400}{500} \frac{25}{113}\right)} = 9.5 \text{ m/s}^2.
\]  
\((D.12)\)

### D.3 AVI guiding eigenfrequency

The springs and the masses of the guiding system are a dynamic system on its own. The system should not be excited in its eigenfrequency. The eigenfrequency of the guiding system be calculated by using the dimensions shown in figure D.4 by:

\[
\omega_n = \frac{1}{2\pi} \sqrt{\frac{k}{m}}.
\]  
\((D.13)\)

As long as the pre-tension force of the two springs is not exceeded by the side forces acting on the guiding system, the stiffness of the guiding system is determined by the contact stiffness between the guiding wheels and the shaft. This stiffness \( k \) can be calculated by:

\[
k = \frac{F}{\delta}.
\]  
\((D.14)\)

Where \( F \) is the force of the springs that tension the wheels to the shaft and \( \delta \) is the impression of the wheel and shaft, which can be calculated using (4.5). Using this formula, the contact stiffness \( k \) equals \( 1.9 \cdot 10^6 \text{ N/m} \).

The eigenfrequencies of the mass \( m \) in combination with with the stiffness \( k \) are:

\[
\omega_{n,1} = \frac{1}{2\pi} \sqrt{\frac{k}{m \left(\frac{75}{113}\right)^2}} = 244 \text{ Hz},
\]  
\((D.15)\)

\[
\omega_{n,2} = \frac{1}{2\pi} \sqrt{\frac{k}{m \left(\frac{113}{188}\right)^2}} = 270 \text{ Hz}.
\]  
\((D.16)\)
D.4 Equations of motion models I & II

Model I

The coordinates of the system can be described by:

\[ z_s = u - \theta l r. \]  
\[ (D.17) \]

The kinetic energy equals:

\[ T = \frac{1}{2} m_s \left( \dot{u} - \dot{\theta}lr \right)^2, \]  
\[ (D.18) \]

and thus

\[ \frac{d}{dt} \left( \frac{\partial T}{\partial \theta} \right) = m_s l^2 \dot{r}^2 \dot{\theta} - m_s l r \ddot{u}. \]  
\[ (D.19) \]

The potential energy is described as:

\[ U = \frac{1}{2} k_a l^2 \dot{\theta}^2, \]  
\[ (D.20) \]

and

\[ \frac{\partial U}{\partial \theta} = k_a l^2 \dot{\theta}. \]  
\[ (D.21) \]
The dissipative energy is:

\[ D = \frac{1}{2} c_a l^2 \dot{\theta}^2, \quad (D.22) \]

and

\[ \frac{\partial D}{\partial \dot{\theta}} = c_a l^2 \dot{\theta}. \quad (D.23) \]

These terms lead to the equations of motion of model I:

\[ m_s l^2 r^2 \ddot{\theta} - m_s l r \ddot{u} + k_a l^2 \theta + c_a l^2 \dot{\theta} - F_{LM} l = T. \quad (D.24) \]

**Model II**

![Diagram of model II](image)

Figure D.6: Test setup model, model II.

The coordinates of model II can be described by:

\[ z_s = -\theta l r. \quad (D.25) \]

The kinetic energy equals:

\[ T = \frac{1}{2} m_s \left( -\dot{\theta} l r \right)^2, \quad (D.26) \]

and from that

\[ \frac{d}{dt} \left( \frac{\partial T}{\partial \dot{\theta}} \right) = m_s l^2 r^2 \ddot{\theta}. \quad (D.27) \]

The potential energy is:

\[ U = \frac{1}{2} k_a (-l \theta - u)^2, \quad (D.28) \]

and

\[ \frac{\partial U}{\partial \theta} = k_a (l^2 \theta + lu). \quad (D.29) \]

The dissipative energy is described by:

\[ D = \frac{1}{2} c_a (-l \dot{\theta} - \dot{u})^2, \quad (D.30) \]
and thus
\[ \frac{\partial D}{\partial \dot{\theta}} = c_a(l^2 \ddot{\theta} + l\dot{\theta}). \]  
(D.31)

The equations of motion of model II can now be written as:
\[ m_s l^2 r^2 \dddot{\theta} + k_a(l^2 \ddot{\theta} + l\dot{\theta}) + c_a(l^2 \ddot{\theta} + l\dot{\theta}) - F_{LM} l = T. \]  
(D.32)
Appendix E

Full model evaluation

E.1 Quarter car vs full vehicle model

The full ambulance model and the quarter ambulance model are compared for the same two scenario’s that are considered in chapter 7:

- A ride over a rough road.
- A speed bump with a 100 mm Watts profile.

The simulations that are made show that the two models have comparable behavior. In table E.1 relevant parameters of interest are given. In figures E.1 and E.2 the stretcher acceleration $\ddot{z}_s$, the linear shaft motor force $F_{LM}$ and the actuator stroke of both models are plotted in the time domain for the ride over the speed bump and the rough road respectively.

As can be seen from the data in the table and the graphs the performance is not identical. The biggest difference is shown in the speed bump simulation. This is mostly due to the wheel base filtering of the road irregularities that is present in the full vehicle model. This results in a smaller required stroke because the disturbances introduced to the bottom of the AVI in the full vehicle model are averaged over the input of the front and rear wheels of the ambulance, and thus less extreme.

<table>
<thead>
<tr>
<th></th>
<th>Speed bump, 25 km/u</th>
<th>Rough road, 50 km/u</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Quarter car</td>
<td>Full car</td>
</tr>
<tr>
<td>$\ddot{z}_s$ (rms)</td>
<td>0.10</td>
<td>0.07</td>
</tr>
<tr>
<td>comfort index</td>
<td>0.043</td>
<td>0.032</td>
</tr>
<tr>
<td>motion sickness index</td>
<td>0.03</td>
<td>0.02</td>
</tr>
<tr>
<td>$\ddot{z}_s$ (peak)</td>
<td>0.26</td>
<td>0.25</td>
</tr>
<tr>
<td>stroke (peak)</td>
<td>40</td>
<td>27</td>
</tr>
<tr>
<td>$F_{LM}$ (peak)</td>
<td>113</td>
<td>79</td>
</tr>
</tbody>
</table>

Table E.1: Simulation data quarter and full ambulance models.
Figure E.1: Quarter and full ambulance model response for speed bump simulation, 25 km/u, solid: full vehicle model, dashed: quarter ambulance model.

Figure E.2: Quarter and full ambulance model response for rough road simulation, 50 km/u, solid: full vehicle model, dashed: quarter ambulance model.
E.2 Acceleration weighting

ISO 2631 [24] covers methods for the measurement of periodic, random and transient vibration with regard to health, comfort and perception of whole body vibration. The measurand is frequency weighted acceleration. The accelerations are weighted because the human body is most sensitive to vibrations in certain frequency regions, mainly between 0.5 and 80 Hz.

For supine subjects, two weighting function are of interest; \( W_k \) is the weighting for vertical vibrations and \( W_f \) is a weighting function which specifically addresses motion sickness. These weighting functions are defined as follows:

\[
W = H_b(s) \cdot H_l(s) \cdot H_t(s) \cdot H_s(s). \tag{E.1}
\]

Where \( H_b \) is a high pass filter, \( H_l \) is a low-pass filter, \( H_t \) is the acceleration-velocity filter and \( H_s \) is the upward step filter. These are defined as:

\[
H_b(s) = \frac{s^2}{s^2 + \frac{\omega_b}{Q_1} s + \omega_1^2}, \tag{E.2}
\]

\[
H_l(s) = \frac{\omega_2^2}{s^2 + \frac{\omega_2}{Q_2} s + \omega_2^2}, \tag{E.3}
\]

\[
H_t(s) = \frac{\omega_4^2}{s^2 + \frac{\omega_4}{Q_4} s + \omega_4^2}, \tag{E.4}
\]

\[
H_s(s) = \frac{s^2}{s^2 + \frac{\omega_s}{Q_5} s + \omega_5^2} \quad \text{and} \quad \omega_s = 2 \pi f_s. \tag{E.5}
\]

The values for these transfer functions are listed in table E.2.

<table>
<thead>
<tr>
<th>( W_k )</th>
<th>( W_f )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f_1 ) (Hz)</td>
<td>( f_2 ) (Hz)</td>
</tr>
<tr>
<td>0.4</td>
<td>0.08</td>
</tr>
<tr>
<td>( 1/\sqrt{2} )</td>
<td>( 1/\sqrt{2} )</td>
</tr>
</tbody>
</table>

Table E.2: Parameters of weighting function, \( f_s \) is the sample frequency. Note that \( \omega = 2 \pi f. \)

These weighting functions are plotted in figure E.3 below.
Figure E.3: Bode diagram of acceleration weighting functions. Solid: $W_k$ vertical vibration weighting, dashed: $W_f$ motion sickness weighting.